CBCS SCHEME

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21MAT11

First Semester B.E./B.Tech. Degree Examination, June/July 2025

Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. With usual notations prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(06 Marks)**

b. Show that the curves $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$ intersect each other orthogonally. **(07 Marks)**

c. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 4$ at point (4, 4). **(07 Marks)**

OR

2 a. Find the angle between the radius vector and the tangent for the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ **(06 Marks)**

b. Find the pedal equation of the curve $\dfrac{2a}{r} = 1 + \cos\theta$. **(07 Marks)**

c. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

Module-2

3 a. Obtain the Maclaurin's expansion of "$\log(1 + e^x)$" upto fourth degree terms. **(06 Marks)**

b. If $u = f\left(xz, \dfrac{y}{z}\right)$ prove that $x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial y} - z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. Show that $f(x, y) = x^3y^2(1 - x - y)$ for x, y # 0 is maximum at the point $\left(\dfrac{1}{2}, \dfrac{1}{3}\right)$ and find maximum value. **(07 Marks)**

OR

4 a. Evaluate : i) $\underset{x\to 0}{\ell t}\,(1+\sin x)^{\cot x}$ ii) $\underset{x\to 0}{\ell t}\left(\dfrac{1}{x}\right)^{\tan x}$ **(06 Marks)**

b. Find the total derivative of $u = xy + yz + zx$ where $x = t\cos t$, $y = t\sin t$, $z = t$. **(07 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$.

Find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Solve $\frac{dy}{dx}+\frac{y}{x}=y^2x$. (06 Marks)

b. Find the orthogonal trajectory of Cardioids $r = a(1-\cos\theta)$. (07 Marks)

c. Solve $p^2 + py - x(x+y) = 0$. (07 Marks)

OR

6 a. Solve $(6x^2 + 4y^3 + 12y)dx + 3x(1+y^2) = 0$ (06 Marks)

b. A cup of coffee at 80°C is placed in a room with temperature 20°C and it cools to 50°C in 5 minutes. Find its temperature after a further interval of 5 minutes. (07 Marks)

c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation and general solution. (07 Marks)

Module-4

7 a. Solve $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=3e^{-x}$. (06 Marks)

b. Solve $(D^2 + 4D + 8)y = x + 1$. (07 Marks)

c. Using method of variation of parameters solve $y''+y=\operatorname{Sec} x$. (07 Marks)

OR

8 a. Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\operatorname{Cos} 2x$ (06 Marks)

b. Solve $4y''-y=e^{2x}$. (07 Marks)

c. Solve the Cauchy's differential equation $x^2y''+xy'+9y=\sin(3\log x)$. (07 Marks)

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$ (06 Marks)

b. Test for consistency and solve

$5x_1 + x_2 + 3x_3 = 20$
$2x_1 + 5x_2 + 2x_3 = 18$
$3x_1 + 2x_2 + x_3 = 14$ (07 Marks)

c. Solve the system of equations
$5x + 2y + z = 12$
$x + 4y + 2z = 15$
$x + 2y + 5z = 20$
Using Gauss – Seidel iteration method. Carryout four iterations taking (1, 0, 3) as initial approximate root. **(07 Marks)**

OR

10 a. Find the rank of the matrix $\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$ **(06 Marks)**

b. Solve the system of equations
$2x + 5y + 7z = 52$
$2x + y - z = 0$
$x + y + z = 9$
using Gauss – Jordan method. **(07 Marks)**

c. Find the dominant eigen value and the corresponding eigen vector of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using Rayleigh's power method, taking initial vector as $[1, 1, 1]^T$. **(07 Marks)**

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