Internal Assessment Test 2 – November 2025

Subject:	Wireless Communication Systems							Code:	BEC703
Date:	29/11/2025	Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	ECE

	Scheme & Solution	Marks	OBE CO	OBE RBT
1.	Compute the MIMO zero-forcing receiver for the channel matrix **H** given as $H = \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix}$	[10]	CO4	L3

Solution: It can be seen that the MIMO channel matrix above is of size 3×2, implying that the number of receive antennas is $r = 3$, while the number of transmit antennas is $t = 2$. Thus, the MIMO system model can be described as

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}}_{\mathbf{n}}$$

Thus, as can be seen from the model above, the transmit vector $\mathbf{x}$ is of dimension 2×1, while the receive and noise vectors $\mathbf{y}$, $\mathbf{n}$ respectively are of dimension 3×1. The above MIMO system model can also be explicitly written to describe the signal received at each receive antenna as

$$y_1 = 2x_1 + 3x_2 + n_1$$

$$y_2 = x_1 + 3x_2 + n_2$$

$$y_3 = 4x_1 + 2x_2 + n_3$$

which basically represents a system of $r = 3$ equations for $t = 2$ unknowns x_1, x_2. To compute the zero-forcing decoder $\mathbf{F}_{ZF}$, we first compute the matrix $(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$. Observe that $\mathbf{H}^H\mathbf{H}$ can be simplified as

$$\mathbf{H}^H\mathbf{H} = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 17 \\ 17 & 22 \end{bmatrix}$$

Hence, the expression for $(\mathbf{H}^H\mathbf{H})^{-1}$ can be simplified as

$$(\mathbf{H}^H\mathbf{H})^{-1} = \frac{1}{21 \times 22 - 17 \times 17} \begin{bmatrix} 22 & -17 \\ -17 & 21 \end{bmatrix}$$

$$= \frac{1}{173} \begin{bmatrix} 22 & -17 \\ -17 & 21 \end{bmatrix}$$

Thus, the zero-forcing receiver matrix $\mathbf{F}_{ZF} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ can be expressed as

$$\mathbf{F}_{ZF} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$$

$$= \frac{1}{173} \begin{bmatrix} 22 & -17 \\ -17 & 21 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \end{bmatrix}$$

$$= \frac{1}{173} \begin{bmatrix} -7 & -29 & 54 \\ 29 & 46 & -26 \end{bmatrix}$$

$$= \begin{bmatrix} -0.04 & -0.17 & 0.31 \\ 0.17 & 0.27 & -0.15 \end{bmatrix} \quad (6.5)$$

Thus, the zero-forcing estimate of the transmit vector is given as

$$\begin{aligned}\hat{\mathbf{x}}_{\text{ZF}} &= \mathbf{F}_{ZF}\mathbf{y} \\ &= \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{y} \\ &= \begin{bmatrix} -0.04 & -0.17 & 0.31 \\ 0.17 & 0.27 & -0.15 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\end{aligned}$$

These expressions can now be explicitly written for the estimate of each of the transmit symbols $\hat{x}_1$, $\hat{x}_2$ as

$$\hat{x}_1 = -0.04y_1 - 0.17y_2 + 0.31y_3$$

$$\hat{x}_2 = +0.17y_1 + 0.27y_2 - 0.15y_3$$

It can also be confirmed that the zero-forcing receiver matrix matrix $\mathbf{F}_{ZF}$ is of dimension 2×3, i.e., $t \times r$.

One of the chief disadvantages of the zero-forcing receivers is *noise amplification*. This can be understood as follows. Consider the SISO wireless system for $r = t = 1$. We have the system model given as

$$y = hx + n$$

Hence, the zero-forcing receiver is given as $f_{\text{ZF}} = (h^*h)^{-1}h^* = h^{-1} = \frac{1}{h}$. Hence, the zero-forcing estimate of the transmitted symbol x is given as

$$\begin{aligned}\hat{x}_{\text{ZF}} &= f_{\text{ZF}} \times y \\ &= \frac{1}{h}(hx + n) \\ &= x + \frac{n}{h} \qquad (6.6)\end{aligned}$$

2.	Elaborate in detail on the multiple access for OFDMA Systems.	[10]	CO3	L2

Orthogonal Frequency-Division Multiple Access (OFDMA) is a multi-user version of the Orthogonal Frequency-Division Multiplexing (OFDM) digital-modulation scheme. This is similar to FDM in concept. However, in the case of OFDM, all of the sub-channels are dedicated to a single data source. The OFDM scheme uses advanced digital signal-processing techniques to distribute the data over multiple carriers at precise frequencies. The precise relationship among the subcarriers is referred to as *orthogonality*. This means that the peaks of the power spectral density of each subcarrier occur at a point at which the power of other subcarriers is zero. With OFDM, the subcarriers can be packed tightly together because there is minimal interference between adjacent subcarriers.

Fig. 8.32 illustrates the concept of OFDM.

Let there be a data stream operating at R bps and an available bandwidth of ($N \times$ $_b$), where N is an integer related to the number of subcarriers, and $_b$ is the base frequency, centered at $_0$. The entire bandwidth could be used to send the data stream, in which case each bit duration would be $1/R$. The alternative is to split the data stream into N substreams, using a serial-to-parallel converter. Each substream has a data rate of R/N bps

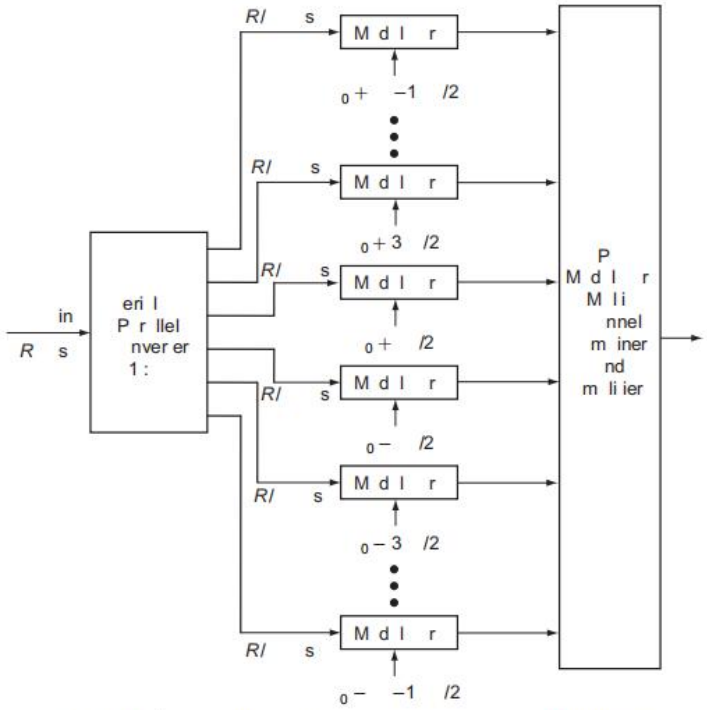


Fig. 8.32 | *Concept of Orthogonal Frequency Division Multiplexing*

and is transmitted on a separate subcarrier, with a spacing between adjacent subcarriers of $_b$. Now the bit duration is N/R. The base frequency, $_b$ is the lowest-frequency subcarrier. All of the other subcarriers are integer multiples of the base frequency, namely 2 $_b$, 3 $_b$, and so on.

For transmission, the set of OFDM subcarriers is further modulated to a higher frequency band. A digital modulation scheme used with OFDM is Quadrature Phase Shift Keying (QPSK). In this case, each transmitted symbol represents two bits. To minimise ISI, data are transmitted in bursts, with each burst consisting of a cyclic prefix followed by data symbols. The cyclic prefix is used to absorb transients from previous bursts caused by multipath. The resulting waveform created by the combined multipath signals is not a function of any sample from the previous burst.

OFDM has several advantages. First, frequency-selective fading affects some subchannels only and not the complete signal. If the data stream is protected by a forward error-correcting code, the impact of frequency-selective fading can be minimised. Moreover, OFDM overcomes intersymbol interference in a multipath environment. ISI has a greater impact at higher data rates because the time duration between symbols is smaller. With OFDM, the data rate is reduced by a factor of N, which increases the symbol period by a factor of N. Thus, if the symbol period is T for the data source stream, the period for the OFDM signals is $N \times T$. This reduces the effect of ISI. Usually, N is chosen so that $N \times T$ is much greater than the root-mean-square delay spread of the wireless fading channel. As a result of these design considerations, it may not be necessary to include equalisers with the use of OFDM.

3.	Explain in detail the MMSE MIMO Receiver and its advantages in MIMO wireless communication.	[10]	CO4	L2

In this section, we develop the *Minimum Mean-Squared Error (MMSE)* receiver for the MIMO wireless communication system. The MMSE receiver is based on a *Bayesian* approach, meaning that the transmit vector $\mathbf{x}$ is assumed to be random in nature. Thus, if $\hat{\mathbf{x}}_{\text{MMSE}}$ denotes the estimated symbol vector, the MMSE receiver minimizes the average or mean of the squared error

$$\mathrm{E}\left\{\|\hat{\mathbf{x}}_{\text{MMSE}} - \mathbf{x}\|^2\right\}$$

Thus, it is aptly named minimum mean-squared error estimator. To illustrate the development of the MMSE receiver, we consider a single-input multiple-output (SIMO) wireless system, i.e., $t = 1$, and generalize the result to the case of a MIMO system. Hence, consider the SIMO system model given as

$$\mathbf{y} = \mathbf{h}x + \mathbf{n}$$

where x is now a scalar transmitted symbol. Thus, the basic problem can be interpreted as estimating the symbol x given the vector $\mathbf{y} = [y_1, y_2, \ldots, y_r]^T$. Let $\mathbf{c} = [c_1, c_2, \ldots, c_r]^T$. One can now define a linear estimator of x as

$$\begin{aligned}\hat{x} &= \mathbf{c}^T\mathbf{y} \qquad (6.7)\\ &= \begin{bmatrix} c_1 & c_2 & \ldots & c_r \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix}\\ &= c_1y_1 + c_2y_2 + \ldots + c_ry_r\end{aligned}$$

Such an estimator as above is termed as a *linear* estimator, since the estimate is a linear function of $\mathbf{y}$. We now wish to find the best or optimal linear estimator $\hat{\mathbf{x}}_{\text{MMSE}}$, which minimizes the mean squared error and hence is also termed the *Linear Minimum Mean Squared Estimator (LMMSE)*. This is frequently also referred to simply as the MMSE, although strictly speaking belongs to the specific class of linear estimators. The average mean squared error is defined as

$$\mathrm{E}\left\{\|\hat{\mathbf{x}} - \mathbf{x}\|^2\right\}$$

Employing the form in Eq. (6.7), the resulting equation can be simplified as

$$\begin{aligned}
\mathrm{E}\left\{\|\hat{\mathbf{x}}-\mathbf{x}\|^2\right\} &= \mathrm{E}\left\{\left(\mathbf{c}^T\mathbf{y}-\mathbf{x}\right)\left(\mathbf{c}^T\mathbf{y}-\mathbf{x}\right)^T\right\} \\
&= \mathrm{E}\left\{\left(\mathbf{c}^T\mathbf{y}-\mathbf{x}\right)\left(\mathbf{y}^T\mathbf{c}-\mathbf{x}^T\right)\right\} \\
&= \mathrm{E}\left\{\mathbf{c}^T\mathbf{y}\mathbf{y}^T\mathbf{c}-x\mathbf{y}^T\mathbf{c}-\mathbf{c}^T\mathbf{y}x+xx^T\right\} \\
&= \mathbf{c}^T\underbrace{\mathrm{E}\left\{\mathbf{y}\mathbf{y}^T\right\}}_{\mathbf{R}_{yy}}\mathbf{c}-\underbrace{\mathrm{E}\left\{x\mathbf{y}^T\right\}}_{\mathbf{R}_{xy}}\mathbf{c}-\mathbf{c}^T\underbrace{\mathrm{E}\left\{\mathbf{y}x\right\}}_{\mathbf{R}_{yx}}+\mathrm{E}\left\{x^2\right\}_{\mathbf{R}_{xx}} \\
&= \mathbf{c}^T\mathbf{R}_{yy}\mathbf{c}-2\mathbf{c}^T\mathbf{R}_{yx}+\mathbf{R}_{xx}
\end{aligned}$$

where the covariance matrix $\mathbf{R}_{yy}$ is defined as $\mathbf{R}_{yy}=\mathrm{E}\left\{\mathbf{y}\mathbf{y}^H\right\}$. Similarly, $\mathbf{R}_{yx}=\mathrm{E}\left\{\mathbf{y}x\right\}$ $=\mathbf{R}_{xy}^T$ and $\mathbf{R}_{xx}=\mathrm{E}\left\{x^2\right\}$. Also note that we have used the fact $\mathbf{c}^T\mathbf{R}_{yx}=\left(\mathbf{c}^T\mathbf{R}_{yx}\right)^T=$ $\mathbf{R}_{xy}\mathbf{c}$. Hence, the average MSE as a function of the receive beamformer $\mathbf{c}$, denoted by $\overline{\mathrm{MSE}}\left(\mathbf{c}\right)$, is given as

$$\overline{\mathrm{MSE}}\left(\mathbf{c}\right)=\mathbf{c}^T\mathbf{R}_{yy}\mathbf{c}-2\mathbf{c}^T\mathbf{R}_{yx}+\mathbf{R}_{xx}$$

Thus, the optimal beamformer $\mathbf{c}$ which minimizes the average or mean squared error can be obtained by differentiating $\overline{\mathrm{MSE}}\left(\mathbf{c}\right)$ with respect to $\mathbf{c}$ and setting equal to zero as

$$\begin{aligned}
\frac{\partial\overline{\mathrm{MSE}}\left(\mathbf{c}\right)}{\partial\mathbf{c}} &= 0 \\
\frac{\partial}{\partial\mathbf{c}}\left(\mathbf{c}^T\mathbf{R}_{yy}\mathbf{c}-2\mathbf{c}^T\mathbf{R}_{yx}+\mathbf{R}_{xx}\right) &= 0 \\
2\mathbf{R}_{yy}\mathbf{c}-2\mathbf{R}_{yx} &= 0 \\
\mathbf{c} &= \mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}
\end{aligned}$$

Thus, the optimal LMMSE beamforming vector $\mathbf{c}$ is given as $\mathbf{c}=\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}$. This is also termed in signal processing as the optimal *Wiener* filter. The above can be generalized in the case of complex vectors by replacing the transpose by the Hermitian operation.

Hence, the MMSE estimate of x is given as

$$\begin{aligned}\hat{\mathbf{x}}_{\text{MMSE}} &= \mathbf{c}^H\mathbf{y} \\ &= \left(\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}\right)^H\mathbf{y} \\ &= \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{y}\end{aligned}$$

We now compute the MMSE receiver for the MIMO wireless system. Consider again the MIMO system model given as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Let the transmit symbols x_i, $1 \leq i \leq t$ be such that each is of power P_d, i.e., $\mathrm{E}\left\{|x_i|^2\right\} = P_d$, with elements on different transmit antennas being uncorrelated, i.e., $\mathrm{E}\left\{x_i x_j^*\right\} = 0$ when $i \neq j$. Hence, the covariance $\mathbf{R}_{xx}$ of the transmit symbols is given as

$$\begin{aligned}\mathbf{R}_{xx} &= \mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\} \\ &= \mathrm{E}\left\{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_t \end{bmatrix}\begin{bmatrix} x_1^* & x_2^* & \dots & x_t^* \end{bmatrix}\right\} \\ &= \begin{bmatrix} \mathrm{E}\left\{|x_1|^2\right\} & \mathrm{E}\{x_1x_2^*\} & \dots & \mathrm{E}\{x_1x_t^*\} \\ \mathrm{E}\{x_2x_1^*\} & \mathrm{E}\left\{|x_2|^2\right\} & \dots & \mathrm{E}\{x_2x_t^*\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{E}\{x_tx_1^*\} & \mathrm{E}\{x_tx_2^*\} & \dots & \mathrm{E}\left\{|x_t|^2\right\} \end{bmatrix} \\ &= \begin{bmatrix} P_d & 0 & 0 & \dots & 0 \\ 0 & P_d & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & P_d \end{bmatrix} \\ &= P_d\mathbf{I}_t\end{aligned}$$

Hence, $\mathbf{R}_{yy}$, the covariance of the receive vector $\mathbf{y}$ can be simplified as

$$\begin{aligned}
\mathbf{R}_{yy} &= \mathrm{E}\left\{\mathbf{y}\mathbf{y}^H\right\} \\
&= \mathrm{E}\left\{(\mathbf{H}\mathbf{x}+\mathbf{n})(\mathbf{H}\mathbf{x}+\mathbf{n})^H\right\} \\
&= \mathrm{E}\left\{\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H} + \mathbf{n}\mathbf{x}^H\mathbf{H}^H + \mathbf{H}\mathbf{x}\mathbf{n}^H + \mathbf{n}\mathbf{n}^H\right\} \\
&= \mathbf{H}\underbrace{\mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\}}_{\mathbf{R}_{xx}}\mathbf{H} + \underbrace{\mathrm{E}\left\{\mathbf{n}\mathbf{x}^H\right\}}_{0}\mathbf{H}^H + \mathbf{H}\underbrace{\mathrm{E}\left\{\mathbf{x}\mathbf{n}^H\right\}}_{0} + \underbrace{\mathrm{E}\left\{\mathbf{n}\mathbf{n}^H\right\}}_{\mathbf{R}_{nn}} \\
&= \underbrace{P_d\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}_r}_{\mathbf{R}_{yy}}
\end{aligned}$$

where we have used the fact that $\mathrm{E}\left\{\mathbf{n}\mathbf{x}^H\right\} = \mathrm{E}\left\{\mathbf{x}\mathbf{n}^H\right\} = \mathbf{0}$, since the noise at the receiver and transmit symbols are uncorrelated, in the above simplification. Further, the cross-covariance matrix $\mathbf{R}_{yx}$ can be simplified as

$$\begin{aligned}
\mathbf{R}_{yx} &= \mathrm{E}\left\{\mathbf{y}\mathbf{x}^H\right\} \\
&= \mathrm{E}\left\{(\mathbf{H}\mathbf{x}+\mathbf{n})\mathbf{x}^H\right\} \\
&= \mathrm{E}\left\{\mathbf{H}\mathbf{x}\mathbf{x}^H + \mathbf{n}\mathbf{x}^H\right\} \\
&= \mathbf{H}\underbrace{\mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\}}_{\mathbf{R}_{xx}} + \underbrace{\mathrm{E}\left\{\mathbf{n}\mathbf{x}^H\right\}}_{0} \\
&= P_d\mathbf{H}
\end{aligned}$$

Thus, the optimal MMSE receiver is given as

$$\begin{aligned}
\mathbf{C} &= \mathbf{R}_{yy}^{-1}\mathbf{R}_{yx} \\
&= \left(P_d\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}P_d\mathbf{H} \\
&= P_d\left(P_d\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}
\end{aligned}$$

Hence, the MMSE estimate $\hat{\mathbf{x}}_{\text{MMSE}}$ of the transmit vector $\mathbf{x}$ is given as

$$\begin{aligned}
\hat{\mathbf{x}}_{\text{MMSE}} &= \mathbf{C}^H\mathbf{y} \\
&= P_d\mathbf{H}^H\left(P_d\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{y} \qquad (6.8)
\end{aligned}$$

We now derive an alternative structure for the MIMO MMSE receiver. Observe that, we have,

$$\begin{aligned}
&P_d\mathbf{H}^H\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{H}^H = P_d\mathbf{H}^H\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{H}^H \\
\Rightarrow &\left(P_d\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}\right)\mathbf{H}^H = \mathbf{H}^H\left(P_d\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right) \\
\Rightarrow &\mathbf{H}^H\left(P_d\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1} = \left(P_d\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}^H
\end{aligned}$$

Thus, the MIMO MMSE receiver in Eq. (6.8) can also be expressed as

$$\begin{aligned}
\hat{\mathbf{x}}_{\text{MMSE}} &= P_d\mathbf{H}^H\left(P_d\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{y} \\
&= P_d\left(P_d\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}^H\mathbf{y} \qquad (6.9)
\end{aligned}$$

Thus, the above expression is an alternative form of implementation of the MIMO MMSE receiver. Observe that the matrix $\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}$ is of dimension $r \times r$, while $P_d\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}$ is $t \times t$ dimensional. Thus, if $r > t$, inversion of the latter matrix is of a lower computation complexity. Since this is frequently the case in MIMO wireless systems, the alternative MIMO MMSE receiver version is more popular for implementation.

4.	Explain the Singular Value Decomposition (SVD) of the MIMO channel with suitable examples.	[10]	CO4	L3

In this section, we will begin to explore the *Singular Value Decomposition* (SVD) of the MIMO channel matrix $\mathbf{H}$, which is a very important tool to understand the behaviour of a MIMO wireless communication system. Consider an $r \times t$ MIMO channel $\mathbf{H}$ with $r \geq t$, i.e., number of receive antennas greater than or equal to the number of transmit antennas. The SVD of the channel matrix $\mathbf{H}$ is given as

$$\mathbf{H} = \underbrace{\begin{bmatrix} \underbrace{u_{11}}_{\mathbf{u}_1} & \underbrace{u_{12}}_{\mathbf{u}_2} & \dots & \underbrace{u_{1t}}_{\mathbf{u}_t} \\ u_{21} & u_{22} & \dots & u_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ u_{r1} & u_{r2} & \dots & u_{rt} \end{bmatrix}}_{\mathbf{U}} \underbrace{\begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_t \end{bmatrix}}_{\mathbf{\Sigma}}$$

$$\underbrace{\begin{bmatrix} v_{11}^* & v_{21}^* & \dots & v_{t1}^* \\ v_{12}^* & v_{22}^* & \dots & v_{t2}^* \\ \vdots & \vdots & \ddots & \vdots \\ v_{1t}^* & v_{2t}^* & \dots & v_{tt}^* \end{bmatrix}}_{\mathbf{V}^H} \begin{matrix} \} \mathbf{v}_1^H \\ \} \mathbf{v}_2^H \\ \\ \} \mathbf{v}_t^H \end{matrix},$$

$$= \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{6.10}$$

where the matrices $\mathbf{U}$, $\mathbf{\Sigma}$, $\mathbf{V}$, which are $r \times t$, $t \times t$ and $t \times t$ dimensional respectively, satisfy important properties, which we examine next. The columns of the matrix $\mathbf{U}$ and $\mathbf{V}$ are unit-norm, i.e., we have

$$\|\mathbf{u}_i\|^2 = \|\mathbf{v}_i\|^2 = 1, \, 1 \leq i \leq t$$

Further, the columns of matrix $\mathbf{U}$ and $\mathbf{V}$ are orthogonal, i.e.,

$$\mathbf{u}_i^H \mathbf{u}_j = \mathbf{v}_i^H \mathbf{v}_j = 0, \, i \neq j, \, 1 \leq i, j \leq t$$

Thus, the columns of matrices $\mathbf{U}$ and $\mathbf{V}$ are orthonormal. As a result, the $t \times t$ dimensional square matrix $\mathbf{V}$ is unitary, i.e.,

$$\mathbf{V}^H \mathbf{V} = \mathbf{V}\mathbf{V}^H = \mathbf{I}_t$$

Further, since if $r = t$, the matrix $\mathbf{U}$ is also a unitary matrix. Otherwise, $\mathbf{U}$ simply satisfies the relation $\mathbf{U}^H \mathbf{U} = \mathbf{I}_t$. Further, the quantities $\sigma_1, \, \sigma_2, \, \dots, \, \sigma_t$ are known as the singular values of the matrix $\mathbf{\Sigma}$. These singular values are non-negative and *ordered*, i.e., each $\sigma_i \geq 0$ and

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_t$$

Finally, an important property of the singular values is that the number of nonzero singular values is equal to the rank of the matrix $\mathbf{H}$. Below, we illustrate some examples of singular value decomposition to give insights into the structure of the SVD.

Let the 2×2 channel matrix $\mathbf{H}$ be given as

$$\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{bmatrix}$$

In this example, it is very intuitive to simplify $\mathbf{H}$ simply as

$$\mathbf{H} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{U}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{bmatrix}}_{\mathbf{\Sigma}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{V}^H}$$

and claim that $\sigma_1 = 1$, $\sigma_2 = \sqrt{5}$ are the singular values of $\mathbf{H}$. However, this is incorrect since $\sigma_1 < \sigma_2$, meaning that the above is not a valid SVD. However, one can recast $\mathbf{H}$ as follows.

$$\mathbf{H} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\mathbf{U}} \underbrace{\begin{bmatrix} \sqrt{5} & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{\Sigma}} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\mathbf{V}^H}$$

where the matrices $\mathbf{U}$, $\mathbf{V}$ are 2×2 permutation matrices which flip the rows and columns of the inner diagonal matrix respectively. It can also be observed that $\mathbf{V}\mathbf{V}^H = \mathbf{V}^H\mathbf{V} = \mathbf{I}_t$. Also, in this case, since $r = t$, we also have $\mathbf{U}\mathbf{U}^H = \mathbf{U}^H\mathbf{U} = \mathbf{I}_t$. Further, $\sigma_1 = \sqrt{5} > \sigma_2 = 1 > 0$. Thus, the singular values are positive and ordered. Hence, this is a valid SVD of the channel matrix $\mathbf{H}$. This example demonstrates how an otherwise seemingly simple diagonal matrix can have an SVD that is not straightforward to compute.

5.	Explain in detail the Alamouti and Space-Time codes.	[10]	CO4	L2

Alamouti coding is a space-time block coding technique used in wireless communication systems with multiple-input multiple-output (MIMO) channels. It was proposed by Prof. Siavash Alamouti in 1998 and provides a simple and effective way to achieve diversity and increase the data rate in wireless communication. The basic idea behind Alamouti coding is to transmit the same information over multiple antennas in such a way that it can be decoded at the receiver using simple linear processing. The Alamouti scheme uses two transmit antennas and one receive antenna, but it can be extended to multiple antennas at both the transmitter and receiver. In the Alamouti scheme, the input data is first split into two parts and each part is transmitted from the two antennas. The transmitted signal at time t is given by:

s(t) = [x1(t) x2(t)] [a1(t) -a2*(t)]…….(1)

where x1(t) and x2(t) are the two parts of the input data, a1(t) and a2(t) are the complex channel gains between the two transmit antennas and the receive antenna at time t, and a2*(t) is the complex conjugate of a2(t).

At the receiver, the received signal is first demodulated and then processed using the following decoding matrix:

[1 0; 0 a2*(t)] [y1(t) y2(t)] = [x1(t) x2(t)]……..(2)

where y1(t) and y2(t) are the received signals at time t.

The decoding matrix takes advantage of the orthogonality between the two transmitted signals to recover the original data. The first row of the matrix simply scales the first received signal by 1 and the second row scales the second received signal by the complex conjugate of the second channel gain, which cancels out the interference caused by the second transmitted signal. Alamouti coding provides diversity gain by transmitting the same information over two antennas and using simple linear processing at the receiver to recover the original data. It is a powerful technique for increasing the data rate and improving the reliability of wireless communication systems, especially in MIMO channels.

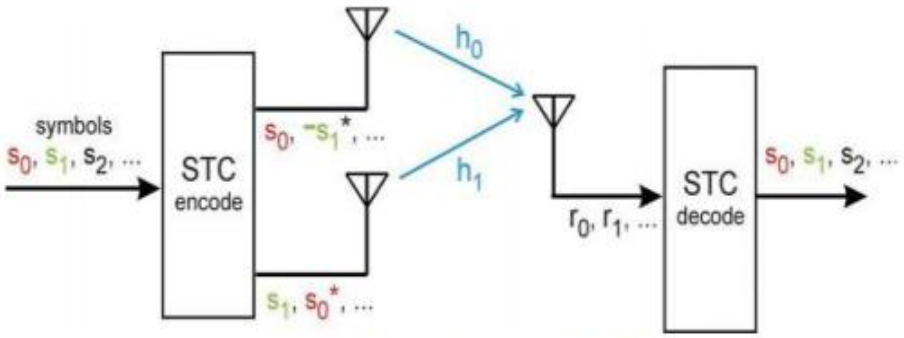


Fig. 1 Simplified Alamouti Space Time Coding (STC) block diagram

A simplified block diagram using Alamouti STC is shown in Fig. 1. In this system, two different symbols are simultaneously transmitted from the two antennas during any symbol period. During the first time period, the first symbol in the sequence, s0, is transmitted from the upper antenna 1 while the second symbol, s1, is simultaneously transmitted from the lower antenna 2. During the next symbol time the signal -s1* is transmitted from the upper antenna and the signal s0* is transmitted from lower antenna. Note that ( )* is the complex conjugate operation.

Table 1 Bit Pattern for two antennas at different time durations

	t+4T	t+3T	t+2T	t+T	t
Antenna 0	....S4	-S3*	S2	-S1*	S0
Antenna 1	....S5	S2*	S3	S0*	S1

At the receiver, a single antenna receives a combination of the two transmitted signals after transmission through the multipath environment. The channel coefficient, h0, represents the magnitude and phase of the transmission path between transmit antenna 1 and the receive antenna. The channel coefficient, h1, represents the path between transmit antenna 2 and the receive antenna. The channel coefficients, h0 and h1, are complex numbers that represent the total amplitude and phase of their respective channels including all multipath effects.

During the first symbol time, the received signal, r0, is the combination of both symbols, s0 and s1, but is modified by the channel coefficients, h0 and h1. During the next symbol period, the receiver measures r1 which contain modified versions of s0 and s1. The received signals, r0 and r1, as a function of the transmitted signals and channels coefficients can be represented as

$r_o = r_o(t) = h_0S_0 + h_1S_1 + n_0$

$r_1 = r_1(t+t) = -h_0S_1^* + h_1S_o^* + n_1$ ..... (3)

In order to recover the actual transmitted symbols, s0 and s1, the receiver requires knowledge of the channel coefficients, h0 and h1. These channel coefficients are often estimated at the receiver by measuring known signals embedded in the transmitted waveforms. Combiner is then used to reconstruct the two combined signals Then it is sent to the maximum likelihood detector

$S_0 = A(h_0^*r_0 + h_1^*r_0)$

$S_1 = A(h_1^*r_1 - h_0^*r_1)$

$A = 1/(|h_o|2 + |h_1|2)$..... (4)

Diversity technique does not improve the system data rate but rather improves the signal quality. The sequence shown in Fig. 1 uses encoding performed in space and time (space–time coding). The encoding may also be done over the space and frequency domains. In this case, instead of two consecutive symbol periods transmitted from two separate antennas, two frequency carriers may be used (space–frequency coding). Utilization of diversity in MIMO channels requires a combination of the transmit and receive diversity described above. The diversity order would then be equal to the product of the number of transmit and receive antennas if the channel between each transmit-receive antenna pair fades independently.

6.	Consider the 2 × 2 MIMO system given below, and describe the various stages of the V Blast Receiver.	[10]	CO4	L3

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} n1 \\ n2 \end{bmatrix}$$

Solution: The channel matrix $\mathbf{H}$ is given as

$$\mathbf{H} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

Therefore, the left inverse of $\mathbf{H}$, i.e., $\mathbf{Q} = \mathbf{H}^{\dagger}$ is given as $\mathbf{Q} = \mathbf{H}^{-1}$ since the matrix $\mathbf{H}$ is square and invertible. Therefore, we have

$$\mathbf{Q} = \mathbf{H}^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

It can be clearly seen that the rows of the matrix $\mathbf{Q}$ are $\mathbf{q}_1^H = \begin{bmatrix} 3 & -2 \end{bmatrix}$, $\mathbf{q}_2^H = \begin{bmatrix} -1 & 1 \end{bmatrix}$. Further, it can be readily seen that $\mathbf{q}_1^H \mathbf{h}_1$ is given as

$$\mathbf{q}_1^H \mathbf{h}_1 = \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$$

and $\mathbf{q}_1^H \mathbf{h}_2$ can be simplified as

$$\mathbf{q}_1^H \mathbf{h}_2 = \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 0$$

Therefore, the first row $\mathbf{q}_1^H$ is orthogonal to $\mathbf{h}_2$, i.e., the second column of the channel matrix $\mathbf{H}$. Therefore, decoding in $\mathbf{q}_1^H$ in the first stage of V-BLAST, we have

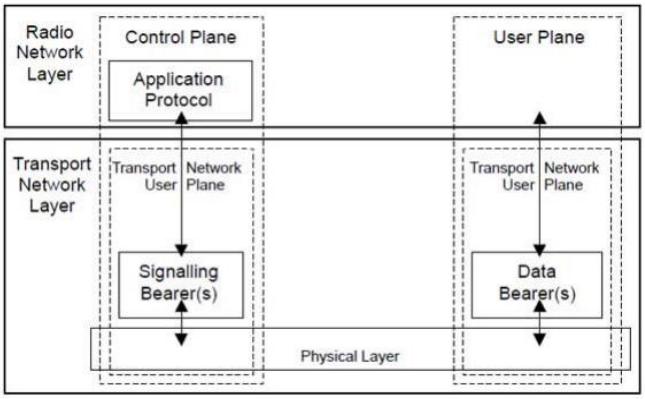
$$\begin{aligned} \tilde{y}_1 &= \mathbf{q}_1^H \mathbf{y} \\ &= \begin{bmatrix} 3 & -2 \end{bmatrix} \left(\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \right). \\ &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\mathbf{q}_1^H \mathbf{n}}_{\tilde{n}_1} \\ &= x_1 + \tilde{n}_1 \end{aligned}$$

Therefore, the symbol x_1 can be detected from $\tilde{y}_1$. The interference caused by this can now be canceled from the received signal $\mathbf{y}$ to form $\tilde{\mathbf{y}}_2$ as

$$\begin{aligned} \tilde{\mathbf{y}}_2 &= \mathbf{y} - \mathbf{h}_1 x_1 \\ &= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_1 \\ &= \begin{bmatrix} 2 \\ 3 \end{bmatrix} x_2 + \mathbf{n} \end{aligned}$$

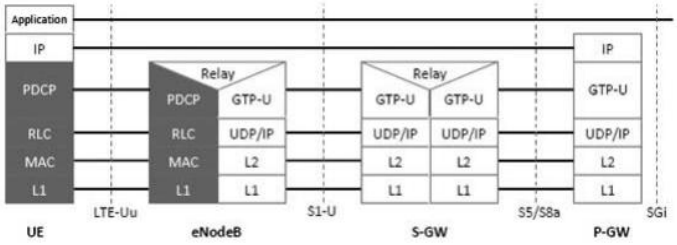
It can now be seen that the above system corresponds effectively to a receive diversity system with 2 receive antennas and a single transmit antenna. In fact, the optimum receiver scheme is to perform maximum ratio combining with the receive beamformer $\mathbf{w} = \begin{bmatrix} 2 & 3 \end{bmatrix}^T$. The diversity order of decoding this stream is, therefore, equal to 2.

7.	Explain the Uplink SC-FDMA Radio resources.	[10]	CO3	L2
	Uplink SC-FDMA (Single-Carrier Frequency-Division Multiple Access) radio resources are the channels and power allocated by a network to user equipment (UE) for transmitting data to the base station. SC-FDMA is a modulation scheme that is efficient for uplink transmissions because it has a low peak-to-average power ratio (PAPR), which helps mobile devices conserve battery power. Resource allocation involves assigning subcarriers (subchannels) to users, often in contiguous blocks, and controlling their transmission power to maximize throughput and meet different quality-of-service (QoS)			

	requirements. **Key aspects of SC-FDMA radio resources** **Low PAPR:** SC-FDMA is designed to have a lower PAPR than OFDM, which is crucial for mobile devices with limited battery power. A lower PAPR allows the amplifier to operate more efficiently, reducing power consumption. **Subcarrier allocation:** The total radio resource is divided into subcarriers, which are then allocated to different users. **Localized SC-FDMA:** In this approach, a user is assigned a contiguous block of subcarriers. This is the most common method for LTE uplink as it offers higher throughput. **Distributed SC-FDMA:** Here, a user's subcarriers are spread out over the entire signal band. This is used less often as it can result in lower throughput. **Resource allocation and scheduling:** The base station's scheduler allocates resources based on various factors: **Channel conditions:** The quality of the radio channel for each user. **User equipment (UE) capability:** What the user's device can support. **Quality of Service (QoS):** Different applications may have different requirements for minimum throughput or other performance metrics. **Constraints:** ● **Exclusivity:** A specific subchannel can only be allocated to a single user at a time to avoid interference. ● **Continuity:** In localized SC-FDMA, the allocated subcarriers must be adjacent to each other. ● **Power control:** In addition to subcarrier allocation, the network also controls the transmission power of each user's device to optimize performance and manage interference.			
8.	Explain in detail the Radio Interface Protocols in LTE.	[10]	CO3	L2
	The radio protocol architecture for LTE can be separated into **control plane** architecture and **user plane** architecture as shown below:  At user plane side, the application creates data packets that are processed by protocols such as TCP, UDP and IP, while in the control plane, the radio resource control (RRC) protocol writes the signalling messages that are exchanged between the base station and the mobile. In both cases, the information is processed by the packet data convergence protocol (PDCP), the radio link control (RLC) protocol and the medium access control (MAC) protocol, before being passed to the physical layer for transmission. User Plane The user plane protocol stack between the e-Node B and UE consists of the following sub-layers: ▪ PDCP (Packet Data Convergence Protocol)			

- RLC (radio Link Control)
- Medium Access Control (MAC)

On the user plane, packets in the core network (EPC) are encapsulated in a specific EPC protocol and tunneled between the P-GW and the eNodeB. Different tunneling protocols are used depending on the interface. GPRS Tunneling Protocol (GTP) is used on the S1 interface between the eNodeB and S-GW and on the S5/S8 interface between the S-GW and P-GW.



Packets received by a layer are called Service Data Unit (SDU) while the packet output of a layer is referred to by Protocol Data Unit (PDU) and IP packets at user plane flow from top to bottom layers.

Control Plane

The control plane includes additionally the Radio Resource Control layer (RRC) which is responsible for configuring the lower layers.

The Control Plane handles radio-specific functionality which depends on the state of the user equipment which includes two states: idle or connected.

Mode	Description
Idle	The user equipment camps on a cell after a cell selection or reselection process where factors like radio link quality, cell status and radio access technology are considered. The UE also monitors a paging channel to detect incoming calls and acquire system information. In this mode, control plane protocols include cell selection and reselection procedures.
Connected	The UE supplies the E-UTRAN with downlink channel quality and neighbour cell information to enable the E-UTRAN to select the most suitable cell for the UE. In this case, control plane protocol includes the Radio Link Control (RRC) protocol.

The protocol stack for the control plane between the UE and MME is shown below. The grey region of the stack indicates the access stratum (AS) protocols. The lower layers perform the same functions as for the user plane with the exception that there is no header compression function for the control plane.