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Internal Assessment Test 2 – November 2025

Sub:	**Theory of Computation**			Sub Code:	BCS503	Branch:	AIML/CSE AIML
Date:	29/11/2025	Duration: 90 min's	Max Marks: 50	Sem/Sec:	V /A,B CSEAIML(A)		OBE

	Answer any FIVE FULL Questions	MARKS	CO	RBT
1.(a)	Construct PDA that accepts the language L={ ww^R | w∈{a+b}* and w^R is the reversal of w}. Write ID for the string aabbaa.	6	CO3	L2, L3
1.(b)	Convert the following CFG to PDA and give the procedure for the same. S→aABB|aAA A→aBB|a B→bBB|A C→a	4	CO3	L3
2	Define CFG and CNF. Convert the given CFG to CNF S→ABC|BaB A→aA|BaC|aaa B→bBb|a|D C→CA|AC D→ϵ	10	CO3	L1, L3
3(a)	State and prove pumping Lemma for context free languages.	5	CO3	L1, L2
3.(b)	Prove that language L={$a^nb^nc^n$ |n>=1} is not context free.	5	CO3	L3

Solution

Sub: Theory of Computation IAT-2 Subcode : BCS503

Dep: AIML/CSE-AIML

Sol: 1.a) PDA - $L = \{ww^R \mid w \in \{a+b\}^*$ & w^R is reversal of $w\}$

$P = (\{q_0, q_1, q_2\}, \{0,1\}, \{0, 1, Z_0\}, \delta, q_0, Z_0, \{q_2\})$

δ:

1. $\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$
2. $\delta(q_0, 1, Z_0) = \{(q_0, 1Z_0)\}$
3. $\delta(q_0, 0, 0) = \{(q_0, 00)\}$
4. $\delta(q_0, 0, 1) = \{(q_0, 01)\}$

$\delta(q_0, 1, 0) = \{(q_0, 10)\}$

$\delta(q_0, 1, 1) = \{(q_0, 11)\}$

$\delta(q_0, \epsilon, Z_0) = \{(q_1, Z_0)\}$

$\delta(q_0, \epsilon, 0) = \{(q_1, 0)\}$

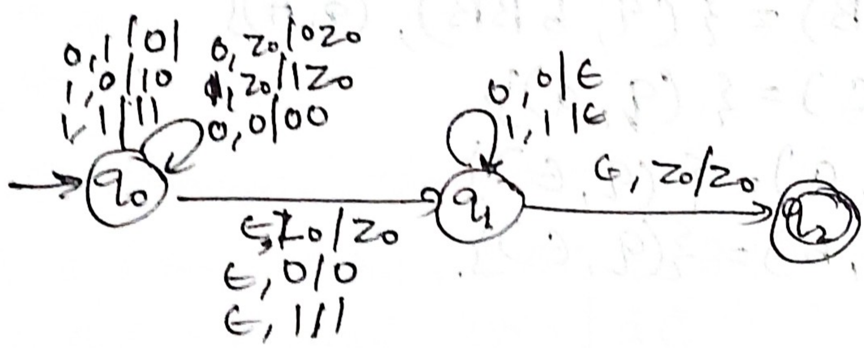
$\delta(q_0, \epsilon, 1) = \{(q_1, 1)\}$

$\delta(q_1, 0, 0) = \{(q_1, \epsilon)\}$

$\delta(q_1, 1, 1) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, Z_0) = \{(q_2, Z_0)\}$

2M



2 M

ID: $(q_0, \underbrace{011}_{w}\ \underbrace{110}_{w^R})$

$(q_0, 011110) \rightarrow (q_1, 011110, Z_0) \rightarrow (q_2, 011110, Z_0)$ 2 M

$\downarrow$

$(q_0, 11110, 0Z_0) \rightarrow (q_1, 11110, 0Z_0)$

$\downarrow$

$(q_0, 1110, 10Z_0) \rightarrow (q_1, 1110, 10Z_0) \rightarrow (q_1, 110, 0Z_0)$

$\downarrow$

$(q_0, 110, 110Z_0) \rightarrow (q_1, 110, 110Z_0) \rightarrow (q_1, 10, 10Z_0)$

$\downarrow$

$(q_0, 10, 1110Z_0)$

$\downarrow$

$(q_0, 0, 11110Z_0)$

$\downarrow$

$(q_0, \epsilon, 011110Z_0)$

$\downarrow$

$(q_1, \epsilon, 011110Z_0)$

Reject

$(q_1, 10, 10Z_0)$

$\downarrow$

$(q_1, 0, 0Z_0)$

$\downarrow$

(q_1, ϵ, Z_0)

$\downarrow$

(q_2, ϵ, Z_0) (Accepted)

1.b CFG → PDA

4 M

$S \to aABB \mid aAA$

$A \to aBB \mid a$

$B \to bBB \mid A$

$C \to a$

$P = (\{q\}, T, V \cup T, \delta, q, S\}$

where transition f^n δ defined as

$\delta(q, \epsilon, S) = \{(q, aABB)\}$

$\delta(q, \epsilon, S) = \{(q, aAA)\}$

$\delta(q, \epsilon, A) = \{(q, aBB), (q, a)\}$

$\delta(q, \epsilon, B) = \{(q, bBB), (q, A)\}$

$\delta(q, \epsilon, C) = \{(q, a)\}$

$\delta(q, a, a) = \{(q, \epsilon)\}$

$\delta(q, b, b) = \{(q, \epsilon)\}$.

2. CFG to CNF

2 M

Definition : A CFG is a 4 tuple $G = (V, T, P, S)$ where

- V - finite set of variable.
- T - finite set of terminals.
- $S \in V$ - start symbol
- P - Production rules represents the recursive def^n of lang.

Each prod^n consists of:

1. head of prod^n : a variable that is being (partially) defined by prod^n.
2. production symbol →
3. body of production - a string of zero or more terminals & variables.

$P \subseteq V \times (V \cup T)^*$.

Def^n of CNF : A specific formate of CFG in which all the production are of the form.

$$A \rightarrow BC \qquad A, B, C \in V - \Sigma$$

or

$$A \rightarrow a \quad , \text{where } a \in \Sigma$$

2M

This form of CFG is called as Chomsky Normal form. (CNF).

CFG to CNF:

6M

$S \rightarrow ABC \mid BaB$

$A \rightarrow aA \mid BaC \mid aaa$

$B \rightarrow bBb \mid a \mid D$

$C \rightarrow CA \mid AC$

$D \rightarrow \epsilon$

1. (a) Eliminate ϵ-prod^n.

$P_1 = S \rightarrow ABC \mid BaB \mid AC \mid aB \mid Ba \mid a$

$A \rightarrow aA \mid BaC \mid aaa \mid aC$

$B \rightarrow bBb \mid a \mid bb.$

$C \rightarrow CA \mid AC.$

2. No unit prod^n.

3. Eliminate useless symbol.

(a) Compute generating symbols.

$P_2 = S \rightarrow BaB \mid aB \mid Ba \mid a$

$A \rightarrow aA \mid aaa$

$B \rightarrow bBb \mid a \mid bb$

(b) Compute reachable symbols.

$P_3 = S \rightarrow BaB \mid aB \mid Ba \mid a$

$B \rightarrow bBb \mid a \mid bb.$

4. Convert of CNF.

$S \rightarrow BXB \mid XB \mid BX$

$B \rightarrow YBY \mid YY \mid a$

$X \rightarrow a$

$Y \rightarrow b$

$\Rightarrow S \rightarrow BU \mid XB \mid BX$

$B \rightarrow YV \mid YY \mid a$

$U \rightarrow XB$

$V \rightarrow BY$

$X \rightarrow a$

$Y \rightarrow b.$

3(a) State and prove. Pumping lemma for CFL. 5 M

Theorem: Let L be a CFL. Then there exists a constant n s.t. if z is any string in L s.t. $|z|$ is at least n, then we can write $z = uvwxy$, subject to the following conditions

1. $|vwx| \leq n$
2. $vx \neq \epsilon$. ∵ v & x are the pumped pieces, atleast one of the string must be non empty.
3. for all $i \geq 0$,

$$uv^iwx^iy \in L$$

i.e. two string v & x may be 'pumped' any no. of times, including 0 the resulting string will be a member of L.

Proof 1. Given CFG $G = (V, T, P, S)$ s.t.

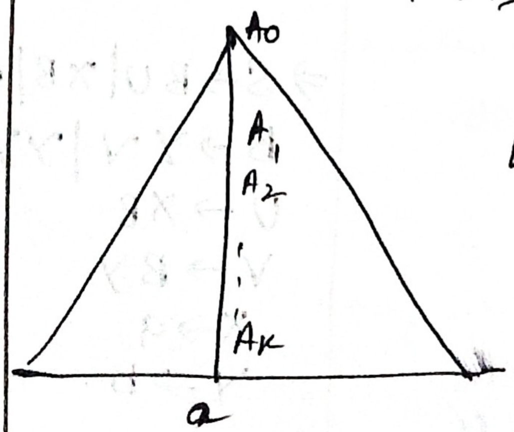
$$L(G) = L - \{\epsilon\}$$

Let G have m - variables.

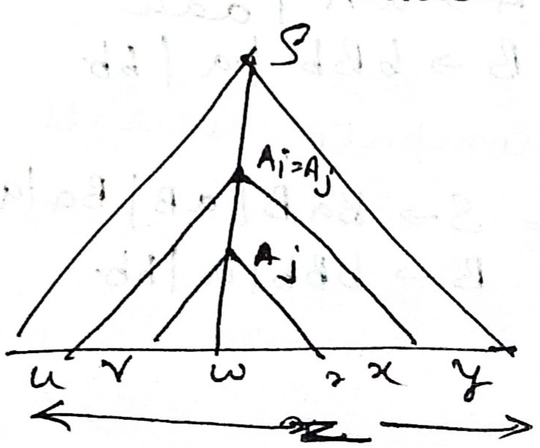
2. choose $n = 2^m$.

3. Suppose that $z \in L$, where $|z| \geq n$.

Let consider the longest path in tree to z with k atleast m, $k \geq m$ & path is of leng $k+1$.

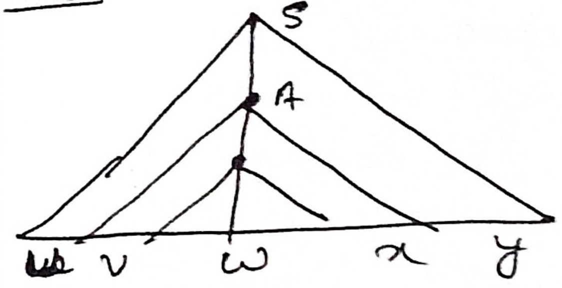


Suppose $A_i = A_j$, where $k - m \leq i < j \leq k$. Then it is possible to divide tree as shown in fig.

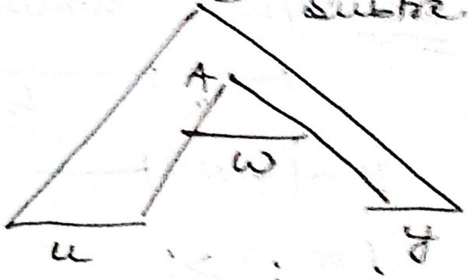


Replacing subtree A_i with Subtree A_j

case i If $A_i = A_j = A$

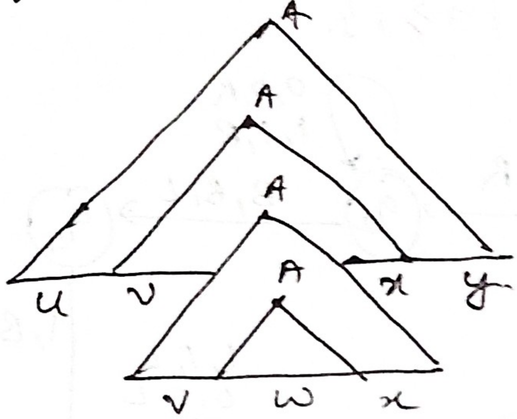


case ii



For string uv^iwx^iy, yield of tree $= uwy$ for $i=0$.

case iii Replace the subtree rooted at A_j by entire subtree rooted at A_i



yield of tree $= uv^2wx^2y$.

Thus, there are parse tree in G for all string of the form uv^iwx^iy. Hence proved the pumping lemma.

3b Sol^n $L = \{a^nb^nc^n \mid n \geq 1\}$ is not context free. 5M

Let assume that $L = \{a^nb^nc^n \mid n \geq 1\}$ is context-free lang.

1. For given n, choose $z = a^nb^nc^n \in L$ & $|z| \geq n$.
2. Using P.L. given a partition.

$z = uvwxy$

St. $|vwx| \leq n$ & $vx \neq \epsilon$.

Then vwx cannot contain both a's & c's

$\because$ last a and first c are seperated by n+1 position

The vwx can either consists of

case 1: all a's or all b's or all c's.

case 2: some a's & some b's

case 3: some b's & some c's

case1 vwx consist of all a's

a b c

u - (vwx) y

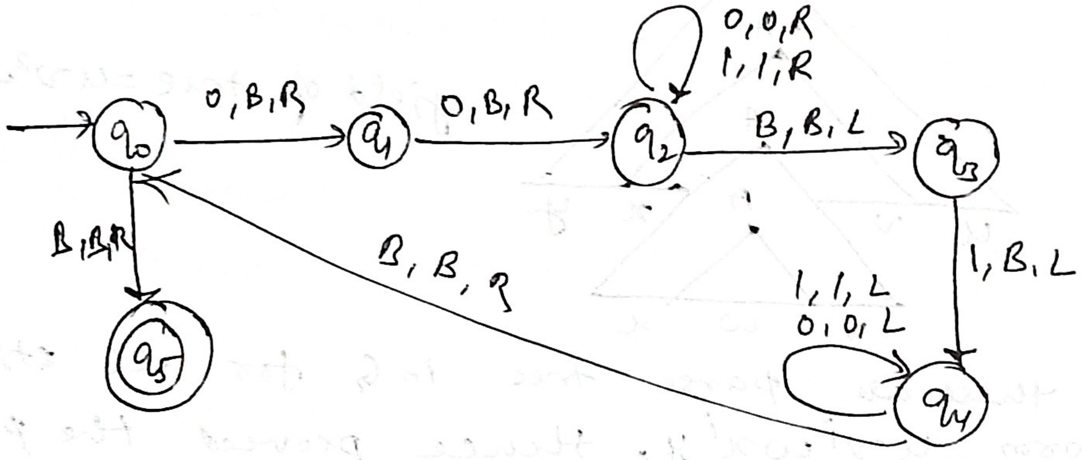
for $i \geq 1$

$z = uv^iwx^iy \notin L$

case 2 same for remaining cases.

Sol^n 4 TM: $L = \{a^{2n}b^n \mid n \geq 1\}$.

5M



δ	0	1	B
→ q_0	q_1,B,R		q_5/B,R
q_1	q_2,B,R		
q_2	q_2,0,R	q_2,1,R	q_3,B,L
q_3		q_4,B,L	
q_4	q_4,0,L	q_4,1,L	
* q_5			

5M

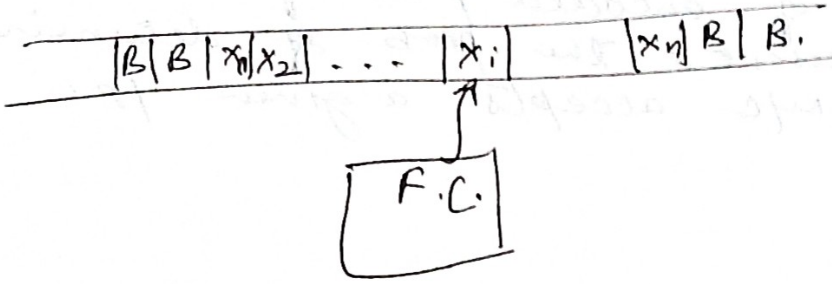
Sol 5 i) Turing Machine and its working. 5 M

TM - used to perform any calculation that can be designed logically by human beings.

- that means if we are able to design a step by step sol^n of any problem. - can design equivalent TM.

TM < DTM, NDTM.

Basic Model of TM.



TM consists of.

1. Finite control
2. Tape.
3. Tape head

(ii) Non Deterministic Turing machine. 5 M

- A Theoritical model of computation where, at every step, ~~at~~ at any step, the m/c can have multiple possible next moves, ~~the~~ rather than a single predetermined one.

  - Multiple transitions
  - Computation tree.
  - Acceptance
  - Guessing

Sol 6. Short note on: 5M

i) Recursive Enumerable language.

- a set of strings for which there exists a TM that will halt and accept any string belonging to the lang. If string is not in lang, the TM may reject and halt or it may run forever in an infinite loop.

5M

(ii) Universal language.

- a lang of encoded pair of a TM & its i/p, which represents the prob of determining if a given M/C accepts a given i/p.