

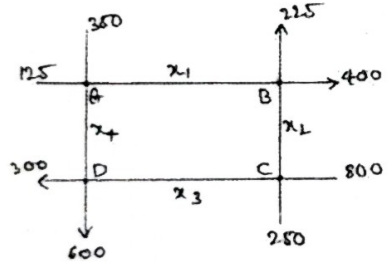
Internal Assessment Test – I November - 2024

Sub:	Mathematics-1 for CSE Stream						Code:	1BMATS101	
Date:	04-11-2025	Duration:	90 mins	Max Marks:	50	Sem:	I	SEC:	A to H (PHY CYCLE)

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	Investigate the values of λ and μ such that the following system may have (i) unique solution, (ii) infinitely many solutions and (iii) no solution. $x + y + z = 6$ $x + 2y + 3z = 10$ $x + 2y + \lambda z = \mu$	[08]	CO3	L3
2	Solve the following system using Gauss elimination method: $x + 2y + z = 3, \quad 2x + 3y + 3z = 10, \quad 3x - y + 2z = 13.$	[07]	CO3	L3
3	Find the rank of the matrix: $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$	[07]	CO3	L3




4	Solve the following system using Gauss Jordan method: $x + y + z = 9, \quad x - 2y + 3z = 8, \quad 2x + y - z = 3.$	[07]	CO3	L3
5	Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to diagonal form.	[07]	CO3	L3
6	If $\vec{F} = \nabla(xy^3z^2)$, find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at $(1, -1, 1)$. Is \vec{F} solenoidal and irrotational?	[07]	CO2	L3
7	If $u=x+y+z$, $v=y+z$ and $z=uvw$. Find the Jacobian of x,y,z with respect to u,v,w ?	[07]	CO1	L3
8	 <p>Find x_1, x_2, x_3 and x_4 to balance the traffic flow.</p>	[07]	CO3	L3

$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$ is the augmented matrix.

$$R_2 \rightarrow -R_1 + R_2, R_3 \rightarrow -R_1 + R_3$$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda - 1 & : & \mu - 6 \end{bmatrix}$$

$$R_3 \rightarrow -R_2 + R_3$$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda - 3 & : & \mu - 10 \end{bmatrix}$$

- (a) *Unique solution* : We must have $\rho[A] = \rho[A:B] = 3$, $\rho[A]$ will be 3 if $(\lambda - 3) \neq 0$ since the other two entries in the last row of A are zero. If $(\lambda - 3) \neq 0$ or $\lambda \neq 3$ irrespective of the value of μ , $\rho[A:B]$ will also be 3.

Hence, the system will have **unique solution** if $\lambda \neq 3$.

- (b) *Infinite solutions* : Here we have $n = 3$ and we need $\rho[A] = \rho[A:B] = r < 3$. We must have $r = 2$ since first row and second row are non zero.

$\rho[A] = \rho[A:B] = 2$ only when the last row of $[A:B]$ is completely zero. This is possible if $\lambda - 3 = 0$, $\mu - 10 = 0$

Hence, the system will have **infinite solution** if $\lambda = 3$ and $\mu = 10$.

- (c) *No solution* : We must have $\rho[A] \neq \rho[A:B]$. By case (a) $\rho[A] = 3$ if

$\lambda \neq 3$ and hence if $\lambda = 3$ we obtain $\rho[A] = 2$.

If we impose $(\mu - 10) \neq 0$ then $\rho[A:B]$ will be 3.

Hence, the system has **no solution** if $\lambda = 3$ and $\mu \neq 10$.

$$x + 2y + z = 3, 2x + 3y + 3z = 10, 3x - y + 2z = 13$$

[Dec 2017]

The augmented matrix of the given system of equations is

$$[A:B] = \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 2 & 3 & 3 & : & 10 \\ 3 & -1 & 2 & : & 13 \end{bmatrix}$$

$$R_2 \rightarrow -2R_1 + R_2, R_3 \rightarrow -3R_1 + R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 0 & -1 & 1 & : & 4 \\ 0 & -7 & -1 & : & 4 \end{bmatrix}$$

$$R_1 \rightarrow 2R_2 + R_1, R_3 \rightarrow -7R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & : & 11 \\ 0 & -1 & 1 & : & 4 \\ 0 & 0 & -8 & : & -24 \end{bmatrix}$$

$$R_1 \rightarrow 3/8 \cdot R_3 + R_1, R_2 \rightarrow 1/8 \cdot R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 2 \\ 0 & -1 & 0 & : & 1 \\ 0 & 0 & -8 & : & -24 \end{bmatrix}$$

We now have, $x = 2, -y = 1, -8z = -24$

$$\Rightarrow x = 2, y = -1, z = 3$$

Thus $(x, y, z) = (2, -1, 3)$ is the required solution.

[5] $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

[June 18]

$\rightarrow R_1 \leftrightarrow R_2 : A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \quad R_3 \rightarrow -3R_1 + R_3, R_4 \rightarrow -R_1 + R_4$

$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix} : R_3 \rightarrow -R_2 + R_3, R_4 \rightarrow -R_2 + R_4$

$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Thus, $\rho(A) = 2$

EXERCISE

WORKED PROBLEMS

[15] Solve the following system of equations by Gauss - Jordan method.

$$x + y + z = 9 \quad ; \quad x - 2y + 3z = 8 \quad ; \quad 2x + y - z = 3 \quad \star \quad \text{[Jan 21]}$$

$$[A : B] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 1 & -2 & 3 & : & 8 \\ 2 & 1 & -1 & : & 3 \end{bmatrix} \quad ; \quad R_2 \rightarrow -R_1 + R_2, \quad R_3 \rightarrow -2R_1 + R_3$$

$$[A : B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & -1 & -3 & : & -15 \end{bmatrix}$$

We use the leading non zero entry in second row (-3) to make the element above and below, 1 and -1 respectively zero. [Take a note of this step in comparison to the Gauss elimination method]

$$R_1 \rightarrow R_2 + 3R_1, R_3 \rightarrow R_2 + (-3)R_3$$

$$[A : B] \sim \begin{bmatrix} 3 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 11 & : & 44 \end{bmatrix}; R_3 \rightarrow 1/11 \cdot R_3$$

$$\sim \begin{bmatrix} 3 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$$

(This step is performed only for convenience to get 1 as the leading entry in the third row).

We use the element 1 in the third row to make the two elements above 2 and 5 zero. $R_1 \rightarrow -5R_3 + R_1, R_2 \rightarrow -2R_3 + R_2$

$$[A : B] \sim \begin{bmatrix} 3 & 0 & 0 & : & 6 \\ 0 & -3 & 0 & : & -9 \\ 0 & 0 & 1 & : & 4 \end{bmatrix} \therefore 3x = 6, -3y = -9, z = 4$$

Thus, $x = 2, y = 3, z = 4$ is the required solution.

[16] Apply Gauss Jordan method to solve it.

[33] Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$

[June 2016, 17, 18]

Let $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$.

ie., $\begin{vmatrix} (-19 - \lambda) & 7 \\ -42 & (16 - \lambda) \end{vmatrix} = 0$

ie., $\lambda^2 + 3\lambda - 304 + 294 = 0$

ie., $\lambda^2 + 3\lambda - 10 = 0$

or $(\lambda - 2)(\lambda + 5) = 0$

ie., $\lambda = 2, -5$ are the eigen values of A .

Now consider, $[A - \lambda I][X] = [0]$

ie., $\begin{bmatrix} (-19 - \lambda) & 7 \\ -42 & (16 - \lambda) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

ie., $(-19 - \lambda)x + 7y = 0$

$-42x + (16 - \lambda)y = 0$

Case - (i) : Let $\lambda = 2$

We get $-21x + 7y = 0$ and $-42x + 14y = 0$

ie., $y = 3x$ or $\frac{y}{3} = \frac{x}{1}$

$\therefore X_1 = (1, 3)'$ is the eigen vector corresponding to $\lambda = 2$.

Case - (ii) : Let $\lambda = -5$

We get $-14x + 7y = 0$ and $-42x + 21y = 0$

ie., $y = 2x$ or $\frac{y}{2} = \frac{x}{1}$

$\therefore X_2 = (1, 2)'$ is the eigen vector corresponding to $\lambda = -5$.

Modal matrix $P = [x_1 \ x_2] = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

We have $|P| = 2 - 3 = -1$ and $P^{-1} = \frac{1}{|P|} (\text{Adj } P)$

$$P^{-1} = - \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$$

Now $P^{-1} AP = D = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -15 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$

Thus $P^{-1} AP = D = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$ is the diagonal matrix.

or $P^{-1} AP = \text{Diag} (2, -5)$

33. If $\vec{F} = \nabla (x y^3 z^2)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the point $(1, -1, 1)$.

>> Let $\phi = x y^3 z^2$

$$\therefore \vec{F} = \nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k = y^3 z^2 i + 3xy^2 z^2 j + 2xy^3 z k$$

$$\begin{aligned} \text{div } \vec{F} &= \nabla \cdot \vec{F} \\ &= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \cdot (y^3 z^2 i + 3xy^2 z^2 j + 2xy^3 z k) \\ &= \frac{\partial}{\partial x} (y^3 z^2) + \frac{\partial}{\partial y} (3xy^2 z^2) + \frac{\partial}{\partial z} (2xy^3 z) \\ &= 0 + 6xyz^2 + 2xy^3 = 2xy(3z^2 + y^2) \end{aligned}$$

$$\therefore \text{div } \vec{F} \text{ at } (1, -1, 1) = -2(3+1) = -8$$

$$\text{Also, } \text{curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^3 z^2 & 3xy^2 z^2 & 2xy^3 z \end{vmatrix}$$

$$= i(6xy^2 z - 6xy^2 z) - j(2y^3 z - 2y^3 z) + k(3y^2 z^2 - 3y^2 z^2) = \vec{0}$$

Thus the required $\text{div } \vec{F} = -8$ and $\text{curl } \vec{F} = \vec{0}$

130. If $x + y + z = u$, $y + z = v$ and $z = uvw$, find the value of $\frac{\partial(x, y, z)}{\partial(u, v, w)}$

Q7)

$$\gg \frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

It is evident that we should have x, y, z in terms of u, v, w .

Consider $x + y + z = u \dots (1)$, $y + z = v \dots (2)$, $z = uvw \dots (3)$

Using (2) in (1) we have, $x + v = u \therefore x = u - v$

Also by using (3) in (2) we have, $y + uvw = v \therefore y = v - uvw$

Thus the given data is modified into the form,

$$x = u - v, y = v - uvw, z = uvw$$

$$\begin{aligned} \therefore \frac{\partial(x, y, z)}{\partial(u, v, w)} &= \begin{vmatrix} 1 & -1 & 0 \\ -vw & (1-uw) & -uv \\ vw & uw & uv \end{vmatrix} \\ &= 1 \left\{ (1-uw)uv - (uw)(-uv) \right\} + 1 \left\{ (-vw)(uv) - (vw)(-uv) \right\} \\ &= uv - u^2vw + u^2vw - uv^2w + uv^2w = uv \end{aligned}$$

Thus $J = uv$

Aliter: Adding R_3 to R_2 we get,

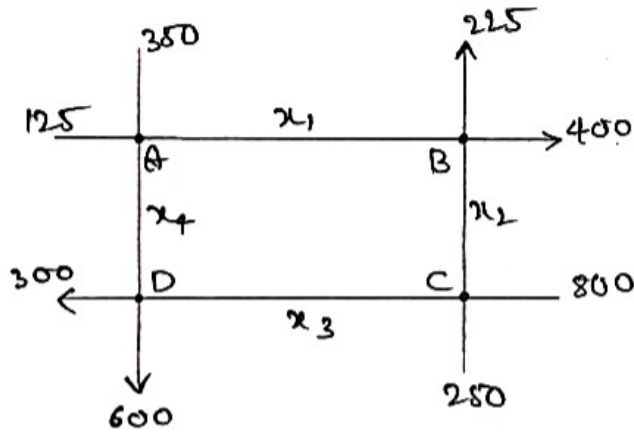
$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ vw & uw & uv \end{vmatrix}$$

On expanding by the second row we have

$$-0 + 1(uv - 0) - 0 = uv$$

5. Assume that the total traffic entering the network equal to the total traffic leaving the network.

① The flow of traffic through a network of streets is shown below:



- (i) Find x_1, x_2, x_3, x_4 to balance the traffic flow.
 (ii) Find the traffic flow at $x_4 = 0$
 (iii) Find the traffic flow at $x_4 = 100$

solⁿ: (i)

Nodes	flow in	flow out	Equations
A	$125 + 350$	$x_1 + x_4$	$x_1 + x_4 = 475$
B	$x_1 + x_2$	$400 + 225$	$x_1 + x_2 = 625$
C	$800 + 250$	$x_2 + x_3$	$x_2 + x_3 = 1050$
D	$x_3 + x_4$	$300 + 600$	$x_3 + x_4 = 900$

The augmented matrix for the linear equations is,

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 475 \\ 1 & 1 & 0 & 0 & 625 \\ 0 & 1 & 1 & 0 & 1050 \\ 0 & 0 & 1 & 1 & 900 \end{array} \right]$$

$$R_2 \rightarrow R_2 - R_1$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 1 & : & 475 \\ 0 & 1 & 0 & -1 & : & 150 \\ 0 & 1 & 1 & 0 & : & 1050 \\ 0 & 0 & 1 & 1 & : & 900 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 1 & : & 475 \\ 0 & 1 & 0 & -1 & : & 150 \\ 0 & 0 & 1 & 1 & : & 900 \\ 0 & 0 & 1 & 1 & : & 900 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 1 & : & 475 \\ 0 & 1 & 0 & -1 & : & 150 \\ 0 & 0 & 1 & 1 & : & 900 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

Rewriting into the linear equations, we have

$$x_1 + x_4 = 475 \quad \Rightarrow \quad x_1 = 475 - x_4 \quad \rightarrow \textcircled{1}$$

$$x_2 - x_4 = 150 \quad \Rightarrow \quad x_2 = 150 + x_4 \quad \leftarrow \textcircled{2}$$

$$x_3 + x_4 = 900 \quad \Rightarrow \quad x_3 = 900 - x_4 \quad \rightarrow \textcircled{3}$$

$\Rightarrow x_4$ is a free variable

from $\textcircled{1}$ we have $x_1 = 475 - x_4$

$$\Rightarrow x_4 \leq 475$$

from $\textcircled{2}$, $x_2 = 150 + x_4$

$$\Rightarrow x_4 \geq -150$$

$$\Rightarrow x_4 \geq 0$$

Also, from $\textcircled{3}$, $x_3 = 900 - x_4$

$$\Rightarrow x_4 \leq 900$$

$$\therefore \boxed{0 \leq x_4 \leq 475}$$

(ii) When $x_4 = 0$,

from $\textcircled{1}$, $\textcircled{2}$ and $\textcircled{3}$ we get

$$\boxed{x_1 = 475, \quad x_2 = 150, \quad x_3 = 900}$$