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Internal Assessment Test 1 – November 2025

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| Sub: | Quantum Physics & Applications | | | | | Sub Code: | 1BPHYS102 | | Branch: | ISE/AIDS/AIML/CSE-AIML |
| Date: | 05/11/2025 | Duration: | 90 mins | Max Marks: | 50 | Sem/Sec: | I Sem / A, B, C, D, E, F, G, H | | | OBE |

**Answer any FIVE FULL Questions**

**Given: c = 3 × $10^8$ m/s; h = 6.625 × $10^{-34}$Js; k = 1.38 × $10^{-23}$ J/K; $m_e$ = 9.1 × $10^{-31}$kg; e = 1.6 × $10^{-19}$C**

| | | MARKS | CO | RBT |
|---|---|---|---|---|
| 1 (a) | Derive the eigen function and energy eigen values for a particle in a one-dimensional potential well of infinite height. | [07] | CO1 | L2 |
| (b) | Find the temperature at which there is 1% probability that an energy state 0.5eV above the Fermi level is occupied? | [03] | CO2 | L3 |
| 2 (a) | Derive time independent Schrodinger wave equation for a particle in three dimensions. | [07] | CO1 | L2 |
| (b) | The concentration of electrons at room temperature in an extrinsic semiconductor is 1.9 x $10^{28}/m^{3}$. If a current of 150 A flows through the semiconductor of thickness 2 mm and the strength of magnetic field perpendicular to it is 4.2 T, calculate the Hall coefficient and the Hall voltage. | [03] | CO2 | L3 |
| 3 (a) | State and explain Heisenberg's uncertainty principle. Describe the broadening of spectral line using uncertainty principle. | [07] | CO1 | L2 |
| (b) | Write about any two failures of classical free electron theory. | [03] | CO2 | L2 |

**PTO**

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**PTO**

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| 4 (a) | Explain the case of a particle in a one dimensional potential well of finite height. Describe how it leads to quantum tunnelling. | [7] | CO1 | L2 |
| (b) | Calculate the energy of an electron whose de-Broglie wavelength is 1.5 Å. | [3] | CO1 | L3 |
| 5 (a) | Discuss the merits of quantum free electron theory. | [6] | CO2 | L2 |
| (b) | Calculate the probability of finding an electron in a one dimensional potential well of width ‘L’ between x= L/3 and x=2L/3 in its second excited state. | [4] | CO1 | L3 |
| 6 (a) | Obtain an expression for the electron concentration for an intrinsic semiconductor. | [6] | CO2 | L2 |
| (b) | The width of a spectral line of wavelength 6200 Å is $10^{-5}$Å. Evaluate the minimum time spent by the electron in the upper energy state between the excitation and de-excitation processes. | [4] | CO1 | L3 |
| 7 (a) | Show that the Fermi level lies the centre of the band gap for an intrinsic semiconductor. | [6] | CO2 | L2 |
| (b) | Define fermi factor. Explain the dependence of Fermi factor on temperature and energy. | [4] | CO2 | L2 |
| 8 (a) | Define Hall effect. Obtain an expression for Hall coefficient in terms of Hall voltage. | [6] | CO2 | L2 |
| 8 (b) | Calculate the de Broglie wavelength of a particle of mass 0.55 MeV/$c^2$ moving with a kinetic energy of 150eV. | [4] | CO1 | L3 |

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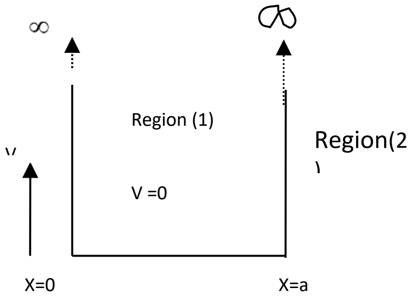
# IAT -1 PHYSICS SOLUTION

## 1A

**Particle in an infinite potential well problem:**

Consider a particle of mass m moving along X-axis in the region from X=0 to X=a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2}+\frac{8\Pi^2 mE\psi}{h^2}=0 \because V=0$$

$$\text{But } k^2=\frac{8\Pi^2 mE}{h^2}$$

$$\therefore \frac{d^2\Psi}{dx^2}+k^2\Psi=0$$

Auxiliary equation is $(D^2+k^2)x=0$
Roots are D = +ik and D = -ik
The general solution is

$$\begin{aligned} x &= Ae^{ikx}+Be^{-ikx} \\ &= A(\cos k\,x+i\sin k\,x)+B(\cos k\,x-i\sin k\,x) \\ &= (A+B)\cos k\,x+i(A-B)\sin k\,x \\ &= C\cos k\,x+D\sin k\,x \end{aligned}$$

The boundary conditions are

1. At x=0, $\Psi = 0 \therefore C = 0$

2. At x=a, $\Psi = 0$

D sin ka = 0 ⇒ka = n$\Pi$ .........(2)

where n = 1, 2 3...

$$\therefore \Psi = D\, sin\left(n\frac{\Pi}{a}\right)x$$

From (1) and (2) E = $\frac{n^2h^2}{8ma^2}$

**To evaluate the constant D:**

Normalisation: For one dimension

$$\int_0^a \Psi^2\, dx = 1$$

$$\int_0^a D^2\, sin^2\left(\frac{n\Pi}{a}\right) xdx = 1$$

But $cos\, 2\,\theta = 1 - 2\, sin^2\,\theta$

$$\int_0^a D^2\,\frac{1}{2}(1 - cos\, 2\,(\frac{n\Pi}{a})x)dx = 1$$

$$\int_0^a \frac{D^2}{2}dx - \int_0^a \frac{1}{2} cos\, 2\,(\frac{n\Pi}{a})x)dx = 1$$

$$\frac{D^2a}{2} - [sin\, 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2\,\frac{a}{2} - 0 = 1$$

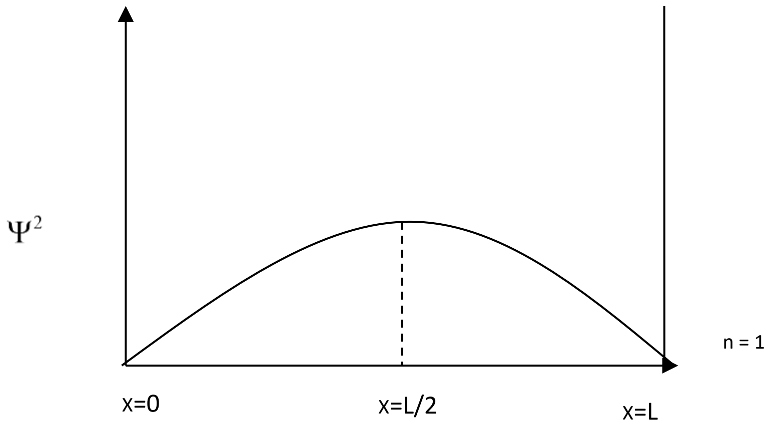
$$D = \sqrt{\frac{2}{a}}$$

$$\therefore \Psi_n = \sqrt{\frac{2}{a}}\, sin\left(n\frac{\Pi}{a}\right)x$$

For n = 1, First state

$$\therefore \Psi_1 = \sqrt{\frac{2}{a}}\, sin\left(1.\frac{\Pi}{a}\right) x$$

The graph of $\Psi^2 \quad versus$x is shown below.

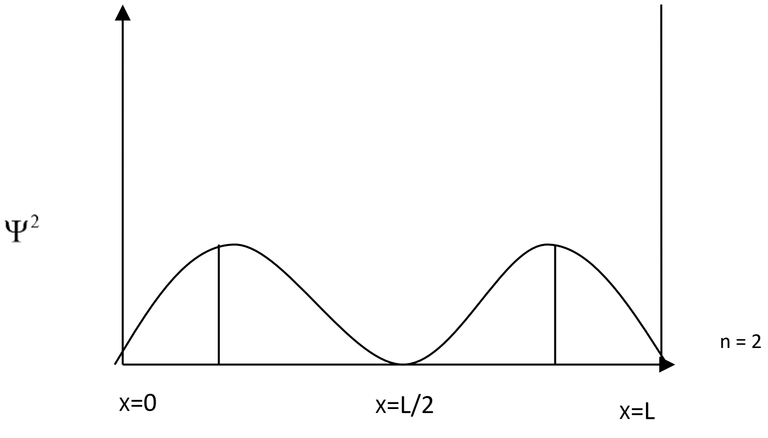


It is seen from the graph that probability density is maximum at the centre for the particle in the first state.

For n = 2, Second state

$$\therefore \Psi_2 = \sqrt{\frac{2}{a}}\, sin\left(2.\frac{\Pi}{a}\right) x$$

The graph of $\Psi^2 \quad versus$x is shown below.



It is seen from the graph that probability density is maximum at x = L / 4 and x = 3L/4 for the particle in the second state.

**Eigen function:** It is the physically acceptable solution to Schrodinger's equation. It represents the matter wave corresponding to a quantum particle in a specific state.

Ex: For a particle in an infinite potential well, the eigen function is $\Psi_n = \sqrt{\frac{2}{a}}\, sin\left(n\frac{\Pi}{a}\right)x$

**Eigen Value:**It represents the energy of a particle in corresponding to its Eigen function.

Eigen value for a particle in an infinite potential well is E = $\frac{n^2h^2}{8ma^2}$

**1B**

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

$$E - E_F = +0.5eV = 0.5x1.6x10^{-19}J$$

$$e^{\frac{E-E_F}{kT}} + 1 = \frac{1}{f(E)}$$

$$e^{\frac{E-E_f}{kT}} = \frac{1}{f(E)} - 1$$

$$\frac{E - E_F}{kT} ln_e\, e = ln\left(\frac{1}{f(E)} - 1\right)$$

$$T = \frac{E - E_F}{k.ln\left(\frac{1}{f(E)} - 1\right)} = \frac{0.5x1.6x10^{-19}}{1.38x10^{-23}ln\left(\frac{1}{0.01} - 1\right)} = 1261K$$

# 2A

## Time independent Schrödinger equation

Differential equation for a wave is

$\frac{d^2\Psi}{dx^2} = \frac{1}{v^2}\frac{d^2\Psi}{dt^2}$……………..(1)

A matter wave can be represented in complex form as

$$\Psi = A\,sin\,k\,x(cos\,w\,t + i\,sin\,w\,t)$$

$$\Psi = A\,sin\,k\,xe^{iwt}$$

Differentiating wrt t

$$\frac{d\Psi}{dt} = iw\,A.sinkxe^{iwt}$$

Differentiating wrt t

$$\frac{d^2\Psi}{dt^2} = iw\,iw\;\;A.sinkxe^{iwt} = -w^2\;\;\psi$$

$$w = 2\pi f \qquad \omega = 4\pi^2 f^2 \qquad v\; = c = f^2\lambda^2$$

Substitute in (1)

$$\frac{d^2\Psi}{dx^2} = \frac{1}{v^2}\frac{d^2\Psi}{dt^2} = \frac{1}{c^2}(-)4\pi^2 f^2\;\;\Psi = -\frac{4\pi^2 f^2}{f^2\lambda^2}\;\;\psi$$

From Debroglie's relation

$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$

$$\frac{d^2\Psi}{dx^2} = -\frac{4\pi^2 p^2}{h^2}\;\;\psi$$

Total energy of a particle

E = Kinetic energy + Potential Energy

$E = \frac{p^2}{2m} + V$

$$p^2 = (E - V)2m$$

$$\frac{d^2\Psi}{dx^2} = -\frac{4\pi^2(E-V)2m}{h^2}\ \psi$$

$$\boxed{\begin{gathered}\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E-V)\Psi}{h^2} = 0 \\ For\ \ 3D \\ \frac{d^2\Psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dx^2} + \frac{8\Pi^2 m(E-V)\Psi}{h^2} \\ = 0\end{gathered}}$$

# 2B

$$V_H = B\frac{I}{wn_e e}$$

$$= \frac{150x4.2}{0.002x1.6x10^{-19}x1.9x10^{28}}$$

$$Hall\ voltage\ V_H = \ 1.03X10^{-4}\ V$$

$$Hall\ coefficient\ R_H = \frac{1}{n_e e} = \frac{1}{1.9\ \mathrm{x}\ 10^{28}x1.6x10^{-19}} = 3.28x10^{-10}\,m^3/C$$

# 3A

**HEISENBERG'S UNCERTAINTY PRINCIPLE:**

The position and momentum of a particle cannot be determined accurately and simultaneously. The product of uncertainty in the measurement of position $(\Delta x)$ and momentum is always greater than or equal to $\frac{h}{2\Pi}$. (1 mark)

$$\boxed{(\Delta x).(\Delta p) \geq \frac{h}{4\Pi}}$$

## Application of - Broadening of Spectral Lines

Whenever an atom is excited to a higher energy state $E_2$, it remains in that state only for a very short duration (lifetime τ). Soon after, it returns to the lower energy state $E_1$, emitting energy in the form of a spectral line. The wavelength of the emitted spectral line is given by:

$$E_2 - E_1 = h\vartheta = h\frac{c}{\lambda}$$

However, in reality, the atoms do not emit radiation at precise wavelengths. Each spectral line is broadened due to various causes (Fig.1).

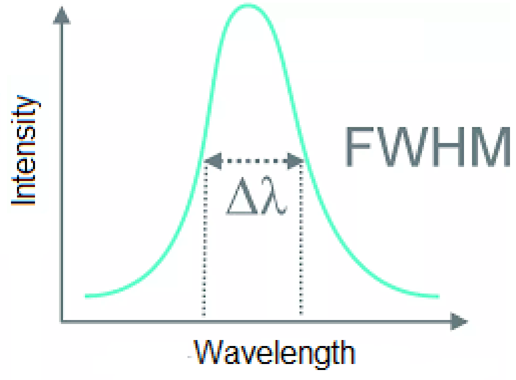


Fig.1 Intensity Vs. wavelength plot

The natural line broadening of a spectral line is a result of Heisenberg's uncertainty principle .According to this principle, there is an uncertainty in the energy state ( $\Delta E$) of a system due to the uncertainty in its life time $(\Delta t)$. The lifetime of the excited states will result in natural broadening of the spectral line.

According to uncertainty principle, energy level above the ground state with energy E and lifetime Δt, has uncertainty in the measurement of energy $\Delta E$ given by

$\Delta E \times \Delta t \geq \frac{h}{4\pi}$ ------- (1)

For an excited state with mean lifetime we take $\Delta t \sim \tau$. Thus , the energy uncertainty is

$\Delta E \geq \frac{h}{4\pi \times \tau}$ ------- (2)

Relation between energy and wavelength is given by

$$E = \frac{hc}{\lambda}$$

Differentiating (taking only magnitudes and ignoring the minus sign, since we're concerned with widths),

$\Delta E = \frac{hc}{\lambda^2} \times \Delta\lambda$ -------- (3)

Substituting eq. (3) in eq. (2) $\frac{hc}{\lambda^2} \times \Delta\lambda \geq \frac{h}{4\pi \times \tau}$

Or $\Delta\lambda \geq \frac{\lambda^2}{4\pi \times \tau \times c}$

When measuring spectral lines, the lifetime of the excited state determines the width of these lines. If a state has a long lifetime, the uncertainty in energy is small, resulting in a narrow spectral line. Conversely, a short lifetime leads to greater energy uncertainty, causing a wider spectral line.

# 3B

**Failures of Classical free electron theory:**

1. Prediction of low specific heats for metals:

Classical free electron theory assumes that conduction electrons are classical particles similar to gas molecules. Hence,they are free to absorb energy in a continuous manner. Hence metals possessing more electrons must have higher heat content. This resulted in high specific heat given by the expression $C_V = 10^{-4}R$.

This was contradicted by experimental results which showed low specific heat for metals.

2. Temperature dependence of electrical conductivity:

From the assumption of kinetic theory of gases

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$

$$\therefore v \propto \sqrt{T}$$

Also mean collision time τ is inversely proportional to velocity,

$$\tau \propto \frac{1}{v}$$

$$\tau \propto \frac{1}{\sqrt{T}}$$

$$\therefore \sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

However experimental studies show that $\sigma \propto \frac{1}{T}$

3. Dependence of electrical conductivity on electron concentration:

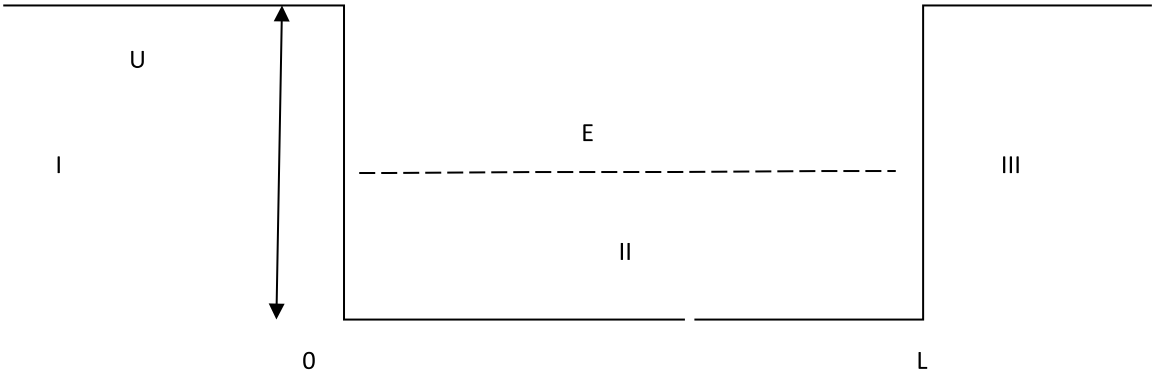
As per free electron theory, $\sigma \propto n$

The electrical conductivity of Zinc and Cadmium are 1.09 x $10^7$ /ohm m and .15 x $10^7$ /ohm m respectively which are very much less than that for Copper and Silver for which the values are 5.88x$10^7$ /ohm m and 6.2 x $10^7$ /ohm m. On the contrary, the electron concentration for zinc and cadmium are 13.1x$10^{28}$ /m$^3$ and 9.28 x$10^{28}$ /m$^3$ which are much higher than that for Copper and Silver which are 8.45x$10^{28}$ /m$^3$ and 5.85 x$10^{28}$ /m$^3$.

These examples indicate that $\sigma \propto n$ does not hold good.

# 4A

## The finite potential well



The finite square well is shown in Figure. A classical particle with energy $E$ less than $U$ is permanently bound to the region $0 < x < L$. Quantum mechanics asserts, however, that there is some probability that the particle can be found *outside* this region. That is, the wave function generally is nonzero outside the well, and so the probability of finding the particle here also is nonzero. For stationary states, the wave function ($x$) is found from the time independent

Schrödinger equation.

For Region I

$$\frac{\partial^2\psi}{\partial x^2} - \frac{8\pi^2 m}{h^2}(E - U)\psi = 0)$$

( negative sign because E is less than U)

The solution is

$$\psi = A \quad e^{kx} + b \quad e^{-kx}$$

Where $k = \frac{8\pi^2 m(E-U)}{h^2}$

For x = -∞

$\psi = A \quad e^{kx}$since $e^{-kx} \Rightarrow \infty$

For Region III

$$\frac{\partial^2\psi}{\partial x^2} - \frac{8\pi^2 m}{h^2}(E - U)\psi = 0)$$

( negative sign because E is less than U)

The solution is

$$\psi = C \quad e^{kx} + D \quad e^{-kx}$$

Where $k = \frac{8\pi^2 m(E-U)}{h^2}$

For x = +∞

$\psi = B \quad e^{-kx}$since $e^{+kx} \Rightarrow \infty$

For Region II

$$\frac{\partial^2\psi}{\partial x^2}+\frac{8\pi^2 m}{h^2}(E-U)\psi=0)$$

( negative sign because E is less than U)

The solution is

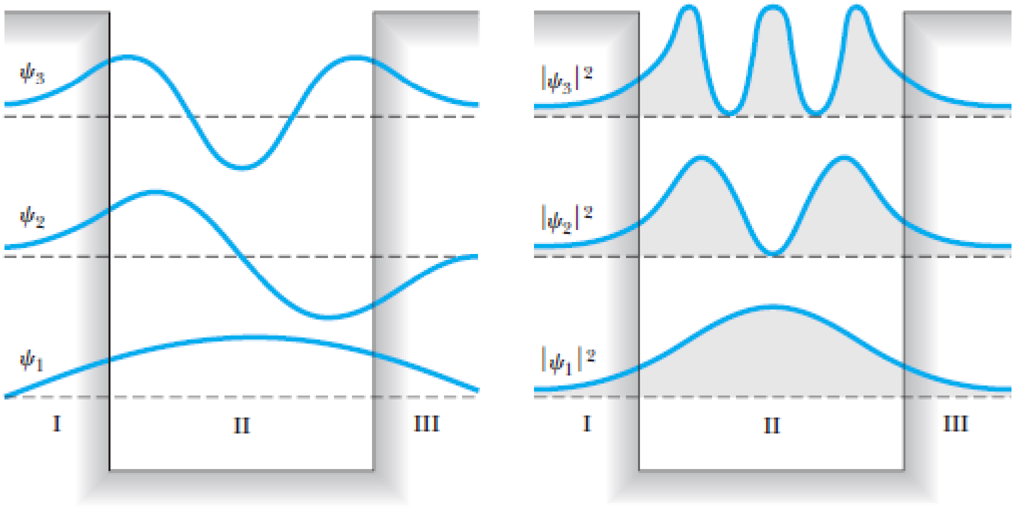
$$\psi = P \quad e^{ikx} + M \quad e^{-ikx} = P \quad \sin k\,x + M \quad \cos k\,x$$

Where $k = \frac{8\pi^2 m(E-U)}{h^2}$

For x = +∞

$\psi = B \quad e^{-kx}$since $e^{+kx} \Rightarrow \infty$

The coefficients *A* and *B* are determined by matching this wave smoothly onto the wave function in the well interior. Here ψ and its first derivative $\frac{\partial\psi}{\partial x}$are continuous at $x = 0$ and again at $x = L$. This can be done only for certain values of *E known as Eigen values*, corresponding to the allowed energies for the bound particle. Figure shows the wave functions and probability densities that result for the three lowest allowed particle energies.



(a) (b)

Note that in each case the waveforms join smoothly at the boundaries of the potential well and the probability is not zero outside the potential well. There exits finite probability that the particle can cross the barrier leading to quantum tunneling.

# 4B

$$\boldsymbol{\lambda = \frac{h}{\sqrt{2mE}}}$$

$$\boldsymbol{E = \frac{h^2}{2m\lambda^2} = 1.07x10^{-17}J}$$

## 5A

**Success of quantum theory:**

**1. Specific heat:**

Classical theory predicted high values of specific heat for metals on the basis of theassumption that all the conduction electrons are capable of absorbing the heat energy as per Maxwell - Boltzmann distribution i.e., $C_V = \frac{3}{2}RT$

But according to the quantum theory, only those electrons occupying energy levels close to Fermi energy ($E_F$) are capable of absorbing heat energy to get excited to higher energy levels. Thus only a small percentage of electrons are capable of receiving the thermal energy and specific heat value becomes small.

It can be shown that $C_V = 10^{-4}RT$.

This is in conformity with the experimental values.

2. **Temperature dependence of electrical conductivity.**

According to classical free electron theory,

Electrical conductivity $\propto \frac{1}{\sqrt{Temperature}}$

Where as from quantum theory

Electrical conductivity $\propto$

$$\frac{1}{collisional\ \ area\ \ of\ \ cross\,section\ \ of\ \ lattice\ \ atoms}\,\alpha\,\frac{1}{vibrational\ \ energy}\,\alpha\,\frac{1}{Temperature}$$

This is in agreement with experimental values.

3. **Dependence of electrical conductivity on electron concentration:**

According to classical theory,

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto n$$

But it has been experimentally found that Zinc which is having higher electron concentration than copper has lower Electrical conductivity.

According to quantum free electron theory,

Electrical conductivity $\sigma = \frac{ne^2}{m}\left(\frac{\lambda}{V_F}\right)$ where $V_F$ is the Fermi velocity.

Zinc possesses lesser conductivity because it has higher Fermi velocity.

| Metal | n | $\sigma$ |
|---|---|---|
| Cu | $8.45\times10^{28}/m^3$ | $6\times10^{7}(\Omega m)^{-1}$ |
| Zn | $13\times10^{28}/m^3$ | $1\times10^{7}(\Omega m)^{-1}$ |

**5B**

$$Pr\,o\,bability = \int_{L/3}^{2L/3} |\psi^2|\,dx = \int \frac{2}{L} sin^2 \frac{3\pi}{L} x.\,dx = \frac{2}{L}\int (\frac{1-cos\,6\,\pi x/L}{2} dx$$

$$= \frac{2}{L}\int \frac{1}{2} dx - \frac{2}{2L}\int \frac{cos\,6\,\pi x/L}{1} dx = \frac{1}{L}[x]_{L/3}^{2L/3} - \frac{1}{L}\left[\frac{sin\,6\,\pi x/L}{2n\pi/L}\right]_{L}^{2L/3}$$

$$= \frac{1}{3} - 0 = 0.33$$

# 6A

### Expression for Electron Concentration in Intrinsic semiconductor

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

For electrons in conduction band,

$$E - E_F >> kT$$

$$Hence$$

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} \approx \frac{1}{e^{\frac{E-E_F}{kT}}}$$

From Fermi-Dirac theory

Density of states for conduction band

$$g(E) = \frac{8\pi\sqrt{2}{m_e}^{\frac{3}{2}}}{h^3}(E - E_C)^{\frac{1}{2}}$$

$$n_e = \int_0^{\infty} n_e(E)dE$$

$$= \int_{E_C}^{\infty} g(E)f(E)dE = \int_{E_C}^{\infty} \frac{8\pi\sqrt{2}{m_e}^{\frac{3}{2}}}{h^3}(E - E_C)^{\frac{1}{2}} \frac{1}{e^{\frac{E-E_F}{kT}}} dE$$

$$= \frac{8\pi\sqrt{2}{m_e}^{\frac{3}{2}}}{h^3}\int_{E_C}^{\infty} x^{\frac{1}{2}} e^{\frac{E_F-E}{kT}} dx$$

$$x = E - E_C$$

$$E = x + E_C$$

$$dx = dE$$

$$n_e = \frac{8\pi\sqrt{2}{m_e}^{\frac{3}{2}}}{h^3}\int_0^{\infty} x^{\frac{1}{2}} e^{\frac{E_F}{kT} - \frac{x+E_C}{kT}} dx$$

$$= \frac{8\pi\sqrt{2}{m_e}^{\frac{3}{2}}}{h^3}\int_0^{\infty} x^{\frac{1}{2}} e^{\frac{E_F-E_C}{kT}} e^{\frac{-x}{kT}} dx$$

$$= \frac{8\pi\sqrt{2}m_e^{\frac{3}{2}}}{h^3} e^{\frac{E_F-E_C}{kT}} \int_0^\infty x^{\frac{1}{2}} e^{\frac{-x}{kT}} dx$$

$$\int_0^\infty x^{\frac{1}{2}} e^{\frac{-x}{kT}} dx = \frac{\sqrt{\pi}}{2}(kT)^{\frac{3}{2}}$$

$$n_e = 2\frac{\pi^{\frac{3}{2}} 2^{\frac{3}{2}} m_e^{\frac{3}{2}} (kT)^{\frac{3}{2}}}{(h^2)^{\frac{3}{2}}} e^{\frac{E_F-E_C}{kT}}$$

$$= 2\left[\frac{2\pi m_e kT}{h^2}\right]^{\frac{3}{2}} e^{\frac{E_F-E_C}{kT}}$$

# 6B

$$\Delta E \times \Delta t = \frac{h}{4\pi}$$

$$lifetime \quad \Delta t = \frac{h}{4\pi.\Delta E}$$

$$E = \frac{hc}{\lambda}$$

$$\Delta E = -\frac{hc}{\lambda^2}\Delta\lambda = \frac{6.62x10^{-34}x3x10^8}{(6200x10^{-10})^2}x10^{-5}x10^{-10}$$

$$lifetime \quad \Delta t = \frac{h}{4\pi.\Delta E} = 1.02\text{x}10^{-7}\text{s}$$

## 7A

**Expression for Fermi Level in Intrinsic Semiconductor**

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C-E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F-E_V}{kT}}$$

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C-E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F-E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f+E_v+E_c-E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f+E_v+E_c}{kT}$$

$$E_f = \frac{E_v+E_c}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

If $m_e$ and $m_h$ are equal,
$E_f = (E_c + E_v)/2$
So, Fermi level is said to be at the centre of energy gap

### 7B

**To show that energy levels below Fermi energy are completely occupied:**

For E **<** $E_F$, at T = 0,

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 1$$

**To show that energy levels above Fermi energy are empty:**

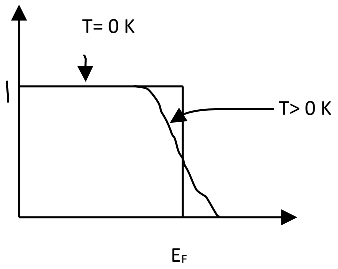
For E $>E_F$, at T=0

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 0$$

**At ordinary temperatures, for E = $E_F$,**

$$f(E) = \frac{1}{2}$$

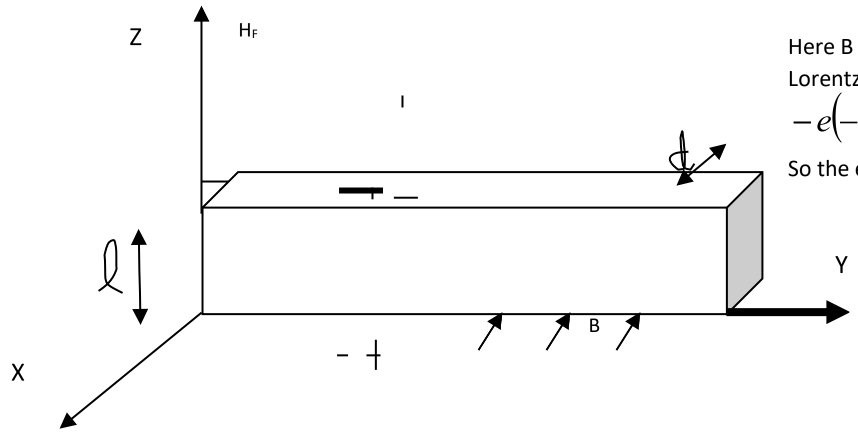
**Fermi energy for T > 0k,** $E_f = E_{f0}\left[1-\frac{\Pi^2}{12}\left(\frac{kT}{E_F}\right)^2\right]$



# 8A

## HALL EFFECT

**Hall effect:** When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.



Here B is along –X , V is along –Y axis
Lorentz force= = e (VXB)
$-e\left(-\hat{j}\ X -\hat{i}\right)=+\hat{k}$
So the electron is deflected along + Z axis

Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential $V_H$-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field $E_y$ is set up.

**Expression for electron concentration:**

At equilibrium, Lorentz force is equal to force due to applied electric field

$$Bev =- e E_H$$

Hall Field $E_H = Bv$

$$Current \quad density \quad J = I/A = neAV/A = n_e ev$$

$$v = \frac{J}{n_e e}$$

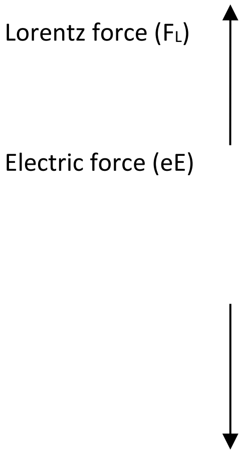
Hence $E_H = B\frac{J}{n_e e}$

$$\frac{E_H}{JB} = \frac{1}{n_e e} = R_H$$

$R_H$ is known as Hall coefficient. It is negative for n type and p type for positiv

Electron concentration $n_e = \frac{BI}{V_H d}$

Hall voltage $V_H = E_H . l = Bvl$



# 8B

$$m = \frac{0.55x10^6 x1.6x10^{-19}}{(3x10^8)^2} = 9.77x10^{-31}kg$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62x10^{-34}}{\sqrt{2x9.77x10^{-31}x100x1.6x10^{-19}}} = 1.18x10^{-10}m$$