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Internal Assessment Test 2 – Nov. 2025

Subject: Digital Signal Processing				Code: BEC502
Date:28/11/25	Duration: 90 mins	Max. Marks: 50	Semester: 05	Branch: ECE

Sl.	Answer any FIVE FULL Questions	Marks	CO	RBT
1a	State and Prove Parseval's theorem for DFTs.	6	CO3	L1
1b	What is the speed improvement factor in calculating DFT of 128 pt DFT of a sequence using direct computation and FFT algorithm?	4	CO3	L1
2	Design an FIR filter using hamming window for $N = 7$. The desired frequency response is given by $$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & \lvert\omega\rvert \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < \lvert\omega\rvert \le \pi \end{cases}$$	10	CO4	L2
3	Develop radix-2 Decimation in Frequency FFT algorithm for $N = 8$.	10	CO3	L2

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4	Realize FIR & IIR filters represented as follows: a) $h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) - \frac{1}{8}\delta(n-3)$ $+\frac{1}{4}\delta(n-4) + \delta(n-5)$ using linear phase realization b) $H(z) = \frac{0.5z^2+z+0.5}{z^2+0.5z+0.4}$ using direct form II realization	10	CO4	L3
5	Consider a FIR filter with impulse response $h(n) = \{1, 2\}$ and input sequence $x(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3\}$. Compute $y(n)$ using overlap-add technique assuming the length of the block is 5.	10	CO3	L2
6	The magnitude squared function of a LPF has $\lvert H(j\Omega)\rvert^2 = \frac{1}{1+16\Omega^4}$, Obtain the analog filter transfer function $H_a(s)$	10	CO5	L2
7	Derive mapping function used for transforming analog filter to digital filter by Bilinear Transformation, preserves the frequency selectivity and stability of analog filter.	10	CO5	L2
8	Design a digital lowpass Butterworth filter with 3dB attenuation at passband edge frequency 1.5KHz, 10dB attenuation at frequency 3KHz and sampling frequency of 8000Hz.	10	CO5	L2

CI CCI HOD

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CI CCI HOD

Internal assessment test - 2.

1

a. Statement.

We know that $x(n) \xleftrightarrow[N]{DFT} X(k)$ and $y(n) \xleftrightarrow[N]{DFT} Y(k)$

then $\sum_{n=0}^{M-1} x(n)\, y^*(n) = \frac{1}{N} \sum_{n=0}^{M-1} X(k)\, Y^*(k)$.

Proof :- By the cross correlational property we have.

$$\tilde{r}_{xy}[l] = \sum_{n=0}^{M-1} x(n)\, y^*(n-l).$$

Put $l=0$

$$\tilde{r}_{xy}[0] = \sum_{n=0}^{M-1} x(n)\, y^*(n)$$

WKT

$$\tilde{r}_{xy}[l] = IDFT\left[R_{xy}(k)\right].$$

$$R_{xy}[k] = \frac{1}{N} \sum_{n=0}^{M-1} X(k)\, Y^*(k).$$

NOW

$$\sum_{n=0}^{M-1} x(n)\, y^*(n) = \frac{1}{N} \sum_{n=0}^{M-1} X(k)\, Y^*(k).$$

Hence Proved.

1b]

=) ~~Nube~~

=) Number of operations in Direct DFT

Number of operation in DFT = $N^2 = 128^2 = 16384$

16384 · Complex multiplication

=) Number of operation in radix-2 FFT

Number of operation in radix-2 FFT = $\frac{N}{2} \log_2 N = \frac{128}{2}$ log

$\log_2 128 = 7$

$\therefore \frac{64}{\cancel{128}_{1}} \times 7 = 448$

Speed Improvement factor

Improvement factor = $\frac{\text{Num of operation in DFT}}{\text{No of operation if FFT}}$

$= \frac{16384}{448} \cong 36.57 \approx 36.6$

$\therefore$ Speed improvement factor = 36.6 //

$\alpha = \frac{M-1}{2}$, $\alpha = 3$

$3 = \frac{M-1}{2}$

$6 = M-1$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-3\pi/4}^{3\pi/4} e^{-j3\omega} e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-3\pi/4}^{3\pi/4} e^{j\omega(n-3)} \, d\omega .$$

$$h_d(n) = \frac{1}{2\pi} \left[\frac{e^{j\omega(n-3)}}{j(n-3)} \right]_{-3\pi/4}^{3\pi/4}$$

$$= \frac{1}{2\pi\,[j(n-3)]} \left[e^{j\frac{3\pi}{4}(n-3)} - e^{-j\frac{3\pi}{4}(n-3)} \right]$$

$$h_d(n) = \frac{1}{2\pi\,[j(n-3)]} \left[2j \sin(n-3)\frac{3\pi}{4} \right] \quad \text{for } n \neq 3.$$

for $n=3$

$$h_d(3) = \frac{1}{2\pi} \int_{-3\pi/4}^{3\pi/4} e^{j\omega(3-3)} \, d\omega$$

$\alpha = \frac{M-1}{2}$, $\alpha = 3$

$3 = \frac{M-1}{2}$

$6 = M-1$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

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$$h_d(3) = \frac{1}{2\pi} \int_{-3\pi/4}^{3\pi/4} e^{j\omega(3-3)} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-3\pi/4}^{3\pi/4} d\omega$$

$$= \frac{1}{2\pi} \left[\frac{3\pi}{4} + \frac{3\pi}{4} \right]$$

$$= \frac{1}{2\pi} \left(2 \cdot \frac{3\pi}{4} \right)$$

$$h_d(3) = \frac{3}{4} \quad \text{for } n = 3.$$

$$h_d(n) = \begin{cases} \frac{\sin[(n-3)\frac{3\pi}{4}]}{\pi(n-3)}, & \text{for } n \neq 3 \\ 3/4, & \text{for } n = 3 \end{cases}$$

for hamming window

$$w(n) = 0.54 - 0.46 \cos\left[\frac{2\pi n}{M-1}\right].$$

n	$h_d(n)$	$w(n) = 0.54 - 0.46\cos\left[\frac{2\pi n}{M-1}\right]$	$h(n) = h_d(n) \cdot w(n)$
0	0.0750	$w(n) = 0.08$	0.006
1	-0.159	0.31	-0.049
2	0.075	0.77	0.0577
3	3/4	1	3/4
4	0.075	0.77	0.0577
5	-0.159	0.31	-0.049
6	0.0750	0.08	0.006

$\therefore h(n) = \{0.006, -0.049, 0.0577, 3/4, 0.0577, -0.049, 0.006\}$.

1. b $N = 128$

Complex multiplications using Direct computation of DFT

$$N^2 = (128)^2 = 16384$$

Complex multiplications using FFT

$$\frac{N}{2}\log_2 N = \frac{128}{2}\log_2 128 = 448$$

Speed improvement factor $= \frac{16384}{448} = 36.57$

Complex additions using direct computation

$$N(N-1) = 128(128-1) = 16256$$

Complex additions using FFT

$$N\log_2 N = 128\log_2 128 = 896$$

Speed improvement factor $= \frac{16256}{896} = 18.14$

4.

(a) given,

$$h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) - \frac{1}{8}\delta(n-3) + \frac{1}{4}\delta(n-4) + \delta(n-5)$$

Apply z-Transform,

$$H(z) = 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2} - \frac{1}{8}z^{-3} + \frac{1}{4}z^{-4} + z^{-5}$$

$$\frac{Y(z)}{X(z)} = 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2} - \frac{1}{8}z^{-3} + \frac{1}{4}z^{-4} + z^{-5}$$

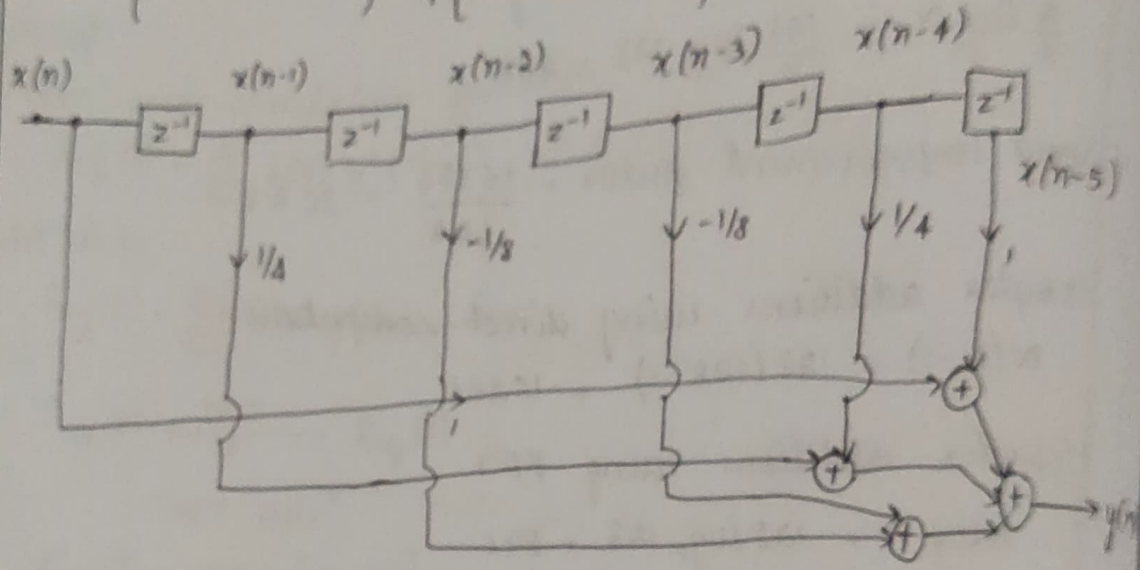
$$Y(z) = X(z) + \frac{1}{4}X(z)z^{-1} - \frac{1}{8}X(z)z^{-2} - \frac{1}{8}X(z)z^{-3} + \frac{1}{4}X(z)z^{-4} + X(z)z^{-5}$$

$$\begin{bmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Apply Inverse Z-Transform,

$$y(n) = x(n) + \frac{1}{4}x(n-1) - \frac{1}{8}x(n-2) - \frac{1}{8}x(n-3) + \frac{1}{4}x(n-4) + x(n-5)$$

$$y(n) = 1\left[x(n) + x(n-5)\right] + \frac{1}{4}\left[x(n-1) + x(n-4)\right] - \frac{1}{8}\left[x(n-2) + x(n-3)\right]$$



Linear phase realization.

(4b)

$$H(z) = \frac{0.5z^2 + z + 0.5}{z^2 + 0.5z + 0.4}$$

$\div$ by z^2

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.5 + z^{-1} + 0.5z^{-2}}{1 + 0.5z^{-1} + 0.4z^{-2}}$$

Apply Inverse Z-Transform,

$$y(n) = x(n) + \frac{1}{4}x(n-1) - \frac{1}{8}x(n-2) - \frac{1}{8}x(n-3) + \frac{1}{4}x(n-4) + x(n-5)$$

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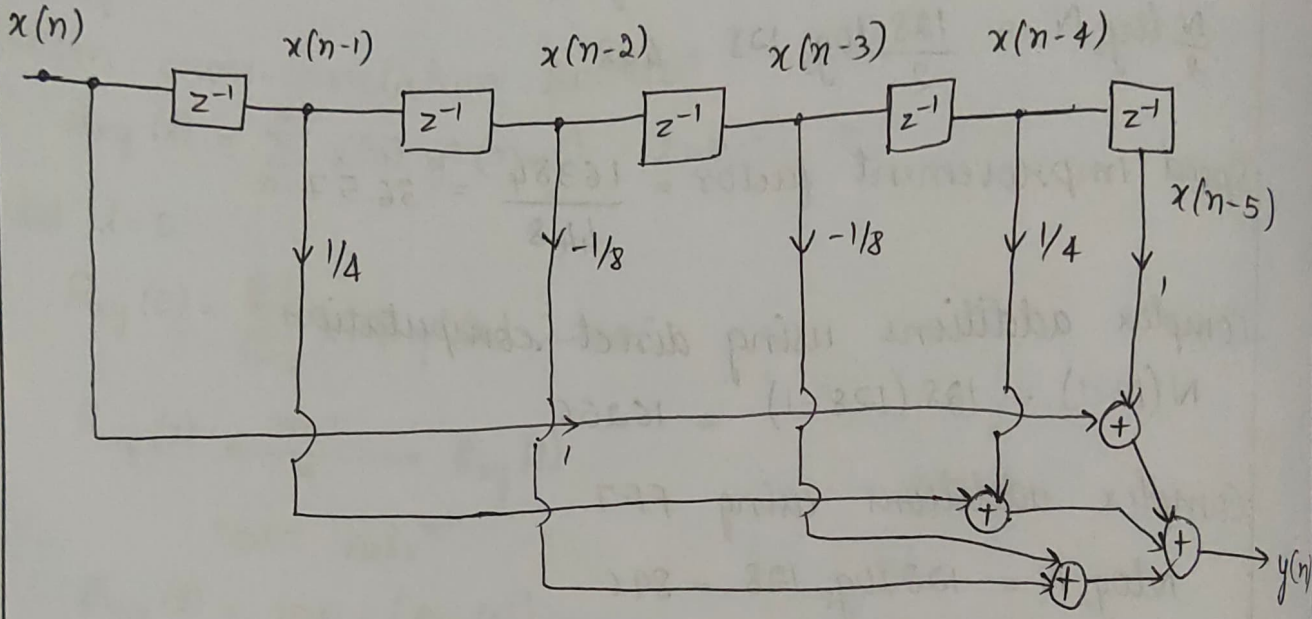

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$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.5 + z^{-1} + 0.5z^{-2}}{1 + 0.5z^{-1} + 0.4z^{-2}}$$

Consider $H(z) = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$

$H_1(z)$ $H_2(z)$

$$H_1(z) = \frac{Y(z)}{W(z)} = 0.5z^{-1} + 0.5z^{-2}$$

$$Y(z) = 0.5W(z) + W(z)z^{-1} + 0.5W(z)z^{-2}$$

Taking Inverse Z Transform,

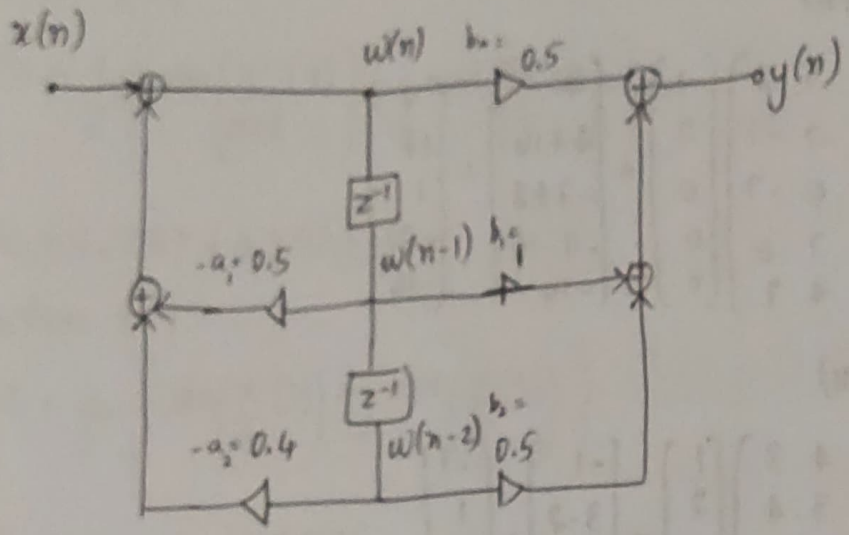
$$\boxed{y(n) = 0.5w(n) + w(n-1) + 0.5w(n-2)}$$

$$H_2(z) = \frac{W(z)}{X(z)} = \frac{1}{1+0.5z^{-1}+0.4z^{-2}}$$

$$W(z) + 0.5W(z)z^{-1} + 0.4W(z)z^{-2} = X(z)$$

Taking Inverse z-Transform.

$$\boxed{x(n) = w(n) + 0.5w(n-1) + 0.4w(n-2)}$$



direct form II realization

(5) $h(n) = \{1, 2\}$ $\quad M = 2$

$x(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3\}$

$N = 5$

$N = L + M - 1$

$5 = L + 2 - 1$

$\boxed{L = 4}$

$x_1(n) = \{1, 4, 3, 0, 0\}$

$x_2(n) = \{7, 4, -7, -7, 0\}$

$x_3(n) = \{-1, 3, 4, 3, 0\}$

$h(n) = \{1, 2, 0, 0, 0\}$

$$H_2(z) = \frac{W(z)}{X(z)} = \frac{1}{1+0.5z^{-1}+0.4z^{-2}}$$

$$W(z) + 0.5W(z)z^{-1} + 0.4W(z)z^{-2} = X(z)$$

Taking Inverse z-Transform,

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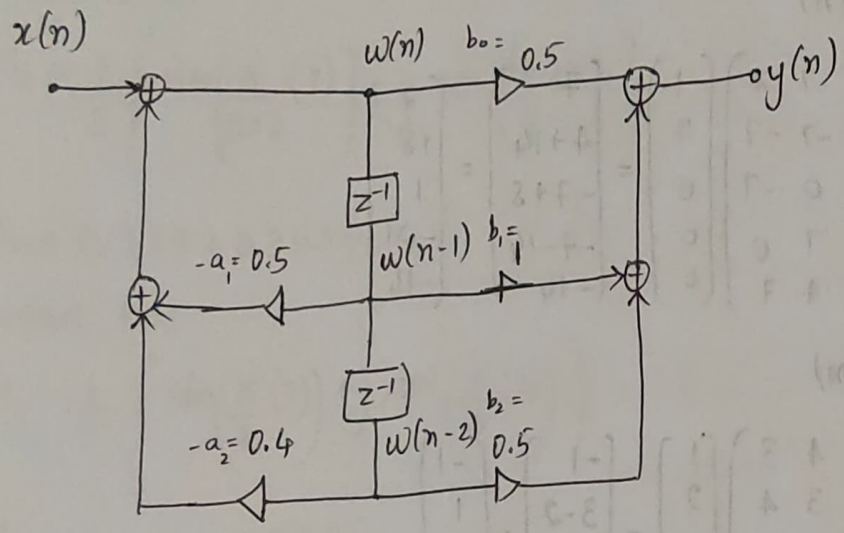

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$N = 5$

$N = L + M - 1$

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$\boxed{L = 4}$

$x_1(n) = \{1, 4, 3, 0, 0\}$

$x_2(n) = \{7, 4, -7, -7, 0\}$

$x_3(n) = \{-1, 3, 4, 3, 0\}$

$h(n) = \{1, 2, 0, 0, 0\}$

$y_1(n) = x_1(n) \textcircled{5} h(n)$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 & 4 \\ 4 & 1 & 0 & 0 & 3 \\ 3 & 4 & 1 & 0 & 0 \\ 0 & 3 & 4 & 1 & 0 \\ 0 & 0 & 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4+2 \\ 3+8 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 11 \\ 6 \\ 0 \end{bmatrix}$$

$y_2(n) = x_2(n) \textcircled{5} h(n)$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \end{bmatrix} = \begin{bmatrix} 7 & 0 & -7 & -7 & 4 \\ 4 & 7 & 0 & -7 & -7 \\ -7 & 4 & 7 & 0 & -7 \\ -7 & -7 & 4 & 7 & 0 \\ 0 & -7 & -7 & 4 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 4+14 \\ -7+8 \\ -7-14 \\ -14 \end{bmatrix} = \begin{bmatrix} 7 \\ 18 \\ 1 \\ -21 \\ -14 \end{bmatrix}$$

$y_3(n) = x_3(n) \textcircled{5} h(n)$

$$\begin{bmatrix} y_3(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 & 4 & 3 \\ 3 & -1 & 0 & 3 & 4 \\ 4 & 3 & -1 & 0 & 3 \\ 3 & 4 & 3 & -1 & 0 \\ 0 & 3 & 4 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3-2 \\ 4+6 \\ 3+8 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 10 \\ 11 \\ 6 \end{bmatrix}$$

$\therefore y(n) = \{1, 6, 11, 6, 7, 18, 1, -21, -15, 1, 10, 11, 6\}$

(6) The magnitude squared funtion of LPF is,

$$|H(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}} \longrightarrow 1$$

given,

$$|H(j\Omega)|^2 = \frac{1}{1 + 16\Omega^4}$$

$$|H(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{(1/2)}\right)^{2\times 2}} \longrightarrow 2$$

From 1 & 2

Order $N=2$ and $\Omega_c = \frac{1}{2}$ rad/sec

Calculation of poles

$$S_k = \Omega_c \left\{ \sin\left(\frac{\pi}{2N}(2k+1)\right) + j\cos\left(\frac{\pi}{2N}(2k+1)\right) \right\} \qquad k = 0, \ldots N-1$$

when $k=0$

$$S_0 = \frac{1}{2}\left\{ \sin\left(\frac{\pi}{2\times 2}(1)\right) + j\cos\left(\frac{\pi}{4}(1)\right) \right\}$$

$$S_0 = 0.353 + 0.353j$$

when $k=1$

$$S_1 = \frac{1}{2}\left\{ \sin\left(\frac{\pi}{4}(3)\right) + j\cos\left(\frac{\pi}{4}(3)\right) \right\}$$

$$S_1 = 0.353 - 0.353j$$

Analog Transfer Function,

$$H_a(s) = \frac{(-s_c)^N}{(s-s_0)(s-s_1)} = \frac{(1/2)^2}{(s-0.353-0.353j)(s-0.353+0.353j)}$$

$$H_a(s) = \frac{0.25}{s^2 - 0.353s + 0.353js - 0.353s + 0.124 - 0.124j - 0.35js + 0.124j + 0.124}$$

$$\boxed{H_a(s) = \frac{0.25}{s^2 + 0.707s + 0.25}}$$

0.124
0.124
0.248
= 0.25

7. Consider Analog Transfer function,

$$H_a(s) = \frac{b}{s+a} \longrightarrow 1$$

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a} \longrightarrow 2$$

$$Y(s)\,s + Y(s)\,a = bX(s)$$

Taking Inverse Laplace,

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

$$y'(t) = bx(t) - ay(t) \rightarrow 3$$

Using Trapezoidal Rule for integration,

$$y(t) = \int_{t_0}^{t} y'(t)\,dt + y(t_0)$$

let the limits be: $t = nT_s$

$$t_0 = nT_s - T_s$$

$$y(nT_s) = \frac{T_s}{2}\left[y'(nT_s) + y'(nT_s - T_s) \right] + y(nT_s - T_s) \rightarrow 4$$

$$(3) \Rightarrow y'(nT_s) = bx(nT_s) - ay(nT_s) \rightarrow 5$$

using 5 in 4

$$y(nT_s) = \frac{T_s}{2}\left[bx(nT_s) - ay(nT_s) + bx(nT_s - T_s) - ay(nT_s - T_s) \right] + y(nT_s - T_s)$$

$$y(nT_s)\left[1 + \frac{aT_s}{2}\right] = y(nT_s - T_s)\left[1 - \frac{aT_s}{2}\right] + \frac{bT_s}{2}\left[x(nT_s) + x(nT_s - T_s)\right]$$

let $y(nT_s) \approx y(n)$

$y(nT_s - T_s) \approx y(n-1)$

$x(nT_s) \approx x(n)$

$x(nT_s - T_s) \approx x(n-1)$

$$y(n)\left(1 + \frac{aT_s}{2}\right) = y(n-1)\left(1 - \frac{aT_s}{2}\right) + \frac{bT_s}{2}\left(x(n) + x(n-1)\right) \rightarrow 6$$

Taking Z Transform,

$$\left[1 + \frac{aT_s}{2}\right) Y(z) = Y(z)z^{-1}\left(1 - \frac{aT_s}{2}\right) + \frac{bT_s}{2}\left[X(z) + X(z)z^{-1}\right] \rightarrow 7$$

$$Y(z)\left[1 + \frac{aT_s}{2} - \left(1 - \frac{aT_s}{2}\right)z^{-1}\right] = X(z)\left[\frac{bT_s}{2}\left(1 + z^{-1}\right)\right] \rightarrow 8$$

8.

$$\frac{y(z)}{x(z)} = \frac{\frac{bT_s}{2}(1+z^{-1})}{1+\frac{aT_s}{2} - \left(1-\frac{aT_s}{2}\right)z^{-1}}$$

$$\frac{y(z)}{x(z)} = \frac{\frac{bT_s}{2}(1+z^{-1})}{(1-z^{-1}) + \frac{aT_s}{2}(1+z^{-1})} \quad \div \text{ by } \frac{T_s}{2}(1+z^{-1})$$

$$\frac{y(z)}{x(z)} = \frac{b}{\frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + a} \rightarrow 9$$

From 9 & 2

$$s = \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \rightarrow 10$$

This is called Bilinear Transformation Relation

let $s = \sigma + j\omega \qquad z = re^{j\omega}$

$$\sigma + j\omega = \frac{2}{T_s}\left(\frac{1-1/re^{j\omega}}{1+1/re^{j\omega}}\right)$$

$$\sigma + j\omega = \frac{2}{T_s}\left(\frac{re^{j\omega}-1}{re^{j\omega}+1}\right)$$

7.

$$\sigma + j\omega = \frac{2}{T_s}\left[\frac{(r\cos\omega - 1) + jr\sin\omega}{(r\cos\omega+1) + jr\sin\omega}\right]\left[\frac{(r\cos\omega+1) - jr\sin\omega}{(r\cos\omega+1) - j(\sin\omega)r}\right]$$

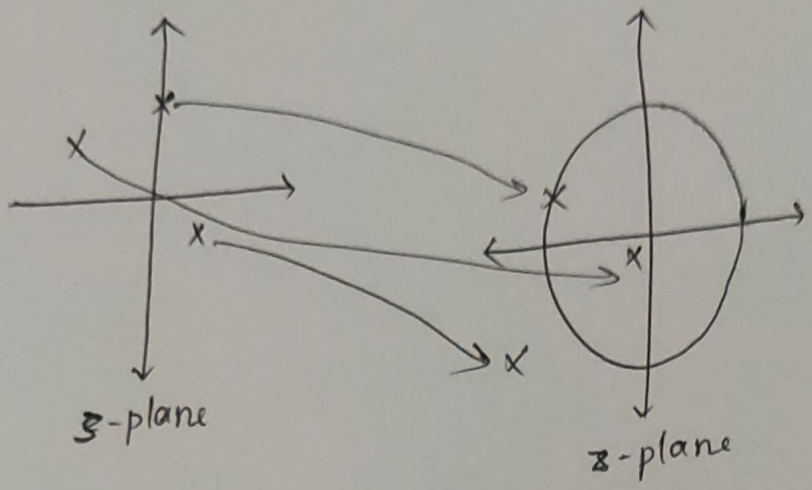
$$= \frac{2}{T_s}\left[\frac{(r^2-1) + 2jr\sin\omega}{(r^2+1) + 2r\cos\omega}\right]$$

$\therefore$ Real Part:

$$\sigma = \frac{2}{T_s}\left[\frac{r^2-1}{(r^2+1)+2r\cos\omega}\right]$$

Imaginary part

$$\omega = \frac{2}{T_s}\left[\frac{2jr\sin\omega}{(r^2+1)+2r\cos\omega}\right]$$



Case 1: $\sigma < 0$

Poles on Left half of s plane

System is stable

Case 2: $\sigma > 0$

Poles on RHS

System is unstable

Case 3: $\sigma = 0$

Poles on the axis

System is marginally stable

6) The magnitude squared function of a LPF has

$$|H(j\Omega)|^2 = \frac{1}{1+16\Omega^4},$$

Obtain the analog filter transfer function $H_a(s)$.

Ans: Magnitude squared response of Butterworth filter is given by:

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} \quad \text{---- (1)}$$

Given,

$$|H(j\Omega)|^2 = \frac{1}{1+16\Omega^4} = \frac{1}{1+\left[\frac{\Omega}{(1/2)}\right]^{2\times 2}} \quad \text{— (2)}$$

Comparing (1) & (2),

$$N = 2, \quad \Omega_c = 1/2 \text{ rad/sec}.$$

Pole locations:

$$S_k = -\Omega_c\left[\sin\left(\frac{(2k+1)\pi}{2N}\right) + j\cos\left(\frac{(2k+1)\pi}{2N}\right)\right]$$

$k = 0, 1$

$$S_0 = -0.3536 + j\,0.3536$$

$$S_1 = S_0^*$$

$$= -0.3536 - j\,0.3536$$

Transfer function:

$$H(s) = \frac{\Omega_c^{\,N}}{(s-S_0)(s-S_1)}$$

$$= \frac{0.25}{s^2 + 0.7072s + 0.25}$$

8) Design a digital lowpass butterworth filter with 3 dB attenuation at passband edge frequency 1.5 kHz, 10 dB attenuation at stopband frequency 3 kHz, and sampling frequency of 8000 Hz.

$A_{PB} = 3$ dB

$A_{SB} = 10$ dB

$F_{PB} = 1.5$ kHz

$F_{SB} = 3$ kHz

$F_S = 8000$ Hz $= 8$ kHz

Digital frequency $\omega_{PB} = \dfrac{\Omega_{PB}}{F_S} = \dfrac{2\pi F_{PB}}{F_S}$

$= \dfrac{2\pi \times 1.5 \text{ kHz}}{8 \text{ kHz}} = 0.375\pi$ //

Digital frequency $\omega_{SB} = \dfrac{2\pi F_{SB}}{F_S} = \dfrac{2\pi \times 3 \text{ kHz}}{8 \text{ kHz}} = 0.75\pi$

$\Omega_{PB}^{new} = \dfrac{2}{T} \tan\left(\dfrac{\omega_{PB}}{2}\right)$ (Assuming $T = 2$ sec)

$= \dfrac{2}{2} \tan\left(\dfrac{0.375\pi}{2}\right) = 0.6682$ rad/sec

$$\Omega_{SB}^{new} = \frac{2}{T}\tan\left(\frac{\omega_{SB}}{2}\right) \quad (\text{Assume } T = 2\,\text{sec})$$

$$= \frac{2}{2}\tan\left(\frac{0.75\pi}{2}\right) = 2.414 \text{ rad/sec}$$

order of the filter (no. of poles):

$$N = \frac{\log_{10}\left(\frac{10^{0.1A_{PB}}-1}{10^{0.1A_{SB}}-1}\right)}{2\log_{10}\left(\frac{\Omega_{PB}^{new}}{\Omega_{SB}^{new}}\right)}$$

$$= \frac{\log_{10}\left(\frac{10^{0.3}-1}{10^{1}-1}\right)}{2\log_{10}\left(\frac{0.6682}{2.414}\right)} = \frac{\log_{10}\left(\frac{0.9953}{9}\right)}{2\log_{10}(0.2768)}$$

$$= \frac{\log_{10}(0.11059)}{2\log_{10}(0.2768)} = 0.8571$$

Cut-off frequency:

$$\Omega_c = \frac{\Omega_{PB}^{new}}{\left[10^{0.1A_{PB}}-1\right]^{1/2N}} = \frac{0.6682}{\left[10^{0.3}-1\right]^{1/2}}$$

$$= \frac{0.6682}{0.9976} = 0.6698 \text{ rad/sec}$$

Pole locations:

$$S_k = \Omega_c\left[-\sin\left(\frac{(2k+1)\pi}{2N}\right) + j\cos\left(\frac{(2k+1)\pi}{2N}\right)\right] \quad k=0$$

$$S_0 = -\sin\left(\frac{\pi}{2}\right) + j\cos\left(\frac{\pi}{2}\right) = -1$$

$$H(s) = \frac{\Omega_c^N}{(s - s_0)}$$

$$H(s) = \frac{0.6698}{(s+1)}$$

$$H_a(s) = H(s)$$

To get digital transfer function,

$$s = \frac{2}{T}\frac{z-1}{z+1} = \frac{z-1}{z+1} \quad (\because T = 2_{sec})$$

$$H(z) = H(s)\Big|_{s = \frac{z-1}{z+1}}$$

$$= \frac{0.6698}{\frac{z-1}{z+1} + 1} = \frac{(z+1) \times 0.6698}{z-1+z+1}$$

$$= \frac{(z+1) \times 0.6698}{z^2} = 0.6698 \times (z^{-1} + z^{-2})$$