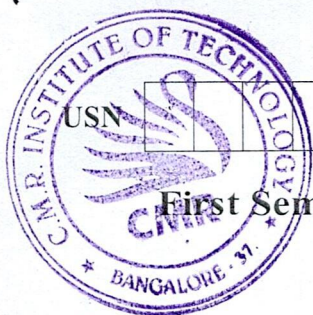


CBCS SCHEME



21MAT11

First Semester B.E./B.Tech. Degree Examination, June/July 2025 Calculus and Differential Equations

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notations prove that $\tan \phi = r \frac{d\theta}{dr}$. (06 Marks)
- b. Show that the curves $r = a(1 + \sin \theta)$ and $r = a(1 - \sin \theta)$ intersect each other orthogonally. (07 Marks)
- c. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 4$ at point (4, 4). (07 Marks)

OR

- 2 a. Find the angle between the radius vector and the tangent for the curve $r^m = a^m (\cos m\theta + \sin m\theta)$. (06 Marks)
- b. Find the pedal equation of the curve $\frac{2a}{r} = 1 + \cos \theta$. (07 Marks)
- c. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$. (07 Marks)

Module-2

- 3 a. Obtain the Maclaurin's expansion of " $\log(1 + e^x)$ " upto fourth degree terms. (06 Marks)
- b. If $u = f\left(xz, \frac{y}{z}\right)$ prove that $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} - z \frac{\partial u}{\partial z} = 0$. (07 Marks)
- c. Show that $f(x, y) = x^3 y^2 (1 - x - y)$ for $x, y \neq 0$ is maximum at the point $\left(\frac{1}{2}, \frac{1}{3}\right)$ and find maximum value. (07 Marks)

OR

- 4 a. Evaluate : i) $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$ ii) $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{\tan x}$ (06 Marks)
- b. Find the total derivative of $u = xy + yz + zx$ where $x = t \cos t, y = t \sin t, z = t$. (07 Marks)
- c. If $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$.
Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (07 Marks)

Module-3

- 5 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. (06 Marks)
- b. Find the orthogonal trajectory of Cardioids $r = a(1 - \cos \theta)$. (07 Marks)
- c. Solve $p^2 + py - x(x + y) = 0$. (07 Marks)

OR

- 6 a. Solve $(6x^2 + 4y^3 + 12y)dx + 3x(1 + y^2) = 0$ (06 Marks)
- b. A cup of coffee at 80°C is placed in a room with temperature 20°C and it cools to 50°C in 5 minutes. Find its temperature after a further interval of 5 minutes. (07 Marks)
- c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation and general solution. (07 Marks)

Module-4

- 7 a. Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 3e^{-x}$. (06 Marks)
- b. Solve $(D^2 + 4D + 8)y = x + 1$. (07 Marks)
- c. Using method of variation of parameters solve $y'' + y = \sec x$. (07 Marks)

OR

- 8 a. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos 2x$ (06 Marks)
- b. Solve $4y'' - y = e^{2x}$. (07 Marks)
- c. Solve the Cauchy's differential equation $x^2y'' + xy' + 9y = \sin(3 \log x)$. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$ (06 Marks)
- b. Test for consistency and solve
- $$\begin{aligned} 5x_1 + x_2 + 3x_3 &= 20 \\ 2x_1 + 5x_2 + 2x_3 &= 18 \\ 3x_1 + 2x_2 + x_3 &= 14 \end{aligned}$$
- (07 Marks)

- c. Solve the system of equations

$$5x + 2y + z = 12$$

$$x + 4y + 2z = 15$$

$$x + 2y + 5z = 20$$

Using Gauss – Seidel iteration method. Carryout four iterations taking (1, 0, 3) as initial approximate root. (07 Marks)

OR

- 10 a. Find the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$$

(06 Marks)

- b. Solve the system of equations

$$2x + 5y + 7z = 52$$

$$2x + y - z = 0$$

$$x + y + z = 9$$

using Gauss – Jordan method.

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(07 Marks)

- c. Find the dominant eigen value and the corresponding eigen vector of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

by using Rayleigh's power method, taking initial vector as $[1, 1, 1]^T$.

(07 Marks)
