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Internal Assessment Test – I November - 2025

Sub:	**Mathematics-1 for ECE Stream**							Code:	1BMATE101
Date:	04-11-2025	Duration:	90 mins	Max Marks:	50	Sem:	I	SEC	**M, N, O, P (CHE CYCLE)**

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	With usual notations prove that. $\cot \varphi = \frac{1}{r}\left(\frac{dr}{d\theta}\right)$	[08]	CO1	L3
2	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$	[07]	CO5	L3
3	Solve the following system using Gauss Seidel method **(perform only 3 iterations)**: $10x + y + z = 12, \quad x + 10y + z = 12, \quad x + y + 10z = 12.$	[07]	CO5	L3

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3	Solve the following system using Seidel method **(perform only 3 iterations)**: $10x + y + z = 12, \quad x + 10y + z = 12, \quad x + y + 10z = 12.$	[07]	CO5	L3

4	Test for consistency and solve 5x + y + 3z=20, 2x + 5y + 2z=18, 3x +2y + z =14;	[07]	CO5	L3
5	Find the pedal Equation for the curve $r^n = a^n \cos n\theta$	[07]	CO1	L3
6	Find the angle between the following pairs of curves r=a(1+sin θ) and r=a(1-sin θ)	[07]	CO1	L3
7	Find the radius of curvature of the curve $x^4 + y^4 = 1$ at the point (1, 1).	[07]	CO1	L3
8	Find the numerically largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ using the initial vector $[1, 0, 0]^T$	[07]	CO5	L3

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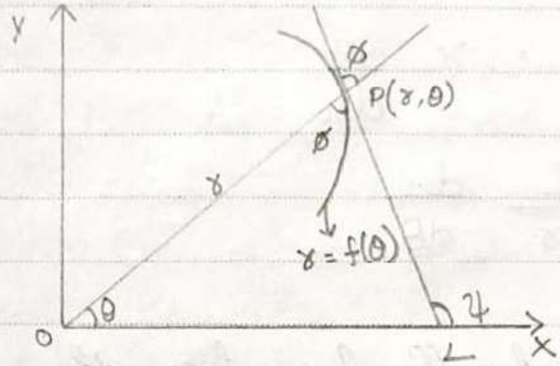
* Angle between radius vector and tangent (ϕ)



- Let $P(r,\theta)$ be any point on the curve $r = f(\theta)$ with angle $\angle XOP = \theta$ and $OP = r$
- Let PL be the tangent to the curve with angle sine and phi be the angle between radius vector and tangent

From fig, $\psi = \theta + \phi$ (7m)

$$\tan(\psi) = \tan(\theta+\phi)$$

$$\tan(\psi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta . \tan\phi}$$

Let $x = r\cos\theta$ and $y = r\sin\theta$

$$\tan\psi = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\tan\psi = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{r\cos\theta + \sin\theta\, r'}{-r\sin\theta + \cos\theta\, r'} \quad \left[\because r' = \frac{dr}{d\theta}\right]$$

$\div$ N & D by $r'\cos\theta$ to the RHS

$$\tan\psi = \frac{\frac{r}{r'} + \tan\theta}{\frac{-r}{r'}\tan\theta + 1}$$

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$$\tan \psi = \frac{\tan\theta + r/r'}{1 - \tan\theta \, r/r'} \longrightarrow (2)$$

Comparing Eq.(1) & Eq.(2)

$$\tan \phi = \frac{r}{r'}$$

$$\tan \phi = \frac{r}{dr/d\theta} = r \frac{d\theta}{dr}$$

or $\cot \phi = \frac{1}{r} \frac{dr}{d\theta}$

DATE ____/____/____

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2. $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$\rightarrow R_1 \leftrightarrow R_2$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 1R_2$

$R_4 \rightarrow R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ The rank of the matrix is 2

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$
$$x = \frac{1}{10}(12 - y - z)$$
$$y = \frac{1}{10}(12 - x - z)$$
$$z = \frac{1}{10}(12 - x - y)$$

Let $x_0 = y_0 = z_0 = 0$

$$x_1 = \frac{1}{10}(12 - 0 - 0) = 1.2$$
$$y_1 = \frac{1}{10}(12 - 1.2 - 0) = 1.08$$
$$z_1 = \frac{1}{10}(12 - 1.2 - 1.08) = 0.972$$

$x_1 = 1.2$ $\quad y = 1.08$ $\quad z = 0.972$

$$x_2 = \frac{1}{10}(12 - 1.08 - 0.972) = 0.9948$$

$$y_2 = \frac{1}{10}(12 - 0.9948 - 0.972) = 1.003$$

$$z_2 = \frac{1}{10}(12 - 0.9948 - 1.003) = 1.0002$$

$x_2 = 0.9948$ $\quad y_2 = 1.003$ $\quad z_2 = 1.0002$

$$x_3 = \frac{1}{10}(12 - 1.003 - 1.0002) = 0.99$$

$$y_3 = \frac{1}{10}(12 - 0.999 - 1.0002) = 1.00001$$

$$z_3 = \frac{1}{10}(12 - 0.999 - 1.00008) = 1.00009$$

$x_3 = 0.999$ $\quad y_3 = 1.00001$ $\quad z_3 = 1.00009$

$$x_4 = \frac{1}{10}(12 - 1.00001 - 1.00009) = 0.9999$$

$$y_4 = \frac{1}{10}(12 - 0.9999 - 1.00009) = 1.000001$$

$$z_4 = \frac{1}{10}(12 - 0.9999 - 1.000001) = 1.000009$$

$x = 0.9999$ $\quad y = 1.000001$ $\quad z = 1.000009$

Test for Consistency and solve

$5x+y+3z=20;\ 2x+5y+2z=18;\ 3x+2y+z=14$

The given SLE can be written as $AX=B$ where

$$A=\begin{bmatrix}5&1&3\\2&5&2\\3&2&1\end{bmatrix}\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix}\quad B=\begin{bmatrix}20\\18\\14\end{bmatrix}$$

Consider the augmented matrix $\left[\begin{array}{ccc|c}5&1&3&20\\2&5&2&18\\3&2&1&14\end{array}\right]=[A|B]$

$R_2\to 5R_2-2R_1$
$R_3\to 5R_3-3R_1$ $\left[\begin{array}{ccc|c}5&1&3&20\\0&23&4&50\\0&7&-4&10\end{array}\right]$

$R_2\to \frac{R_2}{23}$ $\begin{bmatrix}5&1&3&20\\0&1&4/23&50/23\\0&7&-4&10\end{bmatrix}$. $R_3\to R_3-7R_2$. $\begin{bmatrix}5&1&3&20\\0&1&4/23&50/23\\0&0&\frac{-120}{23}&\frac{-120}{23}\end{bmatrix}$

$\Rightarrow$ Rank(A) = Rank $(A|B)=3$ So The system is Consistent.

Now the system can be written as.

$$5x+y+3z=20;$$
$$y+\frac{4z}{23}=\frac{50}{23}$$
$$-\frac{120z}{23}=-\frac{120}{23}$$

$\Rightarrow z=1$ using this in $y+\frac{4z}{23}=\frac{50}{23}$, $y=\frac{50}{23}-\frac{4}{23}=2$

Using y & z in $5x+y+3z=20$, $5x=20-2-3=15\Rightarrow x=3$

$\therefore$ The soln is $x=3,\ y=2;\ z=1$

$r^n = a^n \cos n\theta$

$n \log r = n \log a + \log \cos n\theta$

$\frac{n}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}(-n \sin\theta)$

$\cot\phi = -\tan\theta$

$\cot\phi = \cot(\pi/2 + n\theta)$

$\phi = \pi/2 + n\theta$

Let $p = r\sin\phi$

$p = r\sin(\pi/2 + n\theta)$

$= r\cos n\theta \longrightarrow (2)$

$(2) \rightarrow \cos n\theta = \frac{p}{r}$

$(1) \rightarrow r^n = a^n \quad p/r$

$a^n p = r^{n+1}$

$r = a(1+\sin\theta)$

$r = a(1-\sin\theta)$

$r_1 = a(1+\sin\theta)$

taking log on b.s

$\log r_1 = \log a + \log(1+\sin\theta)$

$$\frac{1}{r_1}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1+\sin\theta}\cdot\cos\theta$$

$$\cot\phi = \frac{\cos\theta}{1+\sin\theta}$$

$$\tan\phi = \frac{1+\sin\theta}{\cos\theta}$$

$r_2 = a(1-\sin\theta)$

log on b.s

$\log r_2 = \log a + \log(1-\sin\theta)$

$$\frac{1}{r_2}\frac{dr}{d\theta} = 0 + \frac{1}{1-\sin\theta}\cdot(-\cos\theta)$$

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$$\cot \phi_2 = \frac{-\cos\theta}{1-\sin\theta}$$

$$\tan \phi_2 = \frac{-(1-\sin\theta)}{\cos\theta}$$

consider $\tan\phi_1 \cdot \tan\phi_2$

$$\frac{1+\sin\theta}{\cos\theta} \times \frac{-(1-\sin\theta)}{\cos\theta}$$

$$= \frac{-(1-\sin^2\theta)}{\cos^2\theta} \qquad [(a+b)(a-b) = a^2-b^2]$$

$$= \frac{-\cos^2\theta}{\cos^2\theta}$$

$$= -1$$

Horizon
Date
Page

3. $x^4 + y^4 = 1$

diff w.r.t x

$$4x^3 + 4y^3 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x^3}{4y^3} = \frac{-x^3}{y^3} \longrightarrow (1)$$

again diff

$$\frac{d^2y}{dx^2} = -\frac{d}{dx}\left(\frac{x^3}{y^3}\right)$$

$$= \frac{-3x^2y^3 - x^3\left(3y^2 \cdot \frac{dy}{dx}\right)}{(y^3)^2} \longrightarrow (2)$$

Substitute (1) in (2)

$$\frac{d^2y}{dx^2} = \frac{-3x^2y^3 - 3x^3y^2\left(\frac{-x^3}{y^3}\right)}{y^6}$$

$$= \frac{-3x^2y^3 + \frac{3x^6}{y}}{y^6}$$

$$= \frac{-3x^2y^4 + 3x^6}{y^7} = -\frac{3x^2(y^4 + x^4)}{y^7}$$

At $(1,1) \Longrightarrow x = 1, \; y = 1$

$$y_1 = \frac{dy}{dx} = \frac{-1^3}{1^3} = -1$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{3(1)^2(1^4+1^4)}{(1)^7} = -\frac{3(2)}{1} = -6$$

radius of curvature

$$\rho = \frac{(1+y_1^2)^{3/2}}{|y_2|}$$

$$= \frac{(1+(-1)^2)^{3/2}}{|-6|} = \frac{[1+1]^{3/2}}{6}$$

$$= \frac{2^{3/2}}{6} = \frac{\cancel{2}^{1}\sqrt{2}}{\cancel{6}_3}$$

$$= \frac{\sqrt{2}}{3}$$

Note : till 4-5 iterations & till 2 consecutive λ values are same

ex: $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$AX^1 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^1 X^1$$

$$[A^1X]^2 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix}$$

$$[AX]^3 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.92 \end{bmatrix}$$

$$[AX]^4 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.92 \end{bmatrix} = \begin{bmatrix} 2.92 \\ 0 \\ 2.84 \end{bmatrix} = 2.92 \begin{bmatrix} 1 \\ 0 \\ 0.97 \end{bmatrix}$$

$$[AX]^5 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.97 \end{bmatrix} = \begin{bmatrix} 2.97 \\ 0 \\ 2.94 \end{bmatrix} = 2.97 \begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix}$$

$\max \lambda = 2.97 \qquad \max X = \begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix}$

Date
Page

At $(1,1) \Longrightarrow x = 1, \ y = 1$

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$$= \frac{2^{3/2}}{6} = \frac{\cancel{2}^{1}\sqrt{2}}{\cancel{6}_{3}}$$

$$= \frac{\sqrt{2}}{3}$$

Horizon

Date
Page

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$$\cot \phi_2 = \frac{-\cos\theta}{1-\sin\theta}$$

$$\tan \phi_2 = \frac{-(1-\sin\theta)}{\cos\theta}$$

consider $\tan\phi_1 \cdot \tan\phi_2$

$$\frac{1+\sin\theta}{\cos\theta} \times \frac{-(1-\sin\theta)}{\cos\theta}$$

$$= \frac{-(1-\sin^2\theta)}{\cos^2\theta} \qquad [(a+b)(a-b) = a^2-b^2]$$

$$= \frac{-\cos^2\theta}{\cos^2\theta}$$

$$= -1$$

$r = a(1+\sin\theta)$

$r = a(1-\sin\theta)$

$r_1 = a(1+\sin\theta)$

taking log on b.s

$\log r_1 = \log a + \log(1+\sin\theta)$

$$\frac{1}{r_1}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1+\sin\theta}\cdot\cos\theta$$

$$\cot\phi = \frac{\cos\theta}{1+\sin\theta}$$

$$\tan\phi = \frac{1+\sin\theta}{\cos\theta}$$

$r_2 = a(1-\sin\theta)$

log on b.s

$\log r_2 = \log a + \log(1-\sin\theta)$

$$\frac{1}{r_2}\frac{dr}{d\theta} = 0 + \frac{1}{1-\sin\theta}\cdot(-\cos\theta)$$

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$r^n = a^n \cos n\theta$

$n \log r = n \log a + \log \cos n\theta$

$$\frac{n}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}(-n \sin\theta)$$

$\cot\phi = -\tan\theta$

$\cot\phi = \cot(\pi/2 + n\theta)$

$\phi = \pi/2 + n\theta$

Let $p = r\sin\phi$

$p = r\sin(\pi/2 + n\theta)$

$= r\cos n\theta \longrightarrow (2)$

$(2) \to \cos n\theta = \frac{p}{r}$

$(1) \to r^n = a^n \; p/r$

$a^n p = r^{n+1}$

Test for Consistency and solve

$5x+y+3z=20$; $2x+5y+2z=18$; $3x+2y+z=14$

The given SLE can be written as $AX=B$ where

$$A=\begin{bmatrix}5&1&3\\2&5&2\\3&2&1\end{bmatrix} \quad X=\begin{bmatrix}x\\y\\z\end{bmatrix} \quad B=\begin{bmatrix}20\\18\\14\end{bmatrix}$$

Consider the augmented matrix $\left[\begin{array}{ccc|c}5&1&3&20\\2&5&2&18\\3&2&1&14\end{array}\right]=[A|B]$

$R_2\to 5R_2-2R_1$
$R_3\to 5R_3-3R_1$ $\left[\begin{array}{ccc|c}5&1&3&20\\0&23&4&50\\0&7&-4&10\end{array}\right]$

$R_2\to \frac{R_2}{23}$ $\begin{bmatrix}5&1&3&20\\0&1&4/23&50/23\\0&7&-4&10\end{bmatrix}$. $R_3\to R_3-7R_2$. $\begin{bmatrix}5&1&3&20\\0&1&4/23&50/23\\0&0&\frac{-120}{23}&\frac{-120}{23}\end{bmatrix}$

$\Rightarrow$ Rank(A) = Rank(A|B) = 3 So The system is Consistent.

Now The system can be written as.

$5x+y+3z=20$;

$y+\frac{4z}{23}=\frac{50}{23}$

$-\frac{120z}{23}=-\frac{120}{23}$

$\Rightarrow z=1$ using this in $y+\frac{4z}{23}=\frac{50}{23}$, $y=\frac{50}{23}-\frac{4}{23}=2$

Using y & z in $5x+y+3z=20$, $5x=20-2-3=15\Rightarrow x=3$

$\therefore$ The soln is $x=3$, $y=2$; $z=1$

DATE ___/___/___ Mo Tu We Th Fr Sa Su

$$x_2 = \frac{1}{10}(12 - 1.08 - 0.972) = 0.9948$$

$$y_2 = \frac{1}{10}(12 - 0.9948 - 0.972) = 1.003$$

$$z_2 = \frac{1}{10}(12 - 0.9948 - 1.003) = 1.0002$$

$x_2 = 0.9948$ $\quad y_2 = 1.003$ $\quad z_2 = 1.0002$

$$x_3 = \frac{1}{10}(12 - 1.003 - 1.0002) = 0.99$$

$$y_3 = \frac{1}{10}(12 - 0.999 - 1.0002) = 1.00001$$

$$z_3 = \frac{1}{10}(12 - 0.999 - 1.00008) = 1.00009$$

$x_3 = 0.999$ $\quad y_3 = 1.00001$ $\quad z_3 = 1.00009$

$$x_4 = \frac{1}{10}(12 - 1.00001 - 1.00009) = 0.9999$$

$$y_4 = \frac{1}{10}(12 - 0.9999 - 1.00009) = 1.000001$$

$$z_4 = \frac{1}{10}(12 - 0.9999 - 1.000001) = 1.000009$$

$x = 0.9999$ $\quad y = 1.000001$ $\quad z = 1.000009$

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

$$x = \frac{1}{10}(12 - y - z)$$

$$y = \frac{1}{10}(12 - x - z)$$

$$z = \frac{1}{10}(12 - x - y)$$

Let $x_0 = y_0 = z_0 = 0$

$$x_1 = \frac{1}{10}(12 - 0 - 0) = 1.2$$

$$y_1 = \frac{1}{10}(12 - 1.2 - 0) = 1.08$$

$$z_1 = \frac{1}{10}(12 - 1.2 - 1.08) = 0.972$$

$x_1 = 1.2 \qquad y = 1.08 \qquad z = 0.972$

DATE ___/___/___

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2. $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$\rightarrow R_1 \longleftrightarrow R_2$

$R_3 \longrightarrow R_3 - 3R_1$

$R_4 \longrightarrow R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - 1R_2$

$R_4 \longrightarrow R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ The rank of the matrix is 2

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$$\tan \psi = \frac{\tan\theta + \gamma/\gamma'}{1 - \tan\theta \, \gamma/\gamma'} \longrightarrow (2)$$

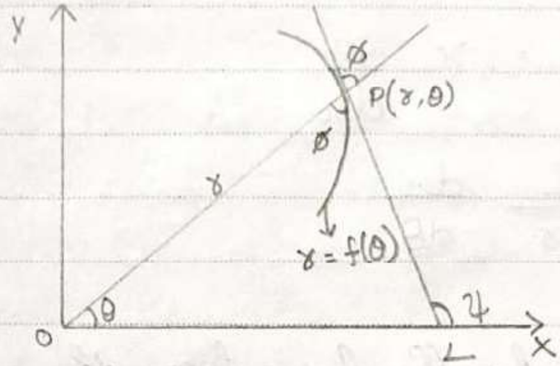
Comparing Eq(1) & Eq(2)

$$\tan \phi = \frac{\gamma}{\gamma'}$$

$$\tan \phi = \frac{\gamma}{dr/d\theta} = \gamma \frac{d\theta}{dr}$$

or $\cot \phi = \frac{1}{\gamma} \frac{dr}{d\theta}$

* Angle between radius vector and tangent (ϕ)



- Let $P(r, \theta)$ be any point on the curve $r = f(\theta)$ with angle $\angle XOP = \theta$ and $OP = r$
- Let PL be the tangent to the curve with angle sire and phi be the angle between radius vector and tangent

From fig, $\psi = \theta + \phi$ (7m)

$$\tan(\psi) = \tan(\theta + \phi)$$

$$\tan(\psi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta . \tan\phi}$$

Let $x = r\cos\theta$ and $y = r\sin\theta$

$$\tan\psi = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\tan\psi = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{r\cos\theta + \sin\theta\, r'}{-r\sin\theta + \cos\theta\, r'} \quad \left[\because r' = \frac{dr}{d\theta}\right]$$

$\div$ N & D by $r'\cos\theta$ to the RHS

$$\tan\psi = \frac{\frac{r}{r'} + \tan\theta}{\frac{-r}{r'}\tan\theta + 1}$$