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INTERNAL ASSESSMENT TEST – II

Sub:	DIGITAL COMMUNICATIONS							Code:	BEC503
Date:	29/ 11 / 2025	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

Answer any 5 full questions

		Marks	CO	RBT
1	a. Discuss pre-envelope and complex envelope of bandpass signals with relevant equations. Plot the spectra of a bandpass signal, its pre-envelope and complex envelope. b. Find the pre-envelope and complex envelope of $x(t) = A_c\,[1 + K_a\,m(t)]\cos(2\pi f_c t)$ where m(t) is a lowpass signal band limited to W Hz and f c>>W	[5+5]	CO1	L3
2	a. Define Hilbert transform. State and prove the properties of Hilbert transform. Plot the magnitude response and phase response of the ideal Hilbert transformer. b. Determine the Hilbert transform of the signal $x(t) = 2 \quad 0 \le t \le T \qquad 0$ otherwise	[5+5]	CO1	L3
3	Consider a (2,1,2) Convolutional encoder with g1=101 and g2=011 . Decode the code sequence {11,01,01,10,00,01,11} using Viterbi algorithm.	[10]	CO2	L2

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4	Derive the expression for error probability of binary QPSK.	[10]	CO2	L1
5	a) Explain M-ary QAM. Draw the constellation diagram for M=4. b) In a PSK system the transmitted binary data is 10^6 bps.PSD of zero mean AWGN is 10^{-10} W/Hz. The amplitude of the received signal is 1μV.Find the probability of error using coherent detection.	[6+4]	CO2	L3
6	Obtain a set of orthonormal basis functions for the following set of signals. $x_1(t) = \begin{cases} 3 \text{ from } 0 \le t \le 3 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_2(t) = \begin{cases} 3 \text{ from } 0 \le t \le 1 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_3(t) = \begin{cases} 3 \text{ from } 1 \le t \le 3 \\ 0 \quad \text{otherwise} \end{cases}$ Express the signals as a linear combination of basis functions. Draw the signal space diagram (Constellation Diagram).	[10]	CO1	L3
7	Consider the (3,1,2) convolutional encoder with generator sequences g(1)=(1 1 0), g(2)= (1 0 1) and g(3)=(1 1 1). a)Draw the encoder block diagram. b) Find the code word corresponding to the information sequence (1 0 1 0 1) using time domain and transform domain approaches.	[10]	CO5	L3

CCI HOD

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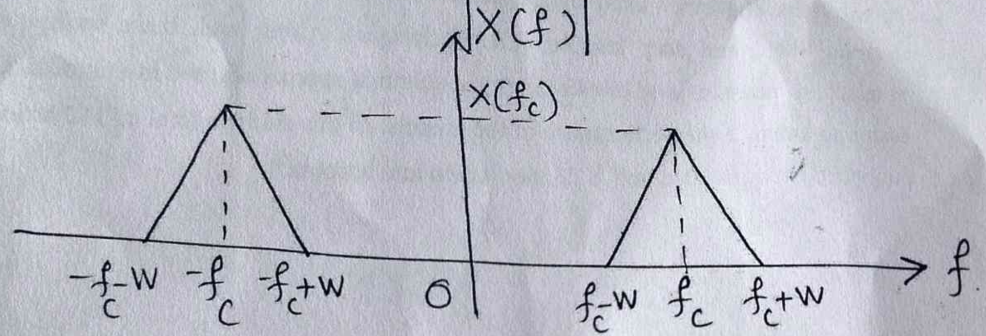
CCI HOD

...ope and complex envelope. (29)

Consider a real valued bandpass signal $x(t)$ with Fourier Transform $X(f)$.

Let $X(f)$ be confined to a band of frequencies $f_c - W \leq |f| \leq f_c + W$.

We assume that $f_c \gg W$.



Note that $|X(f)|$ is symmetric since $x(t)$ is real.

Let us construct a signal $x_+(t)$ such that

$$x_+(t) = x(t) + j\hat{x}(t) \quad \cdots\cdots (1)$$

where $\hat{x}(t)$ is the Hilbert transform of $x(t)$.

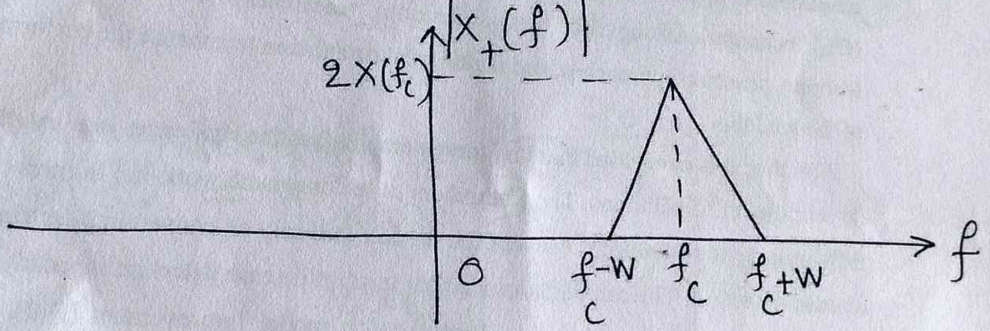
$$\therefore \quad X_+(f) = X(f) + j\hat{X}(f)$$

$$= X(f) + j\left(-j\,\mathrm{sgn}(f)X(f)\right)$$

$$\mathrm{sgn}(f) = \begin{cases} 1 & \text{for } f > 0 \\ 0 & \text{for } f = 0 \\ -1 & \text{for } f < 0 \end{cases} \quad \ldots (3)$$

$\therefore$ $X_+(f)$ can be written as,

$$X_+(f) = \begin{cases} 2X(f) & \text{for } f > 0 \\ X(0) & \text{for } f = 0 \\ 0 & \text{for } f < 0 \end{cases} \quad \ldots (4)$$



Note that $X_+(f)$ contains only +ve freque-ncy components.

Since, $X_+(f)$ is not symmetric, $x_+(t)$ is a complex signal.

$x_+(t)$ is called pre-envelope of $x(t)$ with +ve frequencies. (Also called <u>analytic</u> <u>signal</u>

... a signal $x_-(t)$ such that

$$x_-(t) = x(t) - j\hat{x}(t) \quad \text{--- (5)}$$

$$\therefore X_-(f) = X(f) - j\hat{X}(f)$$

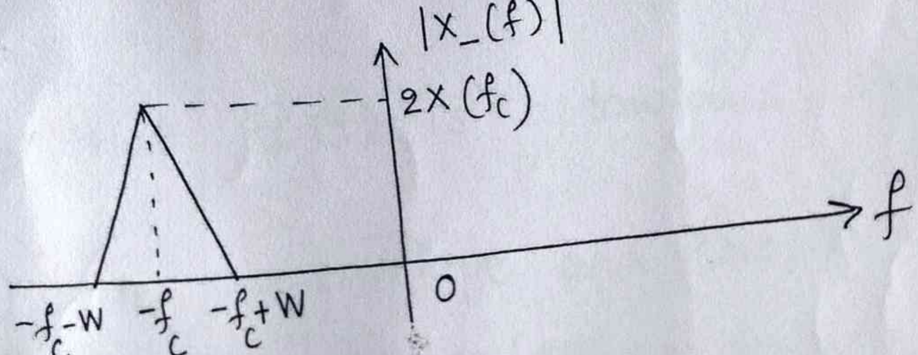
$$= X(f) - j\left[-j\,\mathrm{sgn}(f)X(f)\right]$$

$$= X(f) - \mathrm{sgn}(f)X(f) \quad \text{... (6)}$$

But

$$\mathrm{sgn}(f) = \begin{cases} +1 & \text{for } f > 0 \\ 0 & \text{for } f = 0 \\ -1 & \text{for } f < 0 \end{cases} \quad \text{... (7)}$$

$$\therefore X_-(f) = \begin{cases} 0 & \text{for } f > 0 \\ X(0) & \text{for } f = 0 \\ 2X(f) & \text{for } f < 0 \end{cases} \quad \text{... (8)}$$



Note that $X_-(f)$ contains only $-$ve frequency components.

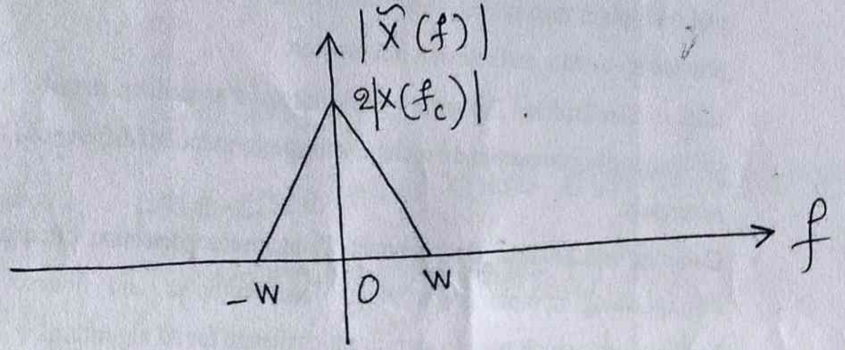
Since $X_-(f)$ is not symmetric, $x_-(t)$ is complex signal.

$x_-(t)$ is called pre-envelope of $x(t)$ with -ve frequencies.

Now, let us construct a signal, $\tilde{x}(t)$, such that,

$$\tilde{x}(t) = x_+(t)\, e^{-j2\pi f_c t} \quad \ldots (9)$$

$$\therefore \tilde{X}(f) = X_+(f+f_c) \quad \ldots (10)$$



We refer to $\tilde{x}(t)$ as complex envelope of $x(t)$.

Note that $\tilde{x}(t)$ is a low-pass signal.

17 Find the pre-envelope and complex envelope of $x(t) = m(t)\cos(2\pi f_c t)$. Assume that $m(t)$ is bandlimited to W Hz and $f_c \gg W$.

$$x(t) = m(t)\cos(2\pi f_c t)$$

$\hat{x}(t) = m(t)\sin(2\pi f_c t)$ (See Problem #8)

Pre-envelope,

$$x_+(t) = x(t) + j\,\hat{x}(t)$$

$$= m(t)\cos(2\pi f_c t) + j\,m(t)\sin(2\pi f_c t)$$

$$= m(t)\left[\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right]$$

$$= m(t)\,e^{j2\pi f_c t}$$

Complex envelope,

$$\tilde{x}(t) = x_+(t)\,e^{-j2\pi f_c t}$$

$$= m(t).$$

complex envelope of

3. State and prove the properties of Hilbert transform.

Property 1: A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ have the same magnitude spectrum.

Proof:

Let $h(t)$ be the impulse response of Hilbert transformer.

$$h(t) = \frac{1}{\pi t} \quad \ldots (1)$$

Frequency response of Hilbert transformer,

$$H(f) = \begin{cases} -j & \text{for } f > 0 \\ 0 & \text{for } f = 0 \\ j & \text{for } f < 0 \end{cases} \quad \ldots (2)$$

$\therefore |H(f)| = 1 \quad \forall f \quad \text{except } f = 0$

$$\hat{x}(t) = x(t) * h(t) \quad \ldots (3)$$

We know that convolution of two functions in time domain is transformed into multiplication of their Fourier transforms.

$$\therefore \hat{X}(f) = X(f)\, H(f)$$

$$\therefore |\hat{X}(f)| = |X(f)|\, |H(f)|$$

$$= |X(f)| \quad \ldots (4)$$

<u>Property 2</u>: If $\hat{x}(t)$ is the Hilbert transform of $x(t)$, then the Hilbert transform of $\hat{x}(t)$ is $-x(t)$.

<u>Proof</u>:

$$\hat{x}(t) = x(t) * h(t) \quad \ldots (1)$$

$$\therefore \hat{X}(f) = X(f)\, H(f)$$

$$= -j\, \mathrm{sgn}(f)\, X(f) \quad \ldots (2)$$

Hilbert transform of $\hat{x}(t)$,

$$\hat{\hat{x}}(t) = \hat{x}(t) * h(t) \quad \ldots (3)$$

$$= \hat{X}(f)\, H(f)$$

(11)

$$= \left[-j\,\mathrm{sgn}(f)\,X(f)\right]\left[-j\,\mathrm{sgn}(f)\right]$$

(Using (2)) ... (4)

But, $j^2 = -1$ and

$$\mathrm{sgn}(f)\,\mathrm{sgn}(f) = 1$$

$\therefore$ (1) can be written as,

$$\hat{\hat{X}}(f) = -X(f) \quad \ldots (5)$$

Taking Inverse FT, we get

$$\hat{\hat{x}}(t) = -x(t) \quad \ldots (6)$$

<u>Property 3</u>: A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ are orthogonal over the entire time interval $(-\infty, \infty)$.

<u>Proof:</u>

To prove that $x(t)$ and $\hat{x}(t)$ are orthogonal, we have to prove that

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0 \quad \ldots (1)$$

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\,\hat{X}^*(f)\,df \quad \cdots (2)$$

We know that,

$$\hat{x}(t) = x(t) * h(t) \quad \cdots (3)$$

$$\therefore \hat{X}(f) = X(f)\,H(f)$$

$$= X(f)\left[-j\,\mathrm{sgn}(f)\right] \quad \cdots (4)$$

$$\therefore \hat{X}^*(f) = j\,\mathrm{sgn}(f)\,X^*(f) \quad \cdots (5)$$

Using (5), (2) can be written as,

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\left[j\,\mathrm{sgn}(f)\,X^*(f)\right] df$$

$$= j\int_{-\infty}^{\infty} \mathrm{sgn}(f)\,|X(f)|^2\,df$$

$$= 0 \quad \cdots (6)$$

$\because$ $\mathrm{sgn}(f)$ is an odd function.

$|X(f)|^2$ is an even function

$\mathrm{sgn}(f)\,|X(f)|^2$ is an odd function.

Hence, $\int_{-\infty}^{\infty} \mathrm{sgn}(f)\,|X(f)|^2\,df = 0$

...ponse of Hilbert transformer. ...ponse and phase

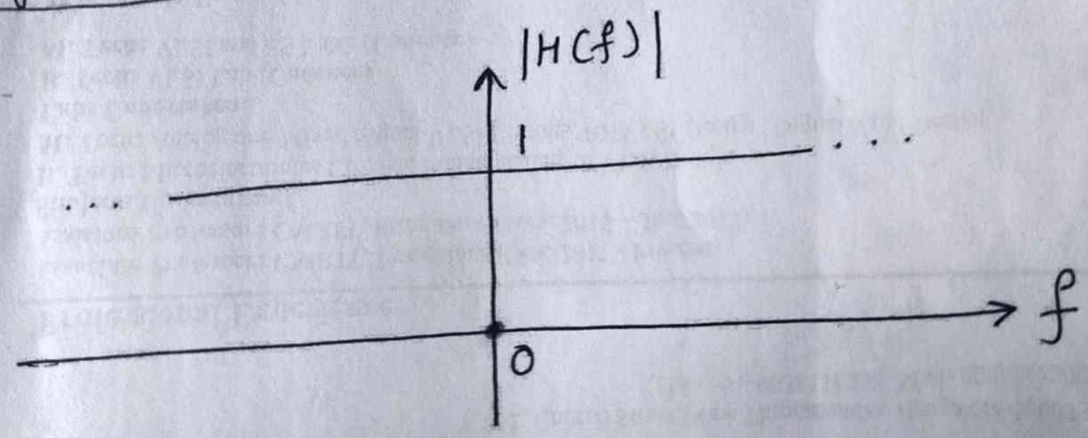
Frequency response of Hilbert transformer is given by

$$H(f) = \begin{cases} e^{-j\frac{\pi}{2}} & \text{for } f>0 \\ 0 & \text{for } f=0 \\ e^{j\pi/2} & \text{for } f<0 \end{cases}$$

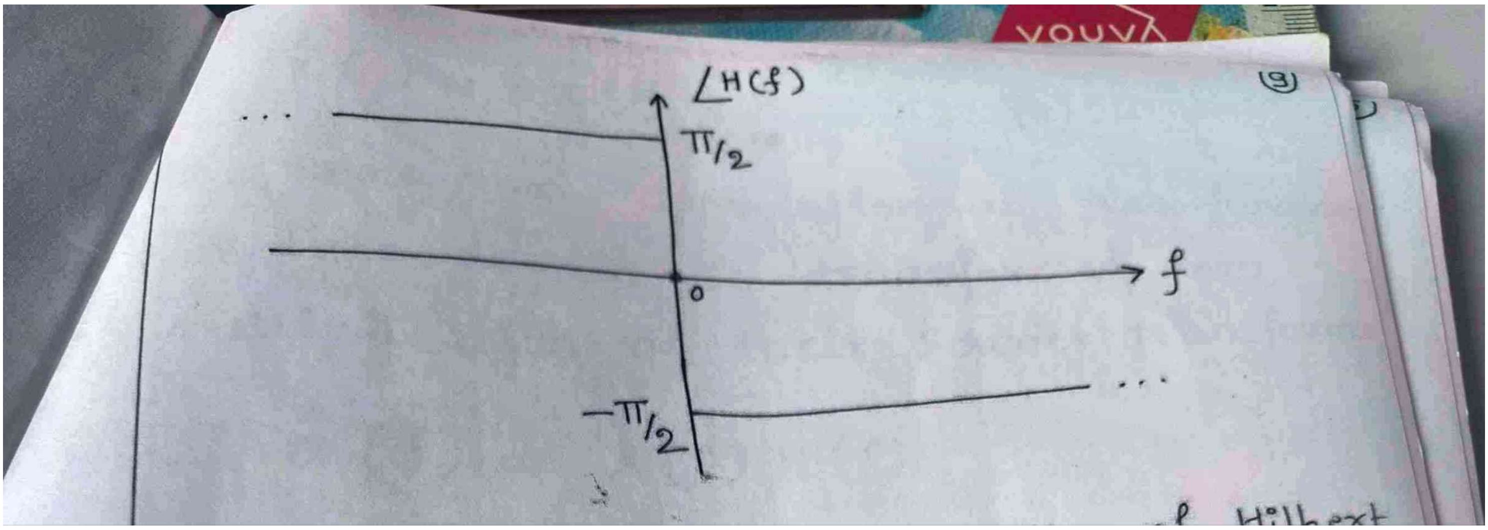
$$\therefore \quad |H(f)| = \begin{cases} 1 & \text{for } f>0 \\ 0 & \text{for } f=0 \\ 1 & \text{for } f<0 \end{cases}$$

$$\angle H(f) = \begin{cases} -\pi/2 & \text{for } f>0 \\ 0 & \text{for } f=0 \\ \pi/2 & \text{for } f<0 \end{cases}$$

Magnitude response:

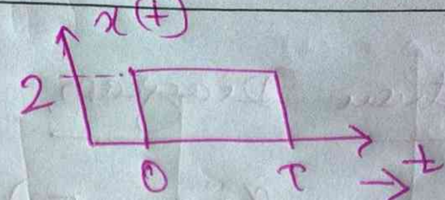


Phase response:



$\angle H(f)$
$\pi/2$
0
f
$-\pi/2$
9

Ques:- 2b



Time Domain Approach,

$$\hat{x}(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

$$= \int_{0}^{T} 2 \cdot \frac{1}{\pi(t-\tau)} d\tau = \frac{2}{\pi} \int_{0}^{T} \frac{1}{(t-\tau)} d\tau$$

Put $t-\tau = u \Rightarrow -d\tau = du$

@ $\tau = 0,\ u = t$ & @ $\tau = T,\ u = t-T$

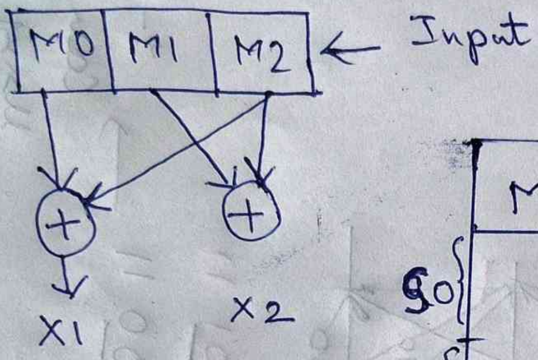
$$\therefore \hat{x}(t) = \frac{2}{\pi} \int_{t}^{t-T} \frac{1}{u}(-du) = -\frac{2}{\pi} \int_{t}^{t-T} \frac{1}{u} du$$

$$= -\frac{2}{\pi} \ln [u]_{t}^{t-T} = \frac{-2}{\pi} \left[\ln(t-T) - \ln(t) \right]$$

$$= \frac{2}{\pi} \ln\left(\frac{t}{t-T} \right) \text{ Ans}$$

Ques:-3 Consider a (2,1,2) Convolutional encoder with g1 = 101 & g2 = 011. Decode the Code Sequence {11, 01, 01, 10, 00, 01, 11} using Viterbi algorithm.

Solⁿ



$X1 = M0 \oplus M2$

$X2 = M1 \oplus M2$

	M0	M1	M2		X1	X2
S0	0	0	0	S0	0	0
	0	0	1	S1	1	1
S1	0	1	0	S2	0	1
	0	1	1	S3	1	0
S2	1	0	0	S0	1	0
	1	0	1	S1	0	1
S3	1	1	0	S2	1	1
	1	1	1	S3	0	0

M0M1 Shows the Current state

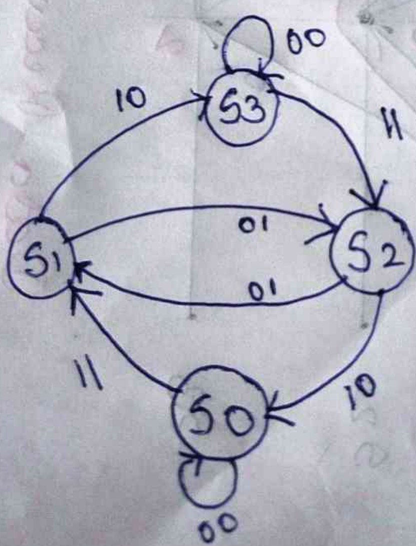
M1M2 Shows next state

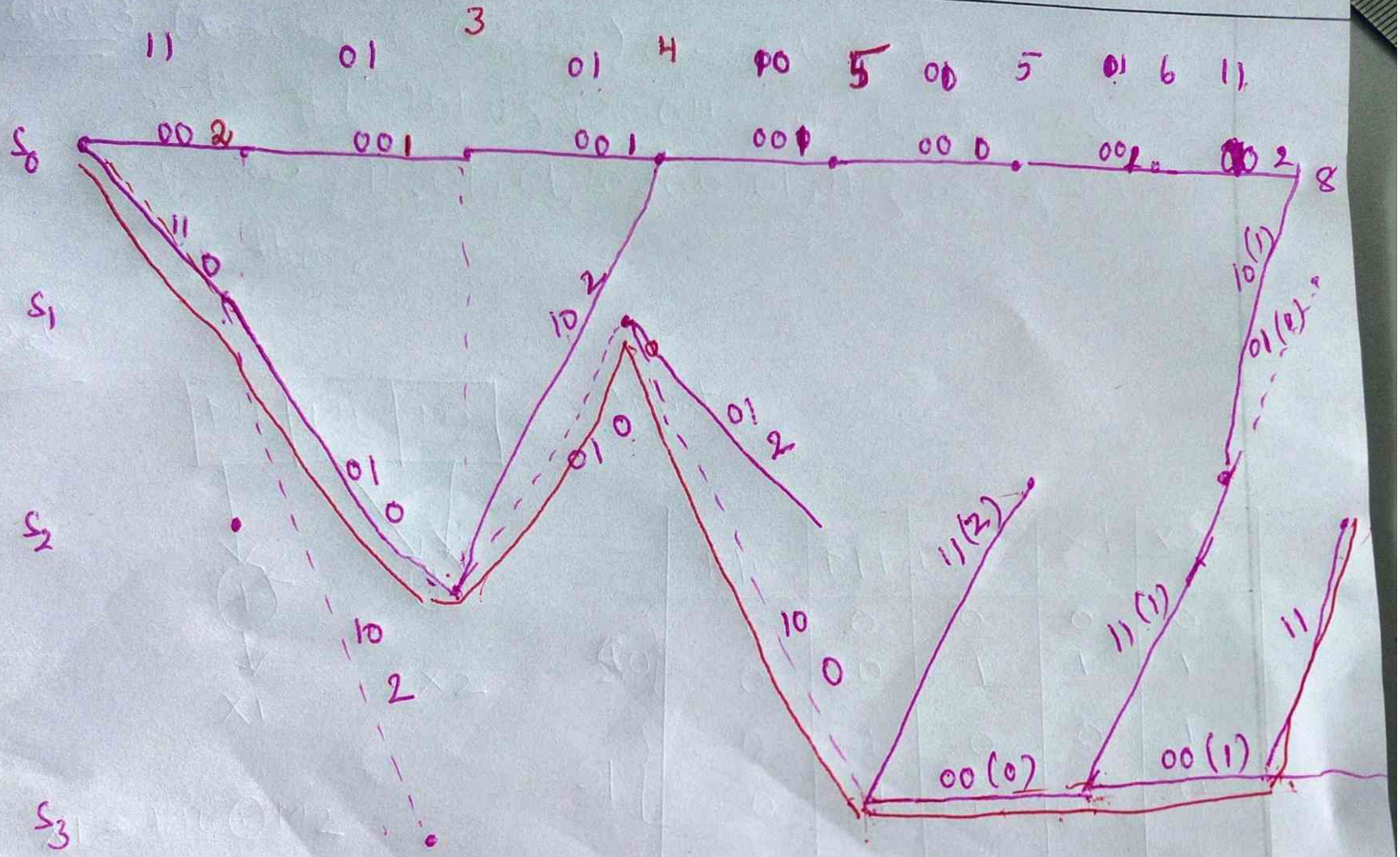
S0 : 00

S1 : 01

S2 : 10

S3 : 11



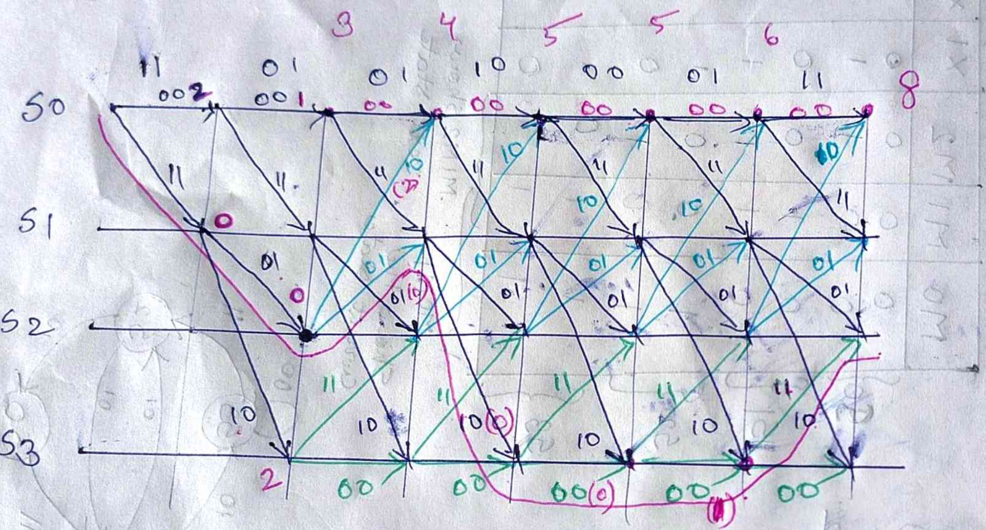


11 01 01 10 00 01 11

11 01 01 10 00 00 11

1 0 1 1 0 1 0

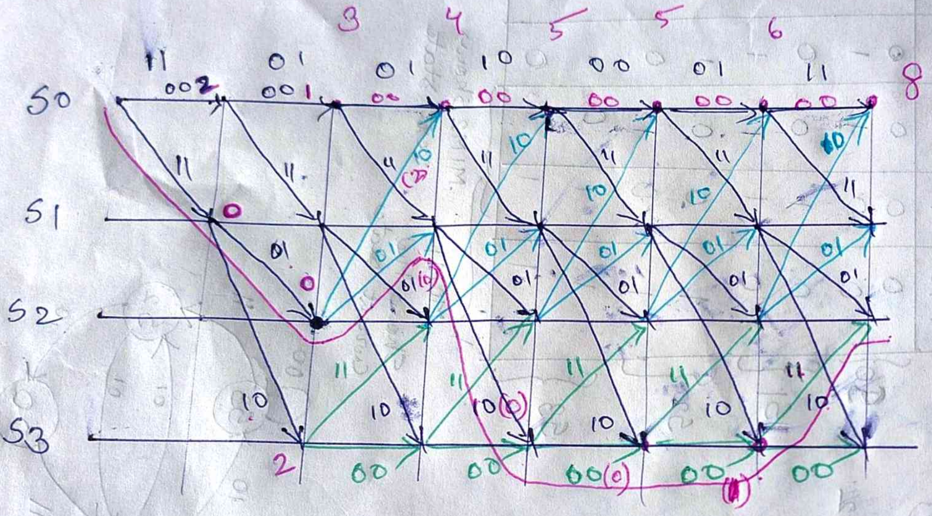
decoding seq



Data Received :- 11 01 01 10 00 <u>01</u> 11

Sequence received from Viterbi Algo. Path :- 11 01 01 10 00 <u>00</u> 11 → 1-Bit Error. Can be corrected using Error Control Coding.

Decoding Sequence :- 1011110 Ans



Data Received :- 11 01 01 10 00 01 11

Sequence received from Viterbi Algo. Path :- 11 01 01 10 00 00 11 → 1-Bit error. Can be corrected using error control coding.

Decoding sequence :- 1011110 Ans

QPSK {Quadrature Phase Shift Keying}

In QPSK, the phase of the carrier takes one of the four equally spaced values such as $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

For this set of values, we may define the transmitted signal as

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], \quad 0 \le t \le T$$

$i = 1, 2, 3, 4$

$f_c = n/T$

n nonzero integer.

Here, T is the symbol duration & E is the Energy of each symbol.

Each possible value of the phase corresponds to a pair of bits (dibit).

$$S_i(t) = \sqrt{\frac{2E}{T}} \cdot \cos\left((2i-1)\frac{\pi}{4}\right) \cdot \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \cdot \sin\left[(2i-1)\frac{\pi}{4}\right] \cdot \sin(2\pi f_c t)$$

$0 \le t \le T$

$i = 1, 2, 3, 4.$

$E = 2E_b$ and $T = 2T_b$

Basis functions are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \cdot \sin(2\pi f_c t), \quad 0 \le t \le T$$

$$\therefore S_i(t) = \sqrt{E} \cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t) - \sqrt{E} \sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2(t), \quad 0 \le t \le T$$

$\therefore$ The coordinates of message points are :-

$$\begin{bmatrix} \sqrt{E} \cdot \cos\left[(2i-1)\pi/4\right] \\ -\sqrt{E} \sin\left[(2i-1)\pi/4\right] \end{bmatrix} \quad i = 1, 2, 3, 4$$

i	Phase	Coordinates	dibits
1	$\frac{\pi}{4}$	$\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	10
2	$\frac{3\pi}{4}$	$-\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	00
3	$\frac{5\pi}{4}$	$-\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	01
4	$\frac{7\pi}{4}$	$\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	11

Probability of Error

Suppose that $S_4(t)$ was transmitted from the block diagram of receiver. We have

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt$$

$$= \int_0^T [S_4(t) + w(t)]\,\phi_1(t)\,dt$$

$$= \int_0^T S_4(t)\cdot\phi_1(t)\,dt + \int_0^T w(t)\cdot\phi_1(t)\,dt$$

$$= \sqrt{\frac{E}{2}} + w_1 \quad \text{--- (1)}$$

↳ Coordinate of $S_4(t)$ with respect to $\phi_1(t)$

Mean of X_1 when $S_4(t)$ was transmitted. i.e. 11 was transmitted.

$$\mu_1 = E[X_1] = \sqrt{\frac{E}{2}} \quad \cdots (2)$$

Variance of x_1 when $S_4(t)$ was transmitted,

$$a_1^2 = \frac{N_0}{2} \quad \text{--- (3)}$$

$$x_2 = \int_0^T x(t) \cdot \phi_2(t)\,dt = \int_0^T [S_4(t) + \omega(t)]\,\phi_2(t)\,dt$$

$$= \int_0^T S_4(t) \cdot \phi_2(t)\,dt + \int_0^T \omega(t) \cdot \phi_2(t)\,dt$$

$$= \sqrt{\frac{E}{2}} + \omega_2 \quad \text{--- (4)}$$

↳ coordinate of $S_4(t)$ w.r.t. $\phi_2(t)$

Mean of x_2 when $S_4(t)$ is transmitted

$$\mu_2 = \sqrt{\frac{E}{2}} \quad \text{--- (5)}$$

Variance of x_2 when $S_4(t)$ is transmitted

$$a_2^2 = \frac{N_0}{2} \quad \text{--- (6)}$$

$\therefore$ PDF of x_1 when $S_4(t)$ was transmitted,

$$f_{x_1}\left(x_1 | 11\right) = \frac{1}{\sqrt{2\pi a_1^2}}\, e^{-\frac{(x_1 - \mu_1)^2}{2a_1^2}}$$

$$= \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{\left(x_1 - \sqrt{E/2}\right)^2}{N_0}} \quad \text{---- (7)}$$

$\therefore$ PDF of x_2 when $S_4(t)$ was transmitted,

$$f_{x_2}\left(x_2 | 11\right) = \frac{1}{\sqrt{2\pi a_2^2}}\, e^{-\frac{\left(x_2 - \sqrt{\frac{E}{2}}\right)^2}{2a_2^2}}$$

$$= \frac{1}{\sqrt{\pi N_0}}\, e^{-\frac{\left(x_2 - \sqrt{\frac{E}{2}}\right)^2}{N_0}} \quad \text{---- (8)}$$

When $S_4(t)$ is transmitted, correct decision is made when $x_1 > 0$ and $x_2 > 0$, so that the received signal point lies in the first quadrature of the signal-space diagram.

$\therefore$ Probability of correct decision when $S_4(t)$ is transmitted,

$$P_c(11) = P(X_1 > 0)\,P(X_2 > 0) \quad \text{---- (9)}$$

$$P(X_1 > 0) = \int_0^\infty f_{X_1}(x_1/11)\,dx_1$$

$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}}\, e^{\frac{-\left(x_1 - \sqrt{\frac{E}{2}}\right)^2}{N_0}} \cdot dx_1 \quad \text{---- (10)}$$

Put $\dfrac{\left(x_1 - \sqrt{E/2}\right)^2}{N_0} = z^2/2$ ---- (11)

$$\frac{x_1 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}} = \frac{z}{\sqrt{2}}$$

$$\therefore\ dx_1 = \sqrt{\frac{N_0}{2}}\,dz \quad \text{---- (12)}$$

when $x_1 = 0$, $z = \sqrt{\frac{E}{N_0}}$ ---- (13)

when $x_1 = \infty$, $z = \infty$ ---- (14)

using (11), (12), (13), (14) we may write (10) as,

$$P(X_1 > 0) = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E}{N_0}}}^{\infty} e^{-z^2/2} \sqrt{\frac{N_0}{2}}\,dz$$

$$P(x_1 > 0) = \frac{1}{\sqrt{2\pi}} \int_{-\sqrt{E/N_0}}^{\infty} e^{-z^2/2} dz$$

$$= Q\left(-\sqrt{\frac{E}{N_0}}\right) = 1 - Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \text{---(15)}$$

Similarly, we can prove that, $P(x_2>0)$ when $S_1(t)$ is transmitted.

$$P(x_2 > 0) = 1 - Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \text{---}\ (16)$$

Using (15) and (16), we may write (9) as.

$$P_c(11) = \left[1 - Q\left(\sqrt{\frac{E}{N_0}}\right)\right]^2$$

$$= 1 + \underbrace{\left[Q\left(\sqrt{\frac{E}{N_0}}\right)\right]^2}_{\text{Negligible}} - 2Q\left(\sqrt{\frac{E}{N_0}}\right)$$

$$P_c(11) \approx 1 - 2Q\left(\sqrt{\frac{E}{N_0}}\right)$$

This is the probability of correct decision when '11' is transmitted.

$\therefore$ Probability of error when '11' is transmitted is given by

$$P_e(11) = 1 - P_c(11)$$

$$P_e(11) = 2Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \text{---}(18)$$

Similarly, we may prove that

$$P_e(00) = P_e(01) = P_e(10) = 2Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \text{---(19)}$$

Assuming equiprobable dibits, we get, average probability of symbol error

$$P_e^{symbol} = 2Q\left(\sqrt{\frac{E}{N_0}}\right) \quad \text{---(20)}$$

But each symbol represents 2 bits.

Hence, average probability of bit error,

$$P_e^{bit} = \frac{1}{2} 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{---(21)}$$

Q-5 a) In QAM, carrier experiences amplitude as well as phase modulation.

M-ary QAM is 2 dimensional & involves 2 basis functions.

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t),$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

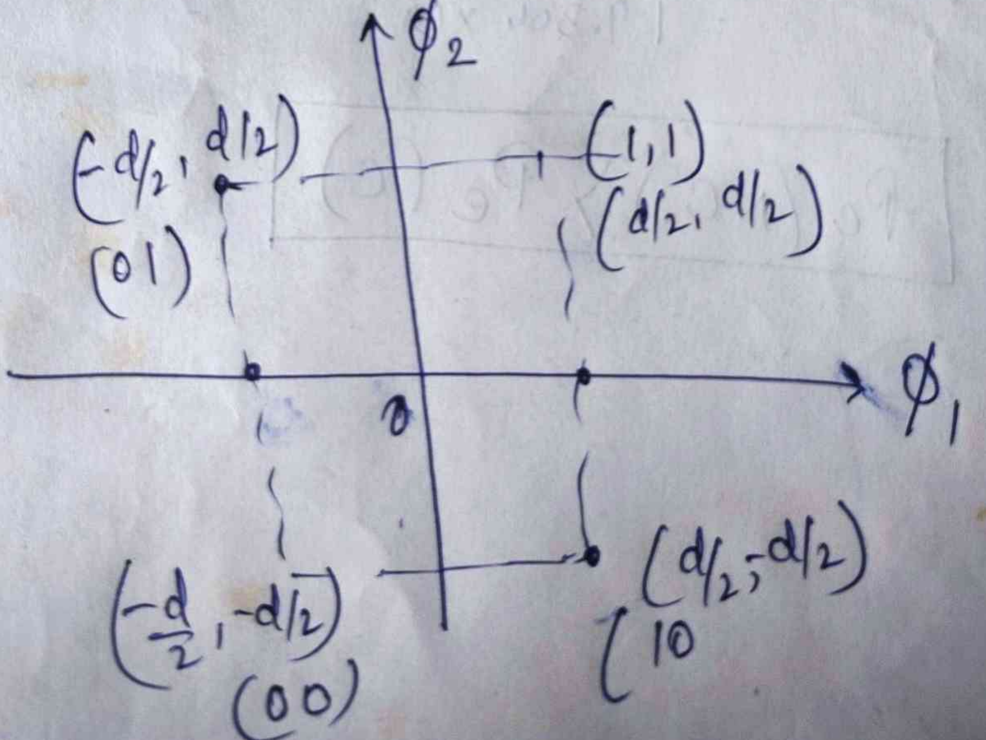
QAM signal is given by :-

$$S_k(t) = \sqrt{\frac{2E_b}{T}}\, a_k \cos 2\pi f_c t - \sqrt{\frac{2E_b}{T}}\, b_k \sin 2\pi f_c t,$$

$$0 \le t \le T$$

$$k = 1, 2, \ldots M.$$

let us consider d is the distance between adjacent signal.



Q-5b)

$R_b = 10^6$ bps $= 10$ Mbps

$\therefore T_b = \frac{1}{10^{+6}} = 1 \times 10^{-6}$ Sec.

PSD $= 10^{-10}$ W/Hz.

$\therefore \frac{N_0}{2} = 10^{-10}$ W/Hz.

Amplitude $= 1\ \mu V$

$\therefore \sqrt{\frac{2E_b}{T_b}} = 1 \times 10^{-6}$.

$$\frac{2E_b}{T_b} = 10^{-12}$$

$$\therefore E_b = \frac{10^{-12} \times 10^{-6}}{2} = 0.5 \times 10^{-18}\ J.$$

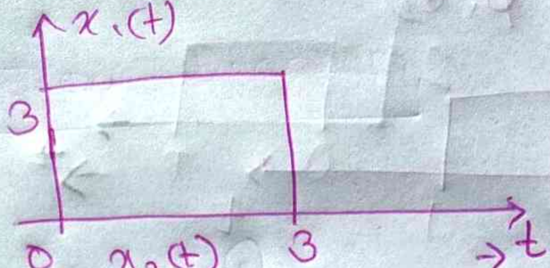
$\therefore$ Pe for PSK coherent detection,

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

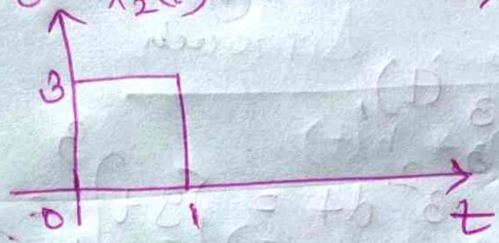
$$= Q\left(\sqrt{\frac{2 \times 0.5 \times 10^{-18}}{10^{-10}}}\right) = Q(7.07 \times 10^{-5})$$

$$\boxed{P_e = 0.4999}$$

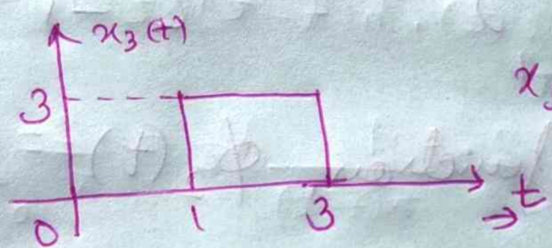
Q-6:- Sol



$x_1(t) = 3 \;,\; 0 \le t \le 3$
$\quad\;\; 0,$ Elsewhere



$x_2(t) = 3 \;,\; 0 \le t \le 1$
$\quad\;\; 0,$ Elsewhere



$x_3(t) = 3 \;,\; 1 \le t \le 3$
$\quad\;\; 0,$ Elsewhere.

Note that:- $x_1(t) = x_2(t) + x_3(t)$

$\therefore$ The given signals are linearly dependent.

Consider $x_2(t)$ & $x_3(t)$.

These 2 are linearly independent.

$\therefore$ There must be 2 basis functions.

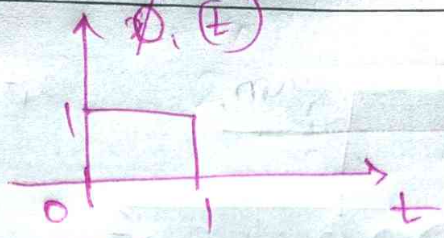
$$\int_0^3 x_2(t) \cdot x_3(t) = 0.$$

$\therefore$ $x_2(t)$ & $x_3(t)$ are orthogonal to each other from 0 to 3.

$\therefore$ Energy of $x_2(t)$,

$$E_2 = \int_0^1 3^2\, dt = 9t\Big]_0^1 = 9$$

$\therefore$ Basis function, $\phi_1(t) = \dfrac{x_2(t)}{\sqrt{9}} = \dfrac{x_2(t)}{3}$

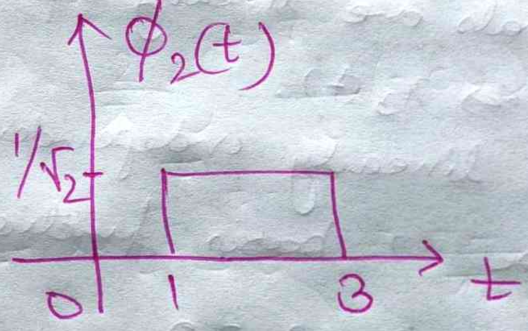


Now,
Energy of $x_3(t)$,

$$\mathcal{E}_3 = \int_1^3 3^2\, dt = \left[9t\right]_1^3 = 9 \times 2 = 18.$$

$\therefore$ Basis function, $\phi_2(t) = \dfrac{x_3(t)}{\sqrt{\mathcal{E}_3}}$

$$= \frac{x_3(t)}{\sqrt{18}} = \frac{x_3(t)}{3\sqrt{2}}$$



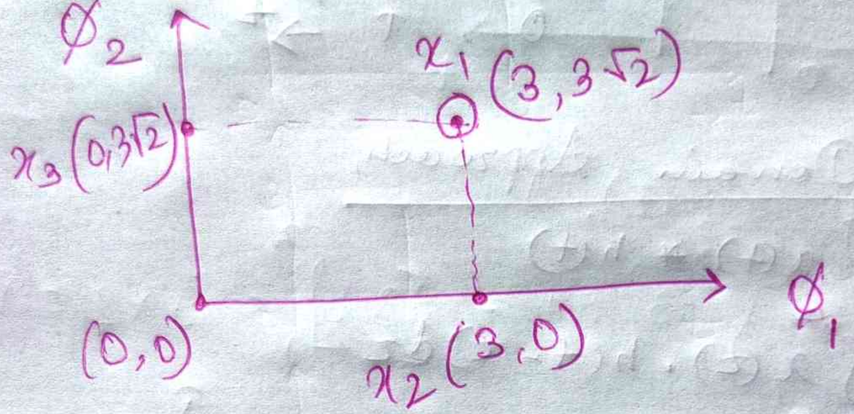
Expressing the signals as a linear combination of basis functions,

$$x_1(t) = x_2(t) + x_3(t) = 3\phi_1(t) + 3\sqrt{2}\,\phi_2(t)$$

$$x_2(t) = 3\,\phi_1(t) + 0\cdot\phi_2(t)$$

$$x_3(t) = 0\cdot\phi_1(t) + 3\sqrt{2}\cdot\phi_2(t)$$

Constellation Diagram,

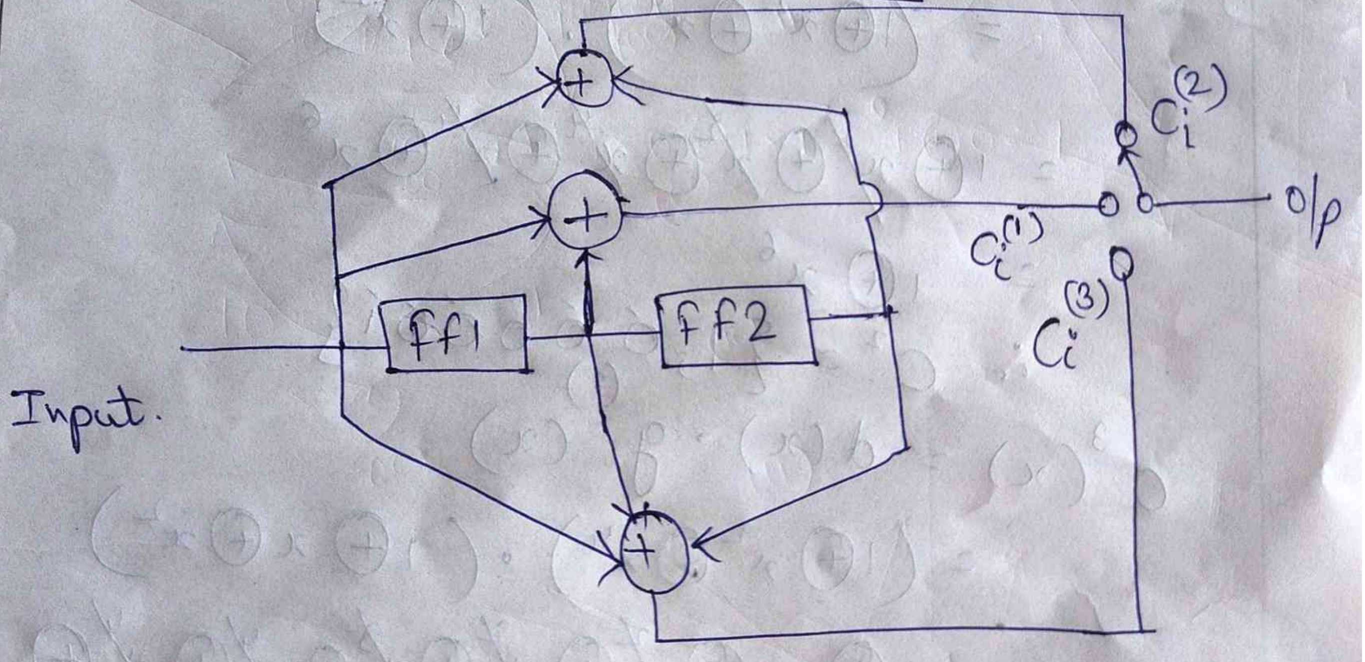


Q-7 A Convolutional Encoder (3,1,2) has generator sequences are $g(1) = (110)$, $g(2) = (101)$ & $g(3) = (111)$.

a) Draw Encoder block diagram.

b) Find Code word Corresponding to the information sequence (10101) using (i) Time domain & (ii) Transform domain approaches.

Solⁿ (a) Encoder Diagram



(b) Transform Domain Approach :-

$$g^{(1)} = 110 \longrightarrow g^{(1)}(x) = 1 \oplus x$$

$$g^{(2)} = 101 \longrightarrow g^{(2)}(x) = 1 \oplus x^2$$

$$g^{(3)} = 111 \longrightarrow g^{(3)}(x) = 1 \oplus x \oplus x^2$$

$D = 10101$

$d(x) = 1 \oplus x^2 \oplus x^4$

$\therefore C^1(x) = d(x) \cdot g^{(1)}(x)$

$= (1 \oplus x^2 \oplus x^4)(1 \oplus x)$

$= 1 \oplus x \oplus x^2 \oplus x^3 \oplus x^4 \oplus x^5$

$C^1 = 1, 1, 1, 1, 1, 1, 0$

$C^2(x) = d(x) \cdot g^{(2)}(x)$

$= (1 \oplus x^2 \oplus x^4) \cdot (1 \oplus x^2)$

$= 1 \oplus \cancel{x^2} \oplus \cancel{x^2} \oplus \cancel{x^4} \oplus \cancel{x^4} \oplus x^6$

$= 1 \oplus x^6$

$\therefore C^2 = \{1, 0, 0, 0, 0, 0, 1\}$

$C^3(x) = d(x) \cdot g^{(3)}(x)$

$= (1 \oplus x^2 \oplus x^4) \cdot (1 \oplus x \oplus x^2)$

$= 1 \oplus x \oplus \cancel{x^2} \oplus \cancel{x^2} \oplus \cancel{x^3} \oplus \cancel{x^4} \oplus \cancel{x^4} \oplus \cancel{x^5} \oplus x^6$

$= 1 \oplus x \oplus x^6$

$C^3 = \{1, 1, 0, 0, 0, 0, 1\}$

$\therefore C = \{111, 101, 100, 100, 100, 100, 011\}$

ii) Time Domain Approach :-

$$c_i^{(1)} = g^{(1)} * m.$$

$$= \sum_{l=0}^{m} g_l^{(1)} m_{i-l} = \sum_{l=0}^{2} g_l^{(1)} m_{i-1}$$

$i=0$:-

$g_l^{(1)}$: 1 1 1

m_{0-l} : 1 0 1 0 1

$c_0^{(1)} = 1 \times 1 = 1$

$i=1$

g_l : 1 1 1

m_{1-l} : 1 0 1 0 1

$c_1^{(1)} : 1 \times 0 \oplus 1 \times 1 = 1 \oplus 0 = 1$

$i=2$

$i=3$ $\quad g_l^3 : \boxed{1\ 1\ 1}$

$\quad m_{3-l} : 1\ 0\ 1\ 0\ 1$

$C_2^{(3)}\ 1\times 0 \oplus 1\times 1 \oplus 1\times 0 = 0 \oplus 1 \oplus 0 = 1$

$i=4$ $\quad g_l^3 : 1\ 1\ 1$

$\quad m_{4-l} : 1\ 0\ 1\ 0\ 1$

$C_2^{(4)} : 1\times 1 \oplus 1\times 0 \oplus 1\times 1 = 1 \oplus 0 \oplus 1 = 1$

Similarly $i=5$, $i=6$ ~~$i=7$~~, we get

CMR

$c_2^{(5)} = 1$; $c_2^{(6)} = 1$; $c_2^{(7)} = 0$;

$$C_i^{(1)} = \{1, 1, 1, 1, 1, 1, 1, 0\}$$

$$c_i^{(2)} = \{1, 0, 0, 0, 0, 0, 1\}$$

$$c_i^{(3)} = \{1, 1, 0, 0, 0, 0, 1\}$$

{ Similary proceeding with Same techniques for other Calculations. }.

$$\therefore \quad C_i = \{111, 101, 100, 100, 100, 100, 011\}$$