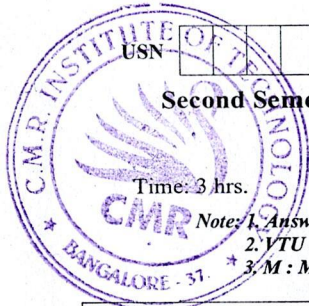




USN

Second Semester B.E./B.Tech. Degree Examination, June/July 2025

Mathematics – II for EEE stream

Time: 3 hrs.

Max. Marks: 100

- Notes: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks, L: Bloom's level, C: Course outcomes.

Module – 1				M	L	C
Q.1	a.	Find the angle between the surface $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1, -2, 1)$	7	L3	CO1	
	b.	Evaluate $\text{curl}(\text{curl} \vec{F})$ and $\text{div}(\text{curl} \vec{F})$ if $\vec{F} = x^2y\mathbf{i} + y^2z\mathbf{j} + z^2x\mathbf{k}$	7	L3	CO1	
	c.	Show that the vector $\vec{F} = \frac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}$ is both solenoidal and irrotational.	6	L2	CO2	
OR						
Q.2	a.	Find the total work done by the force $\vec{F} = 3xy\mathbf{i} - y\mathbf{j} + 2zx\mathbf{k}$ in moving a particle around the circle $x^2 + y^2 = 4$.	7	L3	CO1	
	b.	Using Green's theorem evaluate $\oint_C (xy + y^2)dx + x^2dy$ over the region bounded by the curves $y = x$ and $y = x^2$	7	L2	CO3	
	c.	Using modern mathematical tools, write the code to find gradient of $\phi = x^2y + 2xz - 4$.	6	L2	CO5	
Module – 2						
Q.3	a.	Define a Subspace. Show that any plane passing through the origin is a subspace of $\mathbb{R}^3$.	7	L1	CO1	
	b.	Let V be the vector space of all real valued continuous functions over $\mathbb{R}$. Show that the set W of solutions of differential equations $5\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 2y = 0$ is a subspace of V.	7	L2	CO2	
	c.	Define an Inner Product space. Consider $f(t) = 3t - 5$ and $g(t) = t^2$, the inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$. Find $\langle f, g \rangle$	6	L2	CO2	
OR						
Q.4	a.	Express the vector $(3, 5, 2)$ as a linear combination of the vectors $(1, 1, 0)$, $(2, 3, 0)$, $(0, 0, 1)$ of $V_3(\mathbb{R})$.	7	L2	CO3	

	b.	State Rank-Nullity theorem. Verify the Rank-Nullity theorem for the $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x+2y-z, y+z, x+y-2z)$	7	L2	CO2														
	c.	Using the modern mathematical tool, write the code to represent the reflection transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and to find the image of vector $(10, 0)$ when it is reflected about the y-axis.	6	L1	CO4														
Module – 3																			
Q.5	a.	Find the Laplace transform of, (i) $e^{-3t}(2\cos 5t - 3\sin 5t)$ (ii) $\frac{\cos at - \cos bt}{t}$	7	L3	CO3														
	b.	Given $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$ where $f(t+a) = f(t)$, show that $L[f(t)] = \frac{E}{S} \tanh\left(\frac{aS}{4}\right)$.	7	L2	CO2														
	c.	Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of the Heaviside unit step function and hence find $L[f(t)]$.	6	L3	CO4														
OR																			
Q.6	a.	Find $L^{-1}\left[\frac{2s^2 + 5s - 4}{s^3 + s^2 - 2s}\right]$	7	L3	CO3														
	b.	Using the convolution theorem, find the inverse Laplace Transform of $\frac{s}{(s^2 + a^2)^2}$.	7	L3	CO4														
	c.	Solve $y''' + 2y'' - y' - 2y = 0$, given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform method.	6	L2	CO3														
Module – 4																			
Q.7	a.	By Newton-Raphson method, find the root of $x \tan x + 1 = 0$ which is near to $x = \pi$.	7	L3	CO2														
	b.	Using Newton's forward difference formula find $f(38)$, <table><tr><td>x</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>y</td><td>184</td><td>204</td><td>226</td><td>250</td><td>276</td><td>304</td></tr></table>	x	40	50	60	70	80	90	y	184	204	226	250	276	304	7	L3	CO3
x	40	50	60	70	80	90													
y	184	204	226	250	276	304													
	c.	Use Lagrange's interpolation formula to find y at $x = 10$, given <table><tr><td>x</td><td>5</td><td>6</td><td>9</td><td>11</td></tr><tr><td>y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr></table>	x	5	6	9	11	y	12	13	14	16	6	L3	CO3				
x	5	6	9	11															
y	12	13	14	16															

OR																		
Q.8	a.	Find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places by the method of false position in (2, 3).	7	L4 CO4														
	b.	Find the interpolating polynomial using Newton's dividend difference formula for the following data, <table border="1"><tr><td>x</td><td>2</td><td>4</td><td>5</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>10</td><td>96</td><td>196</td><td>350</td><td>868</td><td>1746</td></tr></table>	x	2	4	5	6	8	10	y	10	96	196	350	868	1746	7	L3 CO4
x	2	4	5	6	8	10												
y	10	96	196	350	868	1746												
	c.	Use Simpson's $\frac{3}{8}$ rule to obtain the approximate value of $\int_0^{0.3} (1 - 8x^3)^{\frac{1}{2}} dx$ by considering 3 equal intervals.	6	L2 CO2														
Module – 5																		
Q.9	a.	Use Taylor's series method, find $y(0.1)$ considering upto fourth degree term if $y(x)$ satisfies the equation, $\frac{dy}{dx} = x - y^2$, $y(0) = 1$.	7	L3 CO3														
	b.	Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$. Compute $y(0.2)$ by taking $h = 0.2$, using Runge-Kutta method of fourth order.	7	L2 CO3														
	c.	Apply Milne's method to compute $y(1.4)$, correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$	6	L2 CO2														
OR																		
Q.10	a.	Using modified Euler's method to find y at $x = 0.2$ given $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$ by taking step size $h = 0.1$.	7	L4 CO4														
	b.	Using the Runge-Kutta method of fourth order find $y(0.1)$ given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ taking $h = 0.1$.	7	L3 CO2														
	c.	Using modern mathematical tools, write the code to find the solution of $\frac{dy}{dx} = x - y^2$ at $y(0.1)$, given that $y(0) = 1$ by Runge-Kutta 4 th order method.	6	L2 CO3														
