NA | Q^n Bank

(1) Initial condition of an Inductor & Capacitor

Inductor

If inductor is present in the ckt at $t=0^-$ we should replace inductor with a wire (S.C)

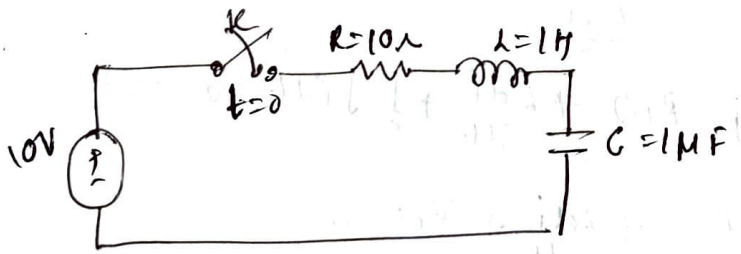
After this if we find current through inductor $i_L(0^-)$ is zero then at $t=0^+$ replace inductor with O.C instead if we find some current through inductor i.e $i_L(0^-) \neq 0$ then at $t=0^+$ replace inductor with a current source with value of $i = i_L(0^-)$

Capacitor

→ If capacitor is present in the ckt at $(t=0^-)$ replace capacitor with O.C at $t=0^+$ if we find charge or voltage across capacitor to be zero i.e $V_c(0^-)=0$ then replace capacitor wire (S.C) at $t=0^+$, if $V_c(0^-) \neq 0$, some voltage was calculated at $t=0^-$, then replace capacitor with voltage source at $t=0^+$

S.T $V_c = V_c(0^-)$

(2)



$i(0^+) = i(0^-) = 0$

at $t=0^+$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i(t)\,dt = 0 \quad (1)$$

$$Ri + L\frac{di}{dt} + V_c(t) = 0$$

putting $t=0^+$

$$Ri(0^+) + L\frac{di(0^+)}{dt} + V_c(0^+) = 10$$

$$0 + L\frac{di(0^+)}{dt} + 0 = 10$$

$$\frac{di}{dt}(0^+) = 10\ A/s$$

differentiate eq^n (1) w.r.t. t

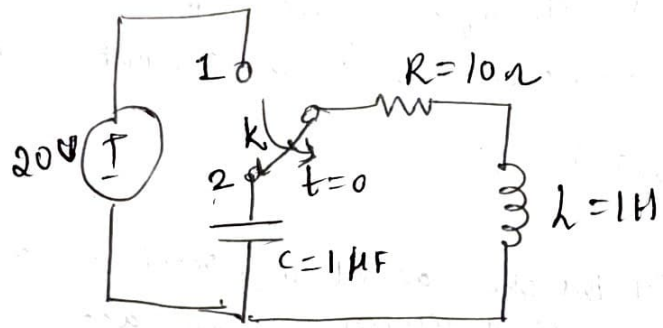
$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i(t)}{C} = 0$$

at $t=0^+$

$$R \times 10 + L\frac{d^2 i}{dt^2}(0^+) + 0 = 0$$

$$\frac{d^2 i}{dt^2} = -100\ A/s^2$$

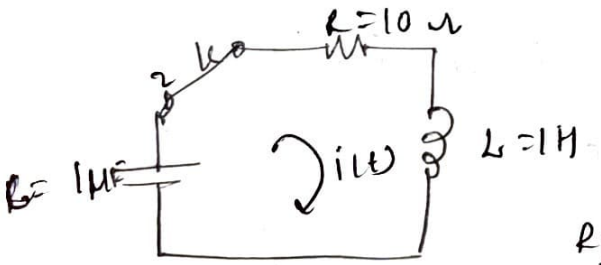
(3)



$$i(0^-) = \frac{20}{10} = 2\ A \quad (S \to 1)$$

$$i_L(0^-) = i_L(0^+) = 2\ A$$

$$v_C(0^-) = v_C(0^+) = 0\ V$$



KVL

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \quad (1)$$

$$Ri + L\frac{di}{dt} + V_C = 0$$

$$L\frac{di}{dt} = -20\ A/s$$

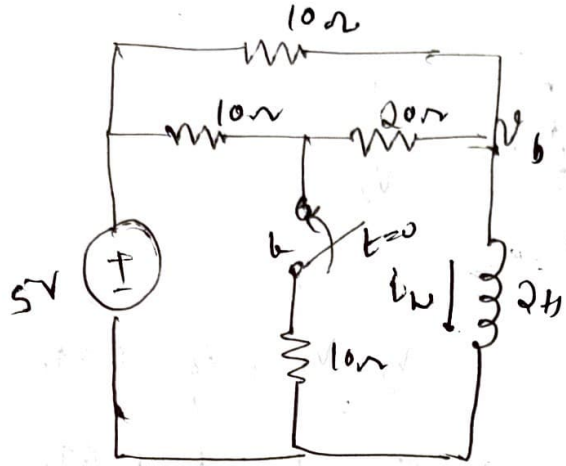
diff (1)

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \left(\frac{i}{C}\right) = 0$$

$$L\frac{d^2 i}{dt^2} = -\frac{2}{C} + 20 \times 10$$

$$\boxed{\frac{d^2 i}{dt^2} = -2 \times 10^6\ A/s^2}$$

(b)



at $t=0^+$ switch is closed

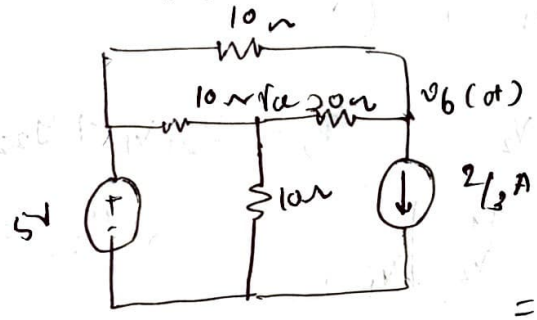
at $t=0^-$

$$\frac{5}{30} + \frac{5}{10} = \frac{2}{3}\ A$$

$i_L(0^-) = \frac{2}{3}\ A = i_L(0^+)$

$$v_a(0^-) = \frac{5 \times 20}{30} = \frac{10}{3}\ V$$

at $t=0^+$



@ node a

$$\frac{v_a(0^+) - 5}{10} + \frac{v_a(0^+)}{10} + \frac{v_a(0^+) - v_b(0^+)}{20} = 0$$

$$v_a(0^+)\left[\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right] - v_b(0^+)\left(\frac{1}{20}\right) = \frac{5}{10}$$

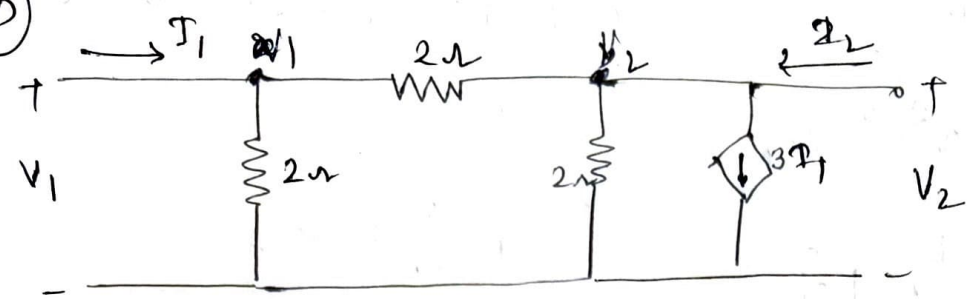
@ node b

$$\frac{v_b(0^+) - v_a(0^+)}{20} + \frac{v_b(0^+) - 5}{10} + \frac{2}{3} = 0$$

$$-v_a(0^+)\left(\frac{1}{20}\right) + v_b(0^+)\left(\frac{1}{20} + \frac{1}{10}\right) = \frac{5}{10} - \frac{2}{3}$$

$$\boxed{v_a(0^+) = \frac{40}{21}\ V}$$

(5)



$$\frac{V_1}{2} + \frac{V_1 - V_2}{2} - I_1 = 0$$

$$V_1 + V_1 - V_2 - 2I_1 = 0$$

$$2V_1 - V_2 - 2I_1 = 0 \quad -(1)$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{2} + 3I_1 = 0$$

$$V_2 - V_1 + V_2 + 6I_1 = 0$$

$$-V_1 + 2V_2 + 6I_1 = 2I_2 \quad -(2)$$

(y)

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad I_2 = y_{21}V_1 + y_{22}V_2$$

$$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$$2V_1 = 2I_1 \qquad V_2 = 2I_1$$

$$y_{11} = \frac{I_1}{V_1} = 1 \qquad \frac{I_1}{V_2} = \frac{1}{2} = y_{12}$$

$$2V_1 = 2I_1$$

$$-V_1 + 6I_1 = I_2$$

$$-V_1 = I_1 - 6I_2$$

$$+V_1 = +5I_2$$

$$\frac{I_2}{V_1} = \frac{1}{5}$$

$$-V_1 + 6V_1 = 2I_2$$

$$+5V_1 = 2I_2$$

$$y_{21} = \frac{I_2}{V_1} = +\frac{5}{2}$$

$$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

$$-V_2 = +2I_1$$

$$I_1 = -\frac{V_2}{2}$$

$$2V_2 - V_2 \times 3 = 2I_2$$

$$-V_2 = 2I_2$$

$$y_{22} = \frac{I_2}{V_2} = -\frac{1}{2}$$

$$y = \begin{bmatrix} 1 & -1/2 \\ 5/2 & -1/2 \end{bmatrix}$$

(2)

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$-V_1 + 2V_2 + 6I_1 = 0$$

$$2V_1 - 2I_1 = V_2$$

$$-V_1 + 4V_1 - 4I_1 + 6I_1 = 0$$

$$3V_1 + 2I_1 = 0$$

$$3V_1 = -2I_1$$

$$\frac{V_1}{I_1} = \frac{-2}{3}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$-V_1 + 2V_2 = 2I_2$$

$$2V_1 = V_2$$

$$V_2$$

$$-V_1 + 2V_1 = 2I_2$$

$$V_1 = 2I_2$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{2}{3}$$

$$2V_2$$

$$2 \times 2V_1$$

$$4V_1 - V_1$$

$$3V_1 = 2I_2$$

$$\frac{V_1}{I_2} = \frac{2}{3}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$-V_1 + 2V_2 + 6I_1 = 0$$

$$V_1 = 2V_2 + 6I_1$$

$$2(2V_2 + 6I_1) - V_2 - 2I_1 = 0$$

$$4V_2 + 12I_1 - V_2 - 2I_1 = 0$$

$$3V_2 + 10I_1 = 0$$

$$3V_2 = -10I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{-10}{3}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$2V_1 - V_2 = 0$$

$$2V_1 = V_2$$

$$-V_1 + 2V_2 = 2I_2$$

$$-\frac{V_2}{2} + 2V_2 = 2I_2$$

$$\frac{3}{2}V_2 = 2I_2$$

$$Z_{22} = \boxed{\left.\frac{V_2}{I_2}\right| = \frac{3}{4}}$$

$$\boxed{\begin{aligned} 2V_1 - V_2 - 2I_1 &= 0 \quad —(1) \\ -V_1 + 2V_2 + 6I_1 &= 2I_2 \quad —(2) \end{aligned}}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$2V_1 = 2I_1$$

$$\frac{V_1}{I_1} = 1 = h_{11}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$2V_1 = V_2$$

$$\boxed{\frac{V_1}{V_2} = \frac{1}{2}} = h_{12}$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$2V_1 = 2I_1$$

$$V_1 = I_1$$

$$-I_1 + 0 + 6I_1 = 2I_2$$

$$5I_1 = 2I_2$$

$$h_{21} = \frac{I_2}{I_1} = \frac{5}{2} =$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$2V_1 = V_2 \quad \text{let } V_1 = V_2/2$$

$$-\frac{V_2}{2} + 2V_2 = 2I_2$$

$$\frac{3V_2}{2} = 2I_2$$

$$h_{22} = \frac{I_2}{V_2} = \boxed{\frac{3}{4}}$$

(4)

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$-V_1 + 2V_2 = -6I_1$$

$$I_1 = \frac{-(-V_1 + 2V_2)}{6}$$

$$2V_1 - V_2 + \frac{1}{3}(-V_1 + 2V_2) = 2V_1 - V_2 - \frac{V_1}{3} + \frac{2}{3}V_2 = 0$$

$$V_1\left(2 - \frac{1}{3}\right) + V_2\left(-1 + \frac{2}{3}\right) = 0$$

$$V_1\left(\frac{5}{3}\right) = V_2\left(\frac{+1}{3}\right)$$

$$A = \boxed{\frac{V_1}{V_2} = \frac{1}{5}}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$2V_1 = 2I_1$$

$$I_1 = V_1$$

$$-V_1 + 6I_1 = 2I_2$$

$$5V_1 = 2I_2$$

$$B = \boxed{\frac{V_1}{I_2} = \frac{-2}{5}}$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$2V_1 = 2I_1$$

$$V_1 = I_1$$

$$-I_1 + 6I_1 = 2I_2$$

$$5I_1 = 2I_2$$

$$D = \boxed{\frac{I_1}{I_2} = -\frac{2}{5}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$V_1 = 2V_2 + 6I_1$$

$$2(2V_2 + 6I_1) - V_2 - 2I_1 = 0$$

$$4V_2 + 12I_1 - V_2 - 2I_1 = 0$$

$$3V_2 + 10I_1 = 0$$

$$3V_2 = -10I_1$$

$$C = \frac{I_1}{V_2} = \boxed{\frac{-3}{10}}$$

(6) $\underline{\underline{h}}$ in terms of $\underline{\underline{Z}}$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$$

$$-Z_{21} I_1 = Z_{22} I_2$$

$$I_2 = \left(\frac{-Z_{21}}{Z_{22}}\right) I_1$$

$$V_1 = Z_{11} I_1 + Z_{12}\left(\frac{-Z_{21}}{Z_{22}}\right) I_1$$

$$V_1 = I_1\left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}}\right)$$

$$h_{11} = \frac{V_1}{I_1} = \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}}\right)$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

$$V_1 = Z_{12} I_2$$
$$I_2 = \frac{V_1}{Z_{12}}$$

$$V_2 = Z_{22}\left(\frac{V_1}{Z_{12}}\right)$$

$$\boxed{\frac{V_1}{V_2} = \frac{Z_{12}}{Z_{22}}} = h_{12}$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$$

$$Z_{21} I_1 = -Z_{22} I_2$$

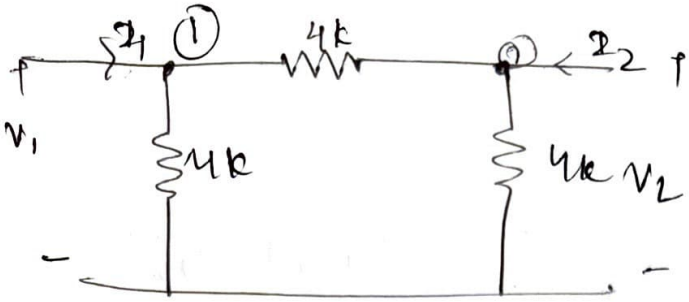
$$h_{21} = \boxed{\frac{I_2}{I_1} = -\frac{Z_{21}}{Z_{22}}}$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

$$V_2 = 0 + Z_{22} I_2$$

$$\boxed{\frac{I_2}{V_2} = \frac{1}{Z_{22}}}$$

(7)



$$\frac{V_1 - V_2}{4} + \frac{V_1}{4} - I_1 = 0$$

$$V_1 - V_2 + V_1 - 4I_1 = 0$$

$$2V_1 - V_2 - 4I_1 = 0$$

$$\frac{V_2 - V_1}{4} + \frac{V_2}{4} - I_2 = 0$$

$$-V_1 + 2V_2 - 4I_2 = 0$$

$$V_1 = AV_2 - BI_2 \qquad I_1 = CV_2 - DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad 2V_2 = V_1$$

$$A = \boxed{\frac{V_1}{V_2} = \frac{2}{1}}$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$-V_1 = 4I_2$$

$$\frac{V_1}{I_2} = 4\,\Omega = B$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad -V_1 + 2V_2 = 0$$

$$2V_2 = V_1$$

$$2V_2 - V_2 - 4I_1 = 0$$

$$3V_2 = 4I_1 \qquad \boxed{\frac{I_1}{V_2} = \frac{3}{4}\,\mho}$$

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$2V_1 - 4I_1 = 0$$

$$2V_1 = 4I_1$$

$$V_1 = 2I_1$$

$$-2I_1 - 4I_2 = 0$$

$$-2I_1 = 4I_2$$

$$\frac{I_1}{I_2} = \frac{4}{2} = -2$$

$$= \boxed{2}$$

$2V_1 - V_2 - 4I_1 = 0$

$-V_1 + 2V_2 - 4I_2 = 0$

$I_1 = y_{11}V_1 + y_{12}V_2 \qquad I_2 = y_{21}V_1 + y_{22}V_2$

$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$

$2V_1 = 4I_1 \qquad -V_2 = 4I_1$

$\frac{I_1}{V_1} = \frac{2}{4} = \underline{\underline{0.5}} \times 10^{-3} \qquad \frac{I_1}{V_2} = \frac{-1}{4}$

$= -0.000259$

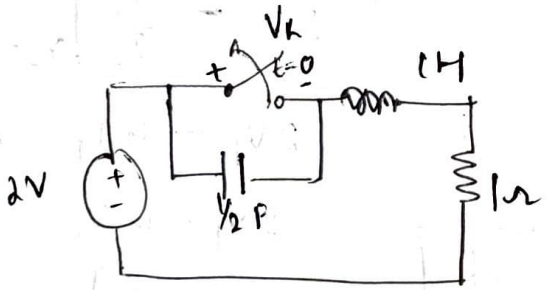
$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

$-V_1 = 4I_2 \qquad 2V_2 = 4I_2$

$-V_1 = 4I_2 \qquad \frac{I_2}{V_2} = 1/2 = 0.00065$

$\frac{I_2}{V_1} = -1/4 = -0.000255$

(8)



at $t = 0^+$ $V_k(0^+) = V_k(0^-) = 0V$

$V(k) = V(c)$

$V(k) = \frac{1}{C}\int i\,dt$

$\frac{dV}{dt} = \frac{1}{C} i(t)$

$i(t) = C\frac{dV(c)}{dt}$

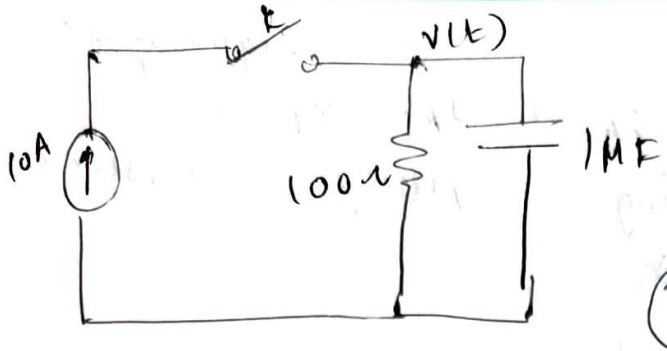
$i(0^+) = C\frac{dV_k(0^+)}{dt}$

$i(0^+) = i(0^-) = \underline{\underline{2A}}$

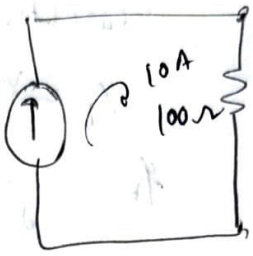
$2 = 1/2 \frac{dV_k(0^+)}{dt}$

$\boxed{\frac{dV(c)}{dt} = 4V/s}$

(4)



at $t=0^-$



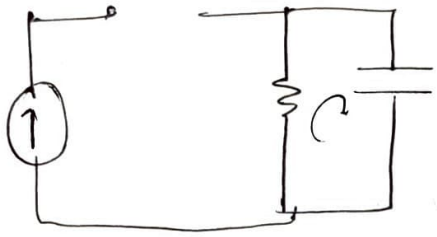
$V(t) = 100 \times 10$

$= \underline{1000V}$

$V(t)\,0^+ = V_{4}(0^-)$

$= \underline{1000V}$

at $t=0^+$



$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{1}{RC}v = 0$$

the coefficient of $v = 1/\tau$

$$\tau = RC$$

$$v(t) = v(0^+)\,e^{-t/\tau}$$

$\tau = 100 \times 1MF \; (R \times C) = 10^{-4}s$

$$v(t) = v(0^+)\,e^{-t/RC}$$

$$= \boxed{1000\,e^{-10000t}\,V}$$

$$\left.\frac{dv}{dt}\right|_{0^+} = -\frac{1}{R\tau} \cdot 1000\,e^{-t/RC}$$

$$= -\frac{1000}{10^{-4}} = \boxed{-1\times10^{-7}\,V/s}$$

$$\left.\frac{d^2v}{dt^2}\right|_{0^+} = \frac{1000}{(RC)^2}\,e^{-t/RC}$$

$$= \frac{1000}{(10^{-4})^2} = \boxed{1\times10^{11}\,V/s^2}$$

(10) **resonant frequency** :- is the frequency at which the inductive reactance equal the capacitive reactance in an RLC ckt resulting in purely resistive impedance

$$X_L = X_C$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

→ impedance is min (series RLC)

→ " max (" RLC)

→ power factor = 1

ii) **Quality factor** (Q-factor)

Q-factor is a measure of the sharpness of resonance and indicates how selective a ckt is

$$Q = 2\pi \times \frac{\text{Energy stored}}{\text{Energy dissipated by RLC ckt}}$$

$$Q = \frac{X_L}{R} \text{ or } \frac{X_C}{R} \quad \text{for } R_{LC} = \frac{|X_L - X_C|}{R}$$

iii) **Selectivity** :- is the ability of a resonant ckt to accept a desired frequency and reject all other frequency

→ ratio of $\boxed{\frac{f_0}{\text{bandwidth}}} = \frac{f_0}{f_2 - f_1}$ → bandwidth $= \frac{R}{2\pi L}$

$$\frac{f_0}{R} \times 2\pi L = \text{selectivity} = \frac{\omega_0 L}{R} = Q_{RL}$$

w.k.T $f_r = \frac{1}{2\pi\sqrt{LC}}$

$$\text{Select} = \frac{2\pi}{2\pi} \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{\sqrt{L}}{\sqrt{C}} \times \frac{1}{R}$$

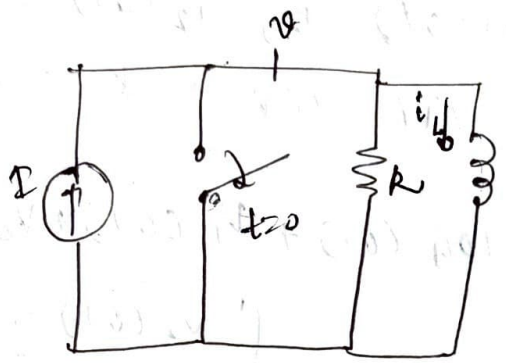
$$= \boxed{\frac{1}{R}\sqrt{\frac{L}{C}}}$$

IV) **Bandwidth** I is the range of freq over which the ckt operates effectively, i.e, b/w the half power (-3 dB) freq

$$BW = f_2 - f_1$$

$$\boxed{f_2 - f_1 = \frac{R}{2\pi L}}$$

11



$i_L(0^-) = i_L(0^+) = 0$

apply KCL at node v(t)

$$I = \frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt$$

diff. w.r.t t

$$0 = \frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t)$$

$v(0^+) = IR = 2\times 200 = 400\,V$

at $t = 0^+$

$$\frac{1}{R}\frac{dv(0^+)}{dt} + \frac{1}{L}v(0^+) = 0$$

$$\frac{1}{200}\frac{dv(0^+)}{dt} + \frac{1}{1}\times 400 = 0$$

$$\frac{dv(0^+)}{dt} = -8\times 10^4\ V/s$$

diff w.r.t.

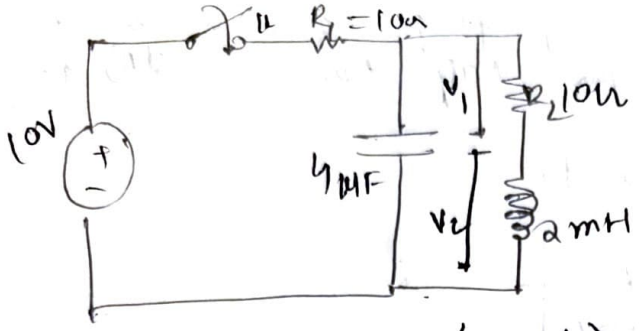
$$\frac{1}{R}\frac{d^2v}{dt^2} + \frac{1}{L}\frac{dv}{dt} = 0$$

$$\frac{1}{200}\frac{d^2v}{dt^2} + \frac{1}{1}\times(-8\times 10^4) = 0$$

12

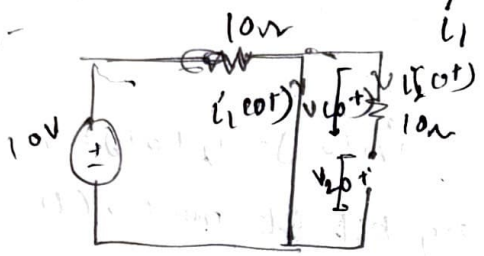
$$\frac{d^2v}{dt^2} = 200\times 8\times 10^4 = 16\times 10^6\ V/s^2$$

12



at $t = 0^-$ S - open

at $t = 0^-$



$$i_1(0^+) = \frac{10}{10} = 1A$$

$$V_1(0^+) = 0, \quad i_2(0^+) = 0$$

Apply KVL 10V → K → 10Ω → 10Ω → 2m

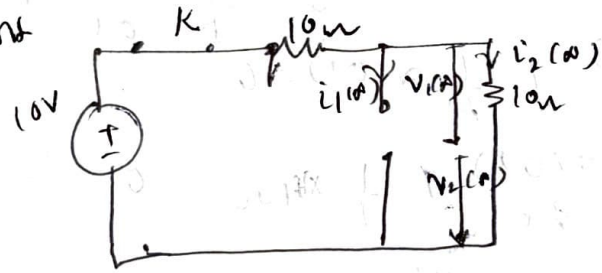
$$-10 + 10i_1(0^+) + V_1(0^+) + V_c(0^+) = 0$$

$$\boxed{V_2(0^+) = 0}$$

(b) at $t = \infty$

C → open

L → short



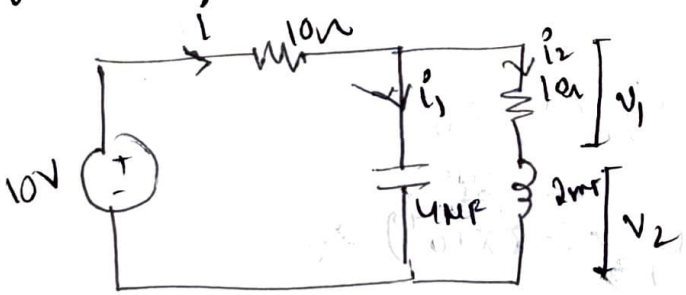
$$i_2(\infty) = \frac{10}{20} = 0.5A$$

$$i_1(\infty) = 0\ A$$

$$V_1(\infty) = 0.5 \times 10 = 5V$$

$$V_2(\infty) = 0V$$

(c) $t \geq 0^+$



$$i_2 = \frac{1}{L}\int_{0^+}^{t} V_2 = \frac{V_1(t)}{R_2}$$

diff.

$$\frac{V_2}{L} = \frac{dV_1}{d(t)} \times \frac{1}{R}$$

$$\frac{dV_1}{d(t)} = 0 \text{ V/sec}$$

apply KVL 10V → K → 10Ω → 4μF

$$-10 + 10i + \frac{1}{C}\int_{0^+}^{t} (i(\tau) - i_2(\tau))\,d\tau = 0$$

diff w.r.t t

$$10\frac{di}{dt} + \frac{1}{C}[i - i_2] = 0$$

(c) $t = 0^+$

$$\frac{di(0^+)}{dt} = \frac{i_2(0^+) - i(0^+)}{C \times 10}$$

$$= \frac{0-1}{10 \times 4 \times 10^{-6}} = -25000 \text{ A/sec.}$$

apply KVL $\quad 10\text{V} \rightarrow R \rightarrow 10\Omega \rightarrow 10\Omega \rightarrow 2\text{mH}$

$$-10 + 10i + 10i_2 + v_L = 0$$

$$10i + v_1 + v_2 = 10$$

diff

$$10\frac{di}{dt} + \frac{dv_1}{dt} + \frac{dv_L}{dt} = 0$$

(c) $t = 0^+$

$$10(-25000) + 0 + \frac{dv_L(0^+)}{dt} = 0$$

$$\frac{dv(0^+)}{dt} = 25 \times 10^4 \text{ V/sec}$$

(d) from part (c)

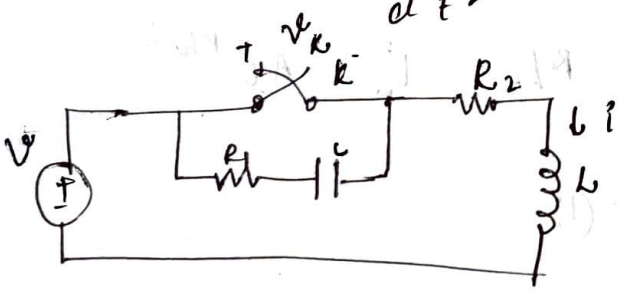
2 times differentiation

$$\frac{1}{L}\int v_2(t)\,dt = \frac{v_1}{10}$$

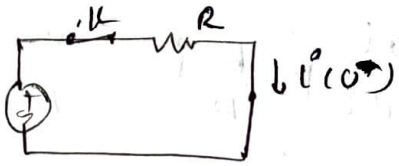
$$\frac{1}{L}\frac{dv_2}{dt} = \frac{1}{10}\frac{d^2 v_1}{dt^2}$$

$$\frac{d^2 v_L(0^+)}{dt^2} = 125 \times 10^7 \text{ V/s}^2$$

(1g)



at $t = 0^-$



at $t = 0^+$ $\quad i(0^+) = \frac{v}{R_2}$ & $V_L(0^-) = 0$

$$v_k = R_1 i + \frac{1}{C}\int i\,dt$$

$$v_k = R_1 i + v_c$$

$$= \frac{v \times R_1}{R_2} + 0$$

$$= \boxed{\frac{v R_1}{R_2} \text{ V}}$$

(b) $v_k = R_1 i + \frac{1}{C}\int i\,dt$ dx

$$\frac{dv_k}{dt} = R\frac{di}{dt} + \frac{i}{C}$$

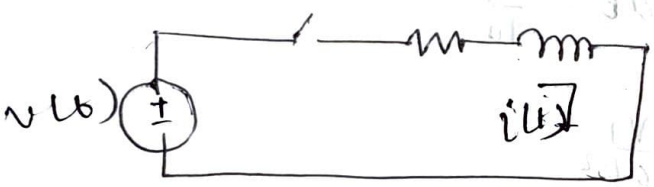
at $t=0^+$

$$\frac{dv_k}{dt} = R_1\frac{di}{dt} + \frac{i(0^+)}{C}$$

$$= R_1(-1) + \frac{1}{C}$$

$$= \frac{1}{C}(-R_1) \ \underline{\underline{V/S}}$$

(14) $\boxed{\text{Analysis RL CKT}}$



apply KVL

$$Ri + L\frac{di}{dt} = v$$

std equ

$$\frac{dy}{dt} + ay = x$$

$$a = R/L \qquad i_N = Ae^{-R/L\,t}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{v}{L} \quad -(1)$$

Sol^n for i

$$i = i_f + i_N$$

i_f is due to forcing fn $\frac{v}{L}$

Let us $v = V_m \sin\omega t$ [AC Excitation]

$$i = \underbrace{i_f \text{ due to } V_m \sin\omega t}_{\text{from standard set of formulas}} + Ae^{-(R/L)t} \quad -(3)$$

$$i_f = D\cos\omega t + E\sin\omega t \quad —(4)$$

so considering Eq(1) and $i \to i_f$,

$$\frac{di_f}{dt} + \frac{R}{L} i_f = \frac{v}{L} \quad —(5)$$

$$\frac{d}{dt}(D\cos\omega t + E\sin\omega t) + \frac{R}{L}(D\cos\omega t + E\sin\omega t) = \frac{v_m}{L}\sin\omega t$$

$$-D\omega\sin\omega t + E\omega\cos\omega t + \frac{R}{L}D\cos\omega t + \frac{RE}{L}\sin\omega t = \frac{v_m}{L}\sin\omega t$$

to find D & E

Equate the coefficient of similar terms

$\sin\omega t$ $\quad -D\omega + \frac{RE}{L} = \frac{v_m}{L} \quad —(5)$ $\quad D = -\frac{v_m\omega L}{(\omega^2L^2+R^2)}$

$$E\omega + \frac{RD}{L} = 0 \quad —(6)$$

$$E = -\frac{RD}{\omega L} \qquad E = \frac{Rv_m}{\omega^2L^2+R^2}$$

$$i_f = \left(\frac{-v_m\omega L}{\omega^2L^2+R^2}\right)\cos\omega t + \left(\frac{v_mR}{\omega^2L^2+R^2}\right)\sin\omega t$$

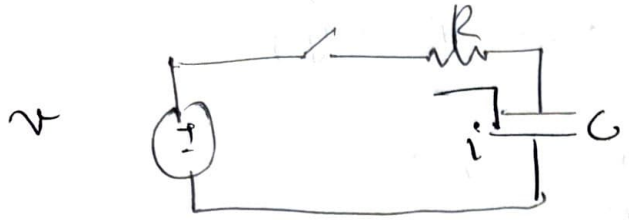
Actual soln for the current is

$$i = \frac{-v_m}{(\omega^2L^2+R^2)}\left[\omega L\cos\omega t - R\sin\omega t\right] + Ae^{-R/L\,t}$$

how to

(18)

R-C



$v = V_0 \sin\omega t$

$v = Ri + \frac{1}{C}\int i\,dt$

Eqs $\frac{dy}{dt} + ay = x$

$$v = Ri + \frac{1}{C}\int i\,dt \quad \text{—(1)}$$

w.r.t t

$$\frac{d}{dt}(V_0 \sin\omega t) = R\frac{di}{dt} + \frac{1}{C}\frac{d}{dt}\left[\int i\,dt\right]$$

$$V_m \omega \cos\omega t = R\frac{di}{dt} + \frac{i}{C}$$

$$\frac{di}{dt} + \frac{i}{RC} = \frac{V_m \omega}{R}\cos\omega t \quad \text{—(2)}$$

$i = i_f + i_N \qquad i_N = Ae^{-t/RC}$

$i_f = [D\cos\omega t + E\sin\omega t]$ → standard set of necessary choice

to find D, E substitute i_f in eq (2)

$$\frac{di_f}{dt} + \frac{i_f}{RC} = \frac{V_m \omega}{R}\cos\omega t$$

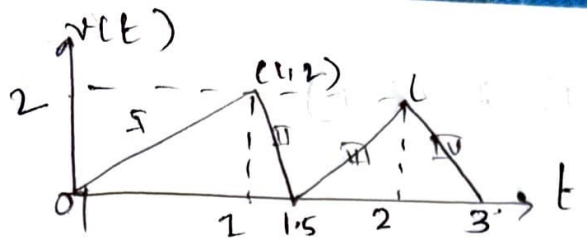
$$-\omega D\sin\omega t + \omega E\cos\omega t + \frac{1}{RC}[D\cos\omega t + E\sin\omega t] = \frac{V_m \omega}{R}\cos\omega t$$

$$\boxed{-\omega D + \frac{E}{RC} = 0} \qquad \boxed{\omega E + \frac{D}{RC} = \frac{V_m \omega}{R}}$$

$$D = \frac{V_m \omega C}{1 + R^2C^2\omega^2} \qquad E = \frac{V_m \omega^2 C^2 R}{1 + R^2C^2\omega^2}$$

$$i = \frac{V_m \omega C}{(1 + R^2C^2\omega^2)}\left[\cos\omega t + \omega CR\sin\omega t\right] + Ae^{-t/RC}$$

(16)



$$m = \frac{2-0}{1-0} = 2$$

$$m_2 = \frac{-2}{1.5-1} = \frac{-2}{0.5} = -4$$

$$m_3 = \frac{2}{0.5} = 4$$

$$m_4 = \frac{-2}{1} = -2$$

$f_1(t) = 2t$

$f_2(t) = -4(t-1)+2$

$f_3(t) = 4(t-1.5)$

$f_4(t) = -2(t-2)+2$

$-4t + 6$

$-4(t-1.5)$

$-4(t-1.5)$

$$= 2t[u(t)-u(t-1)] + [-4(t-1)+2][u(t-1)-u(t-1.5)]$$
$$+[4(t-1.5)][u(t-1.5)-u(t-2)] + [-2(t-2)+2][u(t-2)-u(t-3)]$$

(1) $2[t\,u(t) - (t-1+1)\,u(t-1)]$

$\frac{2}{s}$ $2[t\,u(t) - (t-1)\,u(t-1) - u(t-1)]$

$$= 2\left[\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}\right]$$

(2) $[-4(t-1)+2][u(t-1) - u(t-1.5)]$

$$-4\left[\frac{e^{-s}}{s^2} + (t-1.5+0.5)\,u(t-1.5)\right] + 2u(t-1) - 2u(t-1.5)$$

$$-4\left[\frac{e^{-s}}{s^2} + \frac{e^{-1.5s}}{s^2} - \frac{0.5e^{-1.5}}{s}\right] + \left[\frac{2e^{-s}}{s} - \frac{2e^{-1.5}}{s}\right]$$

term1 $2tu(t) - 2(t-1)u(t-1) - 2u(t-1)$

term-2

$-4(t-1)u(t-1) + 2u(t-1) + 4(t-1.5)u(t-1.5) - 2u(t-1.5)$

term-3

$4(t-1.5)[u(t-1.5) - u(t-2)]$

$\cancel{4(t-1.5)u(t-1.5) - 4(t-2)u(t-2)}$

$4[(t-1.5)u(t-1.5) - (t-2+0.5)(u(t-2))]$

$4[(t-1.5)u(t-1.5) - (t-2)u(t-2) - 0.5u(t-2)]$

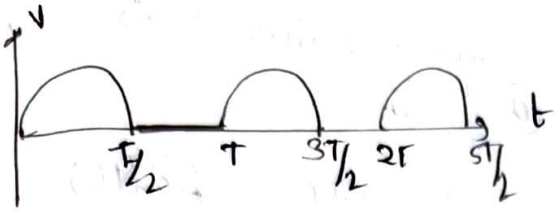
term4 $\{-2(t-2)+2\}[u(t-2) - u(t-3)]$

$-2[(t-2)u(t-2) - (t-3)u(t-3) - u(t-3)] + 2u(t-2) - 2u(t-3)]$

$-2(t-2)u(t-2) + 2(t-3)u(t-3) + \cancel{2u(t-3)} + 2u(t-2) - \cancel{2u(t-3)}$

$$= \frac{2}{s^2}[1 - 3e^{-s} + 4e^{-1.5s} - 3e^{-2s} + e^{-3s}]$$

(16)



$$f_1(t) = \begin{cases} V \sin(\omega t) & 0 < t < T/2 \\ 0 & T/2 < t < T \end{cases}$$

$$\omega = 2\pi/T$$

$$R(s) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

$$\int_0^{T/2} e^{-st}\, V \sin\omega t \, dt$$

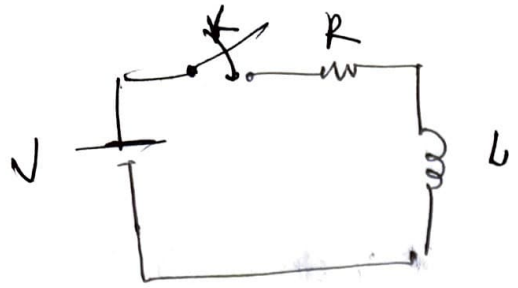
$$\left[\frac{Ve^{-st}}{s^2+\omega^2} (-s \sin\omega t - \omega \cos\omega t) \right]_0^{T/2}$$

$$V \left[\frac{e^{-sT/2}}{s^2+\omega^2} (0 - \omega(-1)) - \frac{1}{s^2+\omega^2} (0 - \omega) \right]$$

$$\frac{V\omega}{s^2+\omega^2} \left(e^{-sT/2} + 1 \right)$$

$$= \boxed{\frac{V\omega}{(s^2+\omega^2)(1-e^{-sT})} \left(1 + e^{-sT/2} + 1 \right)}$$

(12)



when switch k is closed @ $t=0$

$$V = Ri(t) + L\frac{di(t)}{dt}$$

$$i(0^-) = 0$$

take L.T on both sides

$$\frac{V}{s} = RI(s) + L[sI(s) - i(0)]$$

$$\frac{V}{s} = I(s)[R + Ls]$$

$$I(s) = \frac{V}{s(R+Ls)}$$

$$I(s) = \frac{V/L}{s(s+R/L)}$$

using partial fraction

$$\frac{V/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+R/L}$$

$$V/L = A(s+R/L) + B(s)$$

put $s=0$ $\quad A = \frac{V}{L} \times \frac{L}{R} = \boxed{\frac{V}{R}}$

multiply by $s+R/L$ Let $s = -R/L$

$$V/L = 0 + B(-R/L)$$

$$\boxed{B = -V/R}$$

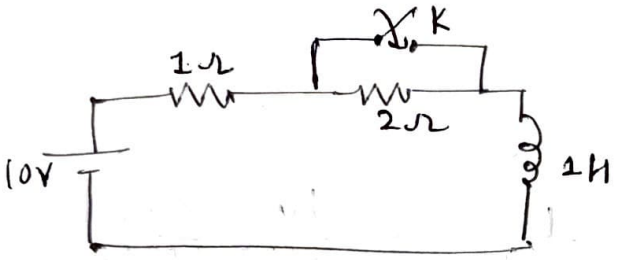
$$I(s) = \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

taking inverse L.T for $i(t)$

$$i(t) = L^{-1}\left\{ \frac{V}{R}\left(\frac{1}{s} - \frac{1}{s+R/L}\right)\right\}$$

$$i(t) = \boxed{\frac{V}{R}\left(1 - e^{-(R/L)t}\right)}$$

18



→ at $t=0^-$

$R_{eq} = 3\Omega$

$i(0^-) = \frac{10}{3}$ A

$i(0^+) = \frac{10}{3}$ A

for $t>0$

$R = 1\Omega$

$$10u(t) = Ri(t) + L\frac{di(t)}{dt}$$

L.T

$$\frac{10}{s} = Ri(s) + L(sI(s) - I(0))$$

$$= \cancel{RI(s)} + \cancel{LsI(s)}$$

$$\cancel{\frac{10}{s}} = \cancel{I(s)(R+Ls)}$$

$$\cancel{I(s) = \frac{10}{s(R+Ls)} = \frac{10/L}{}}$$

$$\frac{10}{s} = RI(s) + LsI(s) - L\left(\frac{10}{3}\right)$$

$$\frac{10}{s} + \frac{10L}{3} = I_s(Ls+R)$$

$$I(s) = \frac{10}{s(Ls+R)} + \frac{10L}{3(Ls+L)}$$

$$I(s) = \frac{10/L}{s(s+R/L)} + \frac{10/3}{(s+R/L)}$$

$$\frac{10}{s(s+1)} + \frac{10/3}{s+1}$$

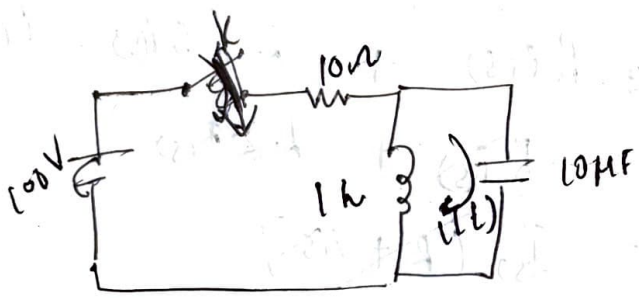
$$\hookrightarrow \frac{10}{s} - \frac{10}{(s+1)}$$

$$I(s) = \frac{10}{s} - \frac{10}{s+1} + \frac{10/3}{s+1}$$

$$\frac{10}{s} + \left(\frac{10}{3} + 10\right)\frac{1}{s+1} = \frac{10}{s} - \frac{20}{3}\cdot\frac{1}{s+1}$$

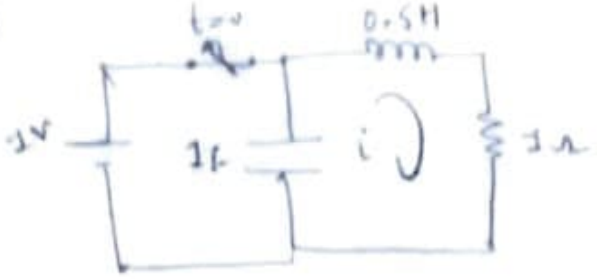
$$\boxed{i(t) = 10 - \frac{20}{3}e^{-t}}$$

19



at $t=0^-$ $\frac{V}{R} = \frac{100}{10} = 10A$

[20]

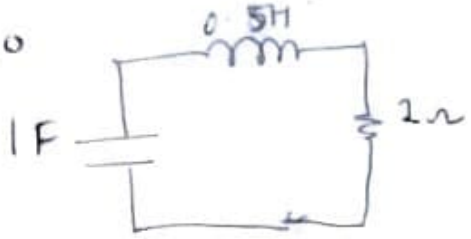


at $t = 0^-$

$i_L = \frac{1}{2} = \boxed{1A}$ $i_L(0^+)$

$v_c(0^-) = \underline{\underline{1V}} = V_c(0^+)$

at $t > 0$



$$0 = \frac{1}{C}\int i\,dt + L\frac{di}{dt} + Ri$$

$$= \frac{1}{CS}I_s + \frac{V_c(0)}{S} + L[I_s S - I_L(0)] + RI_s$$

$$0 = \frac{I(s)}{S} + \frac{1}{S} + I_s + 0.5sI_s - 0.5$$

$$\cancel{I_s} \quad \frac{I_s}{S} + I(s) + 0.5sI(s) = 0.5$$

$$I(s) + sI(s) + 0.5s^2 I(s) = 0.5s$$

$$I(s) = \frac{0.5s}{0.5s^2 + s + 1}$$

$$= \frac{s}{s^2 + 2s + 2}$$

$$\frac{s+1-1}{(s+1)^2+1} = \frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}$$

$$= e^{-t}\cos t - e^{-t}\sin t$$

$$\boxed{e^{-t}(\cos t - \sin t)}$$

21

Initial value theorem

The initial value theorem allows us to find the initial value $x(0)$ directly from L.T $X(s)$ → time domain

$$x(0) = \lim_{s \to \infty} s X(s)$$

Proof

$$L\left\{\frac{d}{dt}(x(t))\right\} = s X(s) - x(0)$$

$$X(s) = L\{x(t)\} = \int_0^\infty x(t) e^{-st} dt \quad (1)$$

$$L\left\{\frac{d}{dt}(x(t))\right\} = s X(s) - x(0) = \int_0^\infty \frac{d}{dt} x(t) e^{-st} dt$$

apply limit $s \to \infty$ to both the sides

$$\lim_{s \to \infty} [s X(s) - x(0)] = \lim_{s \to \infty} \frac{d}{dt} x(t) e^{-st} dt$$

$$\lim_{s \to \infty} s X(s) - x(0) = 0$$

$$x(0) = \lim_{s \to \infty} s X(s)$$

Final value theorem

allows us to find the final value $\lim_{t \to \infty} x(t)$ directly from L.T

If $x(t)$ is a causal signal then

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} s X(s)$$

Proof using time differentiation

$$L\left\{\frac{d}{dt} x(t)\right\} = s X(s) - x(0) \quad (1)$$

$$= \int_0^\infty \frac{d}{dt} x(t) e^{-st} dt = s X(s) - x(0) \quad (2)$$

$$\lim_{s \to 0} \int_0^\infty \frac{d}{dt} x(t) e^{-st} = \lim_{s \to 0} [s X(s) - x(0)]$$

$$\int_0^\infty \frac{d}{dt} x(t)\,dt = \lim_{s\to 0} sX(s) - x(0)$$

$$[x(t)]_0^\infty = \lim_{s\to 0} sX(s) - x(0)$$

$$x(\infty) - \cancel{x(0)} = \lim_{s\to 0} sX(s) - \cancel{x(0)}$$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,X(s)$$

$\boxed{22}$ L.T a) unit step fn b) $\sin\omega t$ c) $\cosh at$ d) $t\cdot\cos(at)$

a) $\boxed{\text{unit step } f^n}$

$$= \int_0^\infty e^{-st} f(t)\,dt$$

$$F(s) = \int_0^\infty 1\cdot e^{-st}\,dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= 0 - \frac{e^{-0}}{-s} = \boxed{\frac{1}{s}}$$

b) $\sin(\omega t)$

$$\cancel{\int_0^\infty \frac{e^{-st}}{s^2+\omega^2}(-s}\quad \int_0^\infty e^{-st}\sin\omega t\cdot dt$$

$$= \left(\frac{e^{-st}}{s^2+\omega^2}(-s\sin\omega t - \omega\cos\omega t)\right)_0^\infty$$

$$= e^{-\infty} - \frac{e^0}{s^2+\omega^2}(-0-\omega) = \boxed{\frac{\omega}{s^2+\omega^2}}$$

c) $\cosh at$ $\frac{1}{2}(e^{at}+e^{-at})$

$$= \frac{1}{2}\{L(e^{at}) + L\{e^{-at}\})$$

$$= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right) = \frac{1}{2}\left(\frac{(s+a)+(s-a)}{s^2-a^2}\right)$$

$$= \boxed{\frac{s}{s^2-a^2}}$$

d) $t \cdot \cos at$

for this we use the differentiation in s-domain

$$\mathcal{L}\{t \cdot f(t)\} = -\frac{d}{ds} F(s)$$

$$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$$

$$= -\frac{d}{ds}\left(\frac{s}{s^2+a^2}\right)$$

$$= -\left[\frac{(s^2+a^2)1 - s(2s)}{(s^2+a^2)^2}\right] = -\left[\frac{s^2+a^2-2s^2}{(s^2+a^2)^2}\right]$$

$$= -\left[\frac{a^2-s^2}{(s^2+a^2)^2}\right] = \frac{s^2-a^2}{(s^2+a^2)^2}$$

$\boxed{23}$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \int_0^\infty f(t)\, e^{-st}\, dt$$

$$\frac{d}{ds} F(s) = \int_0^\infty f(t) \left[\frac{d}{ds} e^{-st}\right] dt$$

$$\frac{d}{ds} F(s) = \int_0^\infty f(t)\, (-t\, e^{-st})\, dt$$

$$\frac{d}{ds} F(s) = -\int_0^\infty (t f(t))\, e^{-st}\, dt$$

the $\int_0^\infty [t f(t)] e^{-st} dt$ is by definition the L.T of the fn $t \cdot f(t)$ ∴

$$\frac{d}{ds} F(s) = -\mathcal{L}\{t \cdot f(t)\}$$

$$\boxed{\mathcal{L}\{t \cdot f(t)\} = -\frac{d}{ds} F(s)}$$

(29) Shifting in 's' Domain

If $\mathcal{L}\{f(t)\} = F(s)$, then multiplying the time domain fn by an exponential e^{at} results in a shift of a in the s-domain

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

Proof

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

$$\mathcal{L}\{e^{at} f(t)\} = \int_0^\infty [e^{at} f(t)]\, e^{-st} dt$$

$$= \int_0^\infty f(t)\, e^{-(s-a)t} dt$$

$$\therefore \mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

ii) Shifting in time (second shifting theorem)

→ If $\mathcal{L}\{f(t)\} = F(s)$ then fn that is delayed in time by a units and "turned on" at $t = a$ has the following transform

$$L\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

$$L\{f(t-a)u(t-a)\} = \int_0^\infty f(t-a)u(t-a)e^{-st}dt$$

$\because$ $u(t-a)=0$ for $t<a$

$$= \int_a^\infty f(t-a)e^{-st}dt$$

Let $x = t-a$ then $dt = dx$ when $t=a, x=0$ when $t=\infty, x=\infty$

put $t = x+a$

$$\int_0^\infty f(x)e^{-s(x+a)}dx$$

$$= e^{-as}\int_0^\infty f(x)e^{-sx}dx$$

$$L\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

25 $\boxed{\text{L.T of differentiation}}$

State $L f(t) = F(s)$ then the L.T of $f'(t)$ is

$$L\{f'(t)\} = sF(s) - f(0)$$

proof

$$L\{f'(t)\} = \int_0^\infty f'(t)e^{-st}dt \rightarrow \int u\,dv = uv - \int v\,du$$

$$= [e^{-st}f(t)]_0^\infty - \int_0^\infty f(t)(-se^{-st})dt$$

$$[0 - f(0)] + s\int_0^\infty f(t)e^{-st}dt$$

$$L\{f'(t)\} = sF(s) - f(0)$$

② **L.T of integration**

Statement: L.T of integral of a fn is given by

$$L\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

from differen

$$L\{g'(t)\} = sG(s) - g(0)$$

put $g'(t) = f(t)$ & $g(0)=0$

$$L\{f(t)\} = sG(s) - 0$$

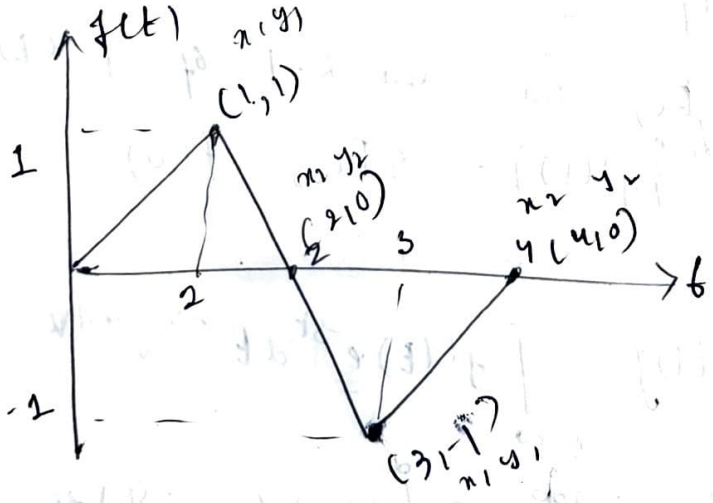
$$F(s) = sG(s)$$

$$G(s) = \frac{F(s)}{s}$$

$$L\int_0^t f(t)\,dt = \frac{F(s)}{s}$$

ans

26



$$f = t(u(t) - u(t-1))$$

$$= tu(t) - (t-1)u(t-1) - \cancel{u(t-1)}$$

$$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} \quad -(1)$$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-1}{0-1} = \frac{x-1}{2-1}$$

$$x(t)-1 = \frac{t-1}{1}$$

$$x(t) = t-1+1$$

$$\frac{y-1}{-1-1} = \frac{x-1}{3-1}$$

$$\frac{y-1}{-2} = \frac{x-1}{2}$$

$$2(y-1) = -2x+2$$

$$2y-2 = -2x+2$$

$$2y = -2x+4$$

$$y = -x+2$$

$$x(t) = -t+2$$

$$(-t+2)\,u(t-1) \qquad (-t+2)\big(u(t-1)-u(t-3)\big)$$

$$--(2-t) - (t-2)\big(u(t-1)-u(t-3)\big)$$

$$= -(t-2+1-1)\big(u(t-1)-u(t-3)\big)$$

$$-[(t-1-1)\,[u(t-1)-u(t-3)]$$

$$-[(t-1)\,u(t-1) - u(t-1) - (t-3+1)\,u(t-3)]$$

$$-\big((t-1)\,u(t-1) - u(t-1 - (t-3)\,u(t-3) - u(t-3)\big)$$

$$-(t-1)\,u(t-1) + \cancel{u(t-1)} + (t-3)\,u(t-3) + \cancel{u(t-3)}$$

III

$$\frac{y+1}{0+1} = \frac{x-3}{4-3} \qquad y = x-4$$

$$y+1 = \frac{x-3}{1}$$

$$y+1 = x-3$$

$$y = x-4 \qquad = (t-4)\big(u(t-3)-u(t-4)\big)$$

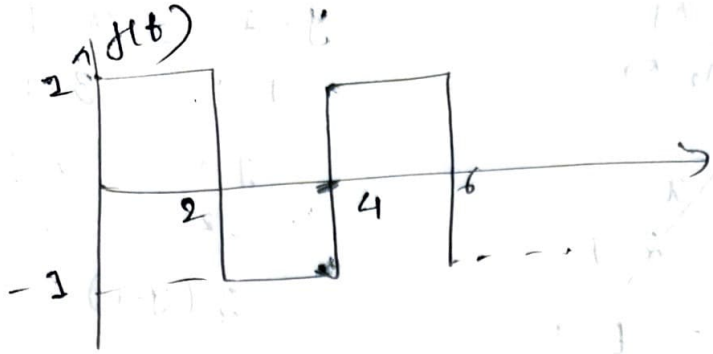
$$(t-3-1)\,u(t-3) - (t-4)\,u(t-4)$$

$$(t-3)\,u(t-3) - \cancel{u(t-3)} - (t-4)\,u(t-4)$$

$$t\,u(t) - 2[(t-1)\,u(t-1)] + 2[(t-3)\,u(t-3)] - (t-4)\,u(t-4)$$

$$\frac{1}{s^2} - \frac{2e^{-s}}{s^2} + 2\frac{e^{-3s}}{s^2} - \frac{e^{-4s}}{s^2} \qquad \left(\frac{1}{1-e^{-4s}}\right)$$

27



$\cancel{u(t) - u(t-2) - u(t-4)} = [u(t) - u(t-2) - u(t-2) + u(t-4)]$

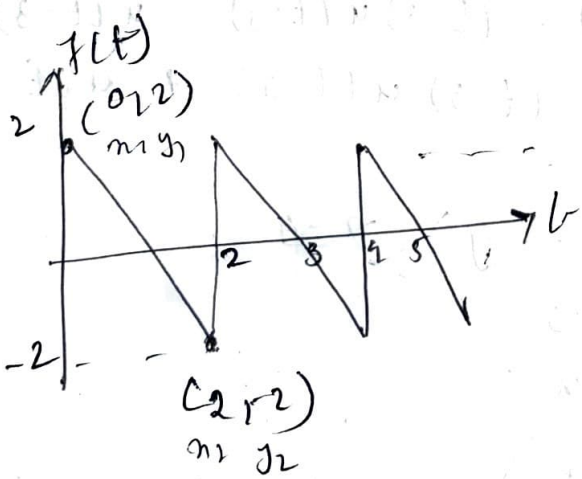
$= u(t) - 2u(t-2) + u(t-4)$

$$\frac{1}{s} - \frac{2e^{-2s}}{s} + \frac{e^{-4s}}{s}$$

$$= \frac{1}{s}(1 - 2e^{-2s} + e^{-4s})$$

$$\boxed{F(s) = \frac{1}{s(1-e^{-4s})}(1 - 2e^{-2s} + e^{-4s})}$$

28



$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{f(t)-2}{-2-2} = \frac{x-0}{2-0}$$

$$\frac{f(t)-2}{-4} = \frac{x}{2}$$

$$f(t)-2 = 2x$$

$$f(t) = 2x+2$$

$$\frac{f(t)-2}{-2} = \frac{t}{1}$$

$$f(t)-2 = -2t$$

$$f(t) = -2t+2$$

$$f_1(t) = (-2t+2)[u(t) - u(t-2)]$$

$$2\{(1-t)(u(t)-u(t-2))\}$$

$$= 2\{(1-t)u(t) + (t-1)u(t-2)\}$$

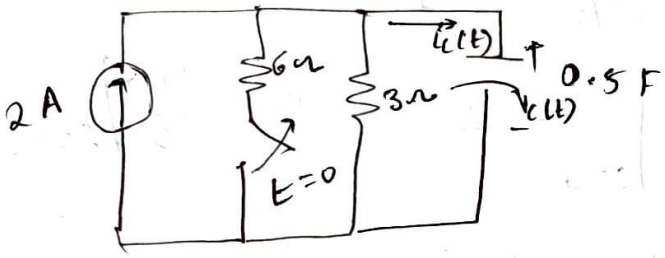
$$2\left\{\frac{1}{s} - \frac{1}{s^2} + (t-1-1+1)\,u(t-2)\right\}$$

$$= 2\left\{\frac{1}{s} - \frac{1}{s^2} + (t-2+1)\,u(t-2)\right\}$$

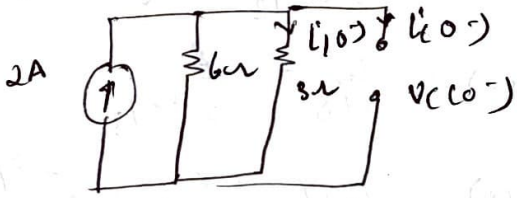
$$= 2\left\{\frac{1}{s} - \frac{1}{s^2} + (t-2)\,u(t-2) + u(t-2)\right\}$$

$$= \frac{2}{s}\left\{\frac{1}{s} - \frac{1}{s^2} + \frac{e^{-2s}}{s^2} + \frac{e^{-2s}}{s}\right\} \Big/ \left(1-e^{-2s}\right)$$

(29)



at $t=0^-$, the S is closed



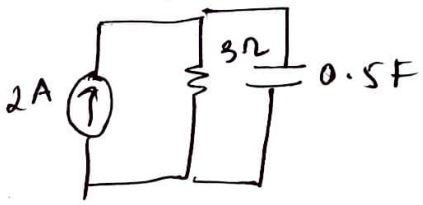
$i_c(0^-) = 0$

$i_1(0^-) = 2 \times \frac{6}{9} = 4/3\ A$

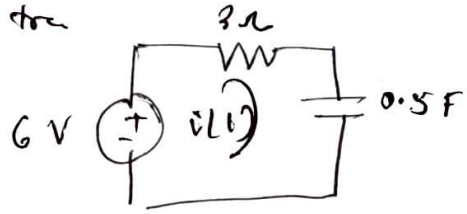
$V_c(0^-) = i_1(0^-) \times 3$

$= \frac{4}{3} \times 3 = 4V$

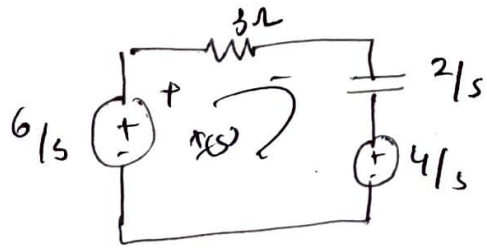
at $t \geq 0^+$ the s is open



source tra →

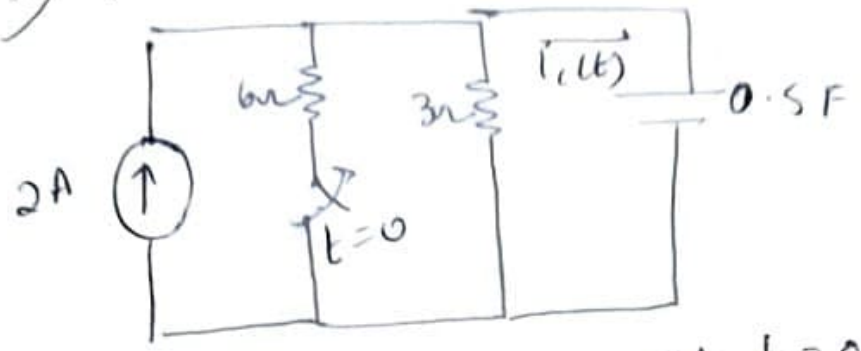


$Z = \frac{1}{Cs} = \frac{1}{0.5s} = \frac{2}{s}$, $\frac{v_c(0^-)}{s} = 4/s$



$(1 - e^{\ldots})$

(29)



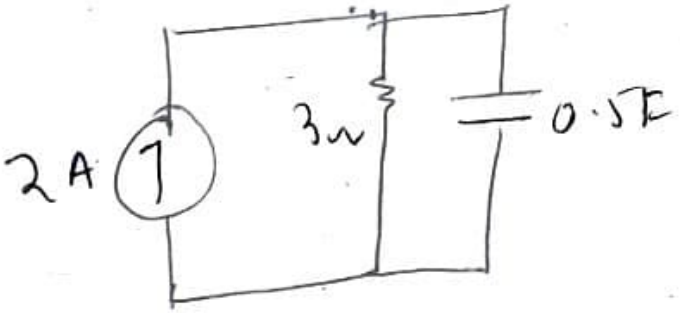
at $t = 0^-$

$R_{eq} = 6 \| 3 = 2\Omega$

$V(0^-) = 2 \times 2 = 4V$

$\boxed{V(0^+) = 4V}$

at $t > 0$



$2 = \frac{V}{3} + C\frac{dv}{dt}$

$2 = \frac{V}{3} + 0.5\frac{dv}{dt}$

Apply L.T

$$\frac{2}{s} = \frac{V(s)}{3} + 0.5\left(sV(s) - 4\right)$$

$$\frac{2}{s} \quad \frac{V_s}{3} + 0.5sV - 2$$

$$\frac{2}{s} + 2 = V\left(\frac{1}{3} + s0.5\right)$$

$$\frac{2+2s}{s} = V_{(s)}\left(\frac{1+1.5s}{3}\right)$$

$$V_{(s)} = \frac{6(1+s)}{s(1+1.5s)}$$

$$V_{(s)} = \frac{6}{s} - \frac{3}{(1+1.5s)}$$

$$= \frac{6}{s} - \frac{2}{s+2/3}$$

$$= \boxed{6 - 2e^{-2/3}\,V}$$

$$i_c(t) = C\frac{dv}{dt}$$

$$= -2\left(-\frac{2}{3}\right)e^{-2/3} = -\frac{4}{3}e^{-2/3\,t}$$

$$= 0.5\left(\frac{4}{3}\right)e^{-2/3\,t}$$

$$i_c(t) = \boxed{\frac{2}{3}e^{-2/3\,t}\,A}, \quad t>0$$