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Internal Assessment Test I – Dec 2025

Sub:	Discrete Mathematics and Graph Theory					Sub Code:	MMC102	Branch:	MCA
Date:	26/12/2025	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	I A & B		OBE

Note: Answer FIVE FULL Questions, choosing ONE full question from each part.

		MARKS	CO	RBT
	PART I			
1	Define a power set with an example. Let U={1,2,3,4,5,6,7,8,9}, A={1,2,3,7}, B={4, 5, 6, 7} and C={1, 3, 6}. Compute $(i)\ \overline{A \cup C}$ (ii) $A \cap B \cap C$ $(iii)\ A - B$ $(iv)\ \bar{A} \cap \bar{B}$.	[10]	CO1	L1
	OR			
2	State and prove De-Morgan's laws for any two sets A and B.	[10]	CO1	L3
	PART II			
3	In a class of 52 students, 30 are studying C, 28 are studying Java and 13 are studying both the languages. (i) How many in this class are studying at least one of these languages? (ii) How many are studying neither of these languages?	[10]	CO1	L3
	OR			
4	Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$.	[10]	CO1	L3
	PART III			
5	State Pigeon-hole Principle. Prove that if 30 books in a library contain a total of 61,237 pages then at least one of the books must contain at least 2042 pages.	[10]	CO1	L2
	OR			

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	OR			

6	Define a tautology. Check whether $\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$ is a tautology, contradiction or contingency.	[10]	CO2	L2
	PART IV			
7	Using laws of logic, prove (i) $[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$ (ii) $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$. **OR**	[10]	CO2	L3
8	Give the direct and indirect proof of "If n is an odd integer then n+11 is an even integer."	[10]	CO2	L2
	PART V			
9	(a) Write the converse, inverse and contrapositive of "If 2 is an integer then 9 is a multiple of 3". (b) Write the negation of "All Americans eat cheese burgers". **OR**	[10]	CO2	L3
10	(a) Define the logical connective 'conjunction' with its truth table. (b) Check whether the following is a valid argument or not. $\neg p \rightarrow q$ $q \rightarrow r$ $\neg r$ $\therefore p$	[10]	CO2	L3

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Discrete Mathematics & Graph Theory - MMC102

1. A Power set of a set is the set of all subsets of that set.

Eg: Let $A = \{1, 2\}$

Then $P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

$A = \{1, 2, 3, 7\}$, $B = \{4, 5, 6, 7\}$, $C = \{1, 3, 6\}$

(i) $A \cup C = \{1, 2, 3, 6, 7\}$

$\overline{A \cup C} = U - (A \cup C)$

$= \{4, 5, 8, 9\}$

(ii) $A \cap B \cap C = \{\}$

(iii) $A - B = \{1, 2, 3\}$

(iv) $\overline{A} = \{4, 5, 6, 8, 9\}$

$\overline{B} = \{1, 2, 3, 8, 9\}$

$\overline{A} \cap \overline{B} = \{8, 9\}$

2. De-Morgan's Laws states that (i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

(i) RHS $= \overline{A} \cap \overline{B} = \{x / x \in \overline{A} \text{ and } x \in \overline{B}\}$

$= \{x / x \notin A \text{ and } x \notin B\}$

$= \{x / x \notin A \cup B\}$

$= \{x / x \in \overline{A \cup B}\}$

$= \overline{A \cup B}$

(ii) RHS $= \bar{A} \cup \bar{B} = \{x / x \in \bar{A} \text{ or } x \in \bar{B}\}$

$= \{x / x \notin A \text{ or } x \notin B\}$

$= \{x / x \notin A \cap B\}$

$= \{x / x \in \overline{A \cap B}\}$

$= \overline{A \cap B}$

3. Let S be the set of all students. Let A and B be the set of all students who study C and Java respectively.

$|S| = 52$, $|A| = 30$, $|B| = 28$, $|A \cap B| = 13$

(i) $|A \cup B| = |A| + |B| - |A \cap B|$

$= 30 + 28 - 13$

$= 45$

No of students who study atleast one of these language is 45.

(ii) $|\overline{A \cup B}| = |U| - |A \cup B|$

$= 52 - 45$

$= 7$

7 students study neither of these languages.

4. Let $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$

Characteristic eqn is $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = 0$$

$\lambda^2 - 3\lambda + 2 = 0 \quad \Rightarrow \lambda = 1, 2$

To find the eigen vectors,

consider $(A-\lambda I)X=0$

$$\begin{pmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} (3-\lambda)x+2y&=0 \\ -x-\lambda y&=0 \end{aligned}\right\} \quad -(1)$$

Put $\lambda=1$ in (1)

$$2x+2y=0 \qquad \Rightarrow x+y=0$$
$$-x-y=0 \qquad\qquad x=-y$$
$$\frac{x}{1}=\frac{y}{-1}$$

$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is the eigen vector corresponding to the eig val $\lambda=1$.

Put $\lambda=2$ in (1)

$$x+2y=0 \qquad x=-2y$$
$$-x-2y=0 \qquad \frac{x}{-2}=\frac{y}{1}$$

$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ is the eigen vector corresponding to $\lambda=2$.

5. ~~If~~ PHP — "If there are m pigeons & n pigeonholes and if $m>n$ then atleast one pigeonhole will have $p+1$ or more pigeons in it where $p=\left\lfloor \frac{m-1}{n} \right\rfloor$."

Let us treat 30 books as pigeon~~boxes~~ holes and 61,237 pages as pigeons. So, $m=61,237$ & $n=30$

$p = \left\lfloor \frac{m-1}{n} \right\rfloor = \left\lfloor \frac{61237-1}{30} \right\rfloor = 2041$

$p+1 = 2042$

So, atleast one book must have 2042 or more pages in it.

6. A compound proposition which is always true regardless of truth values of its components is called a tautology.

p	q	r	$(p\vee q)$	$\overbrace{(p\vee q)\to r}^{①}$	$\neg r$	$(p\vee q)$	$\neg(p\vee q)$	$\overbrace{\neg r\to\neg(p\vee q)}^{②}$	①⟷②
0	0	0	0	1	1	0	1	1	1
0	0	1	0	1	0	0	1	1	1
0	1	0	1	0	1	1	0	0	1
0	1	1	1	1	0	1	0	1	1
1	0	0	1	0	1	1	0	0	1
1	0	1	1	1	0	1	0	1	1
1	1	0	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0	1	1

Since all the entries of the last column are 1's, it is a tautology.

7. (i) $LHS = [(p\vee q)\wedge(p\vee\neg q)]\vee q$

$\Leftrightarrow [p\vee(q\wedge\neg q)]\vee q$ Distributive law

$\Leftrightarrow [p\vee F_0]\vee q$ Inverse Law

$\Leftrightarrow p\vee q$ Identity law

$= RHS$

(ii) $RHS = [(p \to q) \wedge (p \to r)]$

$\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ $\quad\quad p \to q \Leftrightarrow \neg p \vee$

$\Leftrightarrow \neg p \vee (q \wedge r)$ $\quad\quad$ Distributive law

$\Leftrightarrow p \to (q \wedge r)$ $\quad\quad \neg p \vee q \Leftrightarrow p \to q$

$= RHS$

8. Let p: n is an odd integer.

q: n + 11 is an even integer.

Given $p \to q$

(i) Direct proof: Assume p is true.

$\Rightarrow$ n is an odd integer.

$\Rightarrow \quad n = 2k+1 \quad ; \quad k \in Z$

$\Rightarrow \quad n + 11 = 2k+1+11 = 2k+12$

$= 2(k+6)$

$= 2l \quad\quad l = k+6 \in Z$

which is even.

$\therefore$ n+11 is an even integer.

$\Rightarrow$ q is true.

Hence, $p \to q$ is true.

(ii) RHS $= [(p \to q) \wedge (p \to r)]$

$\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ $\qquad p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow \neg p \vee (q \wedge r)$ $\qquad$ Distributive law

$\Leftrightarrow p \to (q \wedge r)$ $\qquad \neg p \vee q \Leftrightarrow p \to q$

$=$ RHS

8. Let p : n is an odd integer.

q : n + 11 is an even integer.

Given $p \to q$

(i) Direct proof : Assume p is true.

$\Rightarrow$ n is an odd integer.

$\Rightarrow n = 2k+1 \quad ; \quad k \in Z$

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$= 2(k+6)$

$= 2l \qquad l = k+6 \in Z$

which is even.

$\therefore$ n+11 is an even integer.

$\Rightarrow$ q is true.

Hence, $p \to q$ is true.

(ii) Indirect proof:

wkt $p \to q \Leftrightarrow \neg q \to \neg p$

Assume $\neg q$ is true.

$\Rightarrow$ $n+11$ is an odd ~~ot~~ integer.

$\Rightarrow n+11 = 2k+1 \quad ; \quad k \in \mathbb{Z}$

$\Rightarrow n = 2k+1-11 = 2k-10$

$= 2(k-5) = 2l \qquad l = k-5 \in \mathbb{Z}$

which is even

$\Rightarrow \neg p$ is true.

$\therefore \neg q \to \neg p$ is true.

Hence, $p \to q$ is true.

9(a) Let p: 2 is an integer

q: 9 is a multiple of 3

Given $p \to q$

Converse: $q \to p$

ie., If 9 is a multiple of 3 then 2 is an integer.

Inverse: $\neg p \to \neg q$

ie., If 2 is not an integer then 9 is not a multiple of 3.

Contrapositive: $\neg q \to \neg p$

ie., If 9 is not a multiple of 3 then 2 is not an integer.

9(b)

Let the Universe be U: set of all Americans.

$p(x)$: x eat cheese burgers.

Given $\forall x, p(x)$

Negation: $\neg [\forall x, p(x)]$

$\exists x, \neg p(x)$

ie., Some Americans do not eat cheese burgers.

10 (a) A conjunction is a compound proposition obtained by inserting the word 'and' between two propositions.

'p and q' is denoted by '$p \wedge q$'.

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

10(b)

$$\begin{array}{l} \neg p \to q \\ q \to r \\ \neg r \\ \hline \therefore p \end{array} \quad \Rightarrow \quad \begin{array}{l} \neg p \to r \\ \neg r \\ \hline \therefore p \end{array}$$

rule of syllogism

This is a valid argument in view of Modus Tollens.