CBCS SCHEME

USN

18MAT11

First Semester B.E./B.Tech. Degree Examination, June/July 2025
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally. **(06 Marks)**

b. Find the radius of curvature of the curve $y = a \log \sec\left(\frac{x}{a}\right)$ at any point (x, y). **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. **(08 Marks)**

OR

2 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Find the pedal equation of the curve $r = ae^{\theta\cot\alpha}$. **(06 Marks)**

c. Find the radius of curvature for the curve $r = a(1 + \cos\theta)$. **(08 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots$ **(08 Marks)**

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}$. **(06 Marks)**

c. Examine the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ for its extreme values. **(06 Marks)**

OR

4 a. If $U = f(x - y, y - z, z - x)$, prove that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. **(06 Marks)**

b. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. **(07 Marks)**

c. A rectangular box, open at the top, is to have a volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction. **(07 Marks)**

Module-3

5 a. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz dx dy$. **(07 Marks)**

b. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, above x-axis. **(07 Marks)**

c. With usual notations, prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$. **(06 Marks)**

OR

6 a. Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ by changing the order of integration. (07 Marks)

b. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates. (07 Marks)

c. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. (06 Marks)

Module-4

7 a. Solve $\frac{dy}{dx} + y\tan x = y^2 \sec x$. (06 Marks)

b. Find the orthogonal trajectories of the family $r^n \cos n\theta = a^n$. (07 Marks)

c. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitution $X = x^2$, $Y = y^2$. (07 Marks)

OR

8 a. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. (06 Marks)

b. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is the parameter. (07 Marks)

c. Solve $xy\left(\frac{dy}{dx}\right)^2 - (x^2+y^2)\frac{dy}{dx} + xy = 0$. (07 Marks)

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing to row-reduced echelon form. (06 Marks)

b. Apply Gauss-elimination method to solve the $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$. (07 Marks)

c. Find numerically largest eigen value and corresponding eigen vector of $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by Rayleigh's power method. Take initial eigen vector $[1,0,0]^T$. Carry out five iterations. (07 Marks)

OR

10 a. Test for consistency and solve the system of equations, $x + y + z = 6$, $x - y + 2z = 5$, $3x + y + z = 8$. (06 Marks)

b. Solve the system of equations by Gauss-Seidel method $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. Carryout three iterations. (07 Marks)

c. Diagonalize the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. (07 Marks)

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