CBCS SCHEME

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1BMATE101

First Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Differential Calculus and Linear Algebra : EEE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. VTU Handbook is permitted.

			M	L	C
		Module – 1			
Q.1	a.	With usual notations, prove that $\tan\phi = r\frac{d\theta}{dr}$.	6	L2	CO1
	b.	Find angle between the following curves $r = a\log\theta$ & $r = \frac{a}{\log\theta}$.	7	L2	CO1
	c.	Show that the radius of curvature of the curve $r^n = a^n\cos n\theta$ varies inversely as r^{n-1}.	7	L2	CO1
		OR			
Q.2	a.	Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point (3a/2,3a/2).	6	L2	CO1
	b.	Show that the following pair of curves cut orthogonally: $r = a(1+\sin\theta)$ & $r = a(1-\sin\theta)$	7	L2	CO1
	c.	Find the pedal equation $r^2 = a^2\sec 2\theta$.	7	L2	CO1
		Module – 2			
Q.3	a.	Expand log(sec x) in powers of x up to the term containing x^4.	6	L2	CO1
	b.	Evaluate *i)* $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$ *ii)* $\lim_{x\to\pi/2}(\sec x)^{\cot x}$.	7	L2	CO1
	c.	Discuss the maxima and minima of the function $f(x,y) = x^3 + y^3 - 3x - 27y + 24$.	7	L3	CO1
		OR			
Q.4	a.	If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.	6	L2	CO1
	b.	If $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$ evaluate Jacobian of u. v, w with respect to x, y, z.	7	L3	CO1
	c.	If $u = x^2 + y^2 + z^2$, $x = e^{2t}$, $y = \cos 3t$, $z = \sin 3t$. Find $\frac{du}{dt}$.	7	L2	CO1
		Module – 3			
Q.5	a.	Solve $(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$.	6	L2	CO1
	b.	Find a solution for the non-linear differential equation $xyp^2 - (x^2 + y^2)p + xy = 0$.	7	L2	CO1
	c.	Solve $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitutions $x^2 = X$, $y^2 = Y$.	7	L2	CO1
		OR			
Q.6	a.	Solve $\frac{dy}{dx} + y\tan x = y^3\sec x$.	6	L2	CO1

	b.	Solve $(xy + y^2 + y)dx + (x^2 + 3xy + 2x)dy = 0$.	7	L2	CO1
	c.	Find the orthogonal trajectories of curve $r^n = a^n \sin n\theta$.	7	L3	CO1
		Module – 4			
Q.7	a.	Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 0$.	6	L2	CO1
	b.	Solve $x^2\frac{d^2y}{dx^2} - 2y = x^2 + \frac{1}{x}$.	7	L2	CO1
	c.	Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = \tan x$.	7	L2	CO1
		OR			
Q.8	a.	Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 12x^2$.	6	L2	CO1
	b.	Solve $y'' - 6y' + 9y = 5e^{3x} - \sin 2x$.	7	L2	CO1
	c.	Solve $(3x + 2)^2y'' + 3(3x + 2)y' - 36y = 8x$.	7	L2	CO1
		Module – 5			
Q.9	a.	Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$	6	L2	CO2
	b.	Solve the following system of equations by Gauss seidal $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$, Carry out three iteration.	7	L2	CO2
	c.	Find the dominant eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by power method. With initial Eigen vector $[1\ 0\ 0]^T$. Carry out four iterations.	7	L3	CO2
		OR			
Q.10	a.	Solve by Gauss elimination method $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$.	6	L2	CO2
	b.	Investigate the values of λ and μ, such that the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have i) Unique solution ii) No solution iii) Infinitely many solution.	7	L3	CO2
	c.	Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.	7	L2	CO2

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Module 1

Q1(a) Question

With usual notations, prove that for a polar curve

$$\tan\phi \;=\; r\,\frac{d\theta}{dr},$$

where ϕ is the angle between the tangent and the radius vector.

Solution

Let the polar curve be $r = f(\theta)$.

$$x = r\cos\theta, \qquad y = r\sin\theta$$

Differentiate w.r.t. θ:

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta,$$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta.$$

Hence slope of tangent,

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}.$$

Let ψ be the angle made by the tangent with the x–axis.

$$\tan\psi = \frac{dy}{dx}.$$

Radius vector makes angle θ with x–axis, hence angle between tangent and radius vector,

$$\phi = \psi - \theta.$$

Using tangent subtraction formula,

$$\tan\phi = \frac{\tan\psi - \tan\theta}{1 + \tan\psi\tan\theta}.$$

Substituting and simplifying,

$$\tan\phi = \frac{r}{\frac{dr}{d\theta}} = r\,\frac{d\theta}{dr}.$$

$$\tan\phi = r\,\frac{d\theta}{dr}$$

Q1(b) Question

Find the angle between the curves

$$r = a\log\theta \quad and \quad r = \frac{a}{\log\theta}.$$

Solution

Angle between curves is the angle between tangents:

$$\tan\phi = r\frac{d\theta}{dr}.$$

Curve 1: $r = a\log\theta$

$$\log\theta = \frac{r}{a}$$

Differentiate:

$$\frac{1}{\theta}\frac{d\theta}{dr} = \frac{1}{a} \Rightarrow \frac{d\theta}{dr} = \frac{\theta}{a}.$$

Hence,

$$\tan\phi_1 = r\frac{\theta}{a}.$$

Substitute $r = a\log\theta$:

$$\tan\phi_1 = \theta\log\theta.$$

Curve 2: $r = a\log\theta$

$$\log\theta = \frac{a}{r}.$$

Differentiate:

$$\frac{1}{\theta}\frac{d\theta}{dr} = -\frac{a}{r^2} \Rightarrow \frac{d\theta}{dr} = -\frac{a\theta}{r^2}.$$

Thus,

$$\tan\phi_2 = r\left(-\frac{a\theta}{r^2}\right) = -\frac{a\theta}{r}.$$

Substitute $r = a\log\theta$:

$$\tan\phi_2 = -\theta\log\theta.$$

Angle between curves:

$$\tan\alpha = \left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}\right|.$$

$$\tan\alpha = \frac{\theta\log\theta - (-\theta\log\theta)}{1-(\theta\log\theta)^2}.$$

$$\tan\alpha = \frac{2\theta\log\theta}{1-\theta^2(\log\theta)^2}$$

Q1(c) Question

Show that the radius of curvature of the curve

$$r^n = a^n\cos n\theta$$

varies inversely as $r^{\,n-1}$.

Solution

Radius of curvature in polar form:

$$\rho = \frac{\left(r^2+\left(\frac{dr}{d\theta}\right)^2\right)^{3/2}}{\left|r^2+2\left(\frac{dr}{d\theta}\right)^2 - r\frac{d^2r}{d\theta^2}\right|}.$$

Given:

$$r^n = a^n\cos n\theta.$$

Differentiate:

$$nr^{n-1}\frac{dr}{d\theta} = -a^n n\sin n\theta.$$

$$\frac{dr}{d\theta} = -\frac{a^n\sin n\theta}{r^{n-1}}.$$

Using $a^n\cos n\theta = r^n$,

$$\frac{dr}{d\theta} = -r\tan n\theta.$$

Second derivative:

$$\frac{d^2r}{d\theta^2} = -\frac{d}{d\theta}(r\tan n\theta)$$

$$= -\left(\frac{dr}{d\theta}\tan n\theta + rn\sec^2 n\theta\right).$$

Substitute $\frac{dr}{d\theta} = -r\tan n\theta$:

$$\frac{d^2r}{d\theta^2} = r\tan^2 n\theta - rn\sec^2 n\theta.$$

Substitute into curvature formula and simplify using

$$\sec^2 n\theta = 1 + \tan^2 n\theta,$$

we obtain

$$\rho = \frac{r}{n+1}\sec n\theta.$$

Since

$$\cos n\theta = \frac{r^n}{a^n} \Rightarrow \sec n\theta = \frac{a^n}{r^n},$$

$$\rho = \frac{a^n}{n+1}\frac{1}{r^{\,n-1}}.$$

$$\rho \propto 1r^{\,n-1}$$

Q2(a) Question

Find the radius of curvature of the curve

$$x^3 + y^3 = 3axy$$

at the point $(\frac{3a}{2}, \frac{3a}{2})$.

Solution

Given curve:

$$x^3 + y^3 = 3axy.$$

Differentiate implicitly w.r.t. x:

$$3x^2 + 3y^2 \frac{dy}{dx} = 3a\left(x\frac{dy}{dx} + y\right).$$

Divide by 3:

$$x^2 + y^2 y' = a(xy' + y),$$

where $y' = dydx$.

Rearrange:

$$y'(y^2 - ax) = ay - x^2.$$

$$y' = \frac{ay - x^2}{y^2 - ax}.$$

At $(x, y) = (3a2, 3a2)$:

$$y' = \frac{a \cdot 3a2 - (3a2)^2}{(3a2)^2 - a \cdot 3a2} = \frac{3a^2 2 - 9a^2 4}{9a^2 4 - 3a^2 2} = \frac{-3a^2 4}{3a^2 4} = -1.$$

Second derivative:

Differentiate

$$x^2 + y^2 y' = a(xy' + y).$$

$$2x + 2yy' + y^2 y'' = a(xy'' + y' + y').$$

$$2x + 2yy' + y^2 y'' = a(xy'' + 2y').$$

Rearrange for y'':

$$y''(y^2 - ax) = a(2y') - 2x - 2yy'.$$

Substitute $y' = -1$ and point values:

$$y''\left(9a^2 4 - 3a^2 2\right) = a(-2) - 2 \cdot 3a2 - 2 \cdot 3a2(-1).$$

LHS factor:

$$y'' \cdot 3a^2 4.$$

RHS:

$$-2a - 3a + 3a = -2a.$$

Thus,

$$y'' = \frac{-2a}{3a^2 4} = -\frac{8}{3a}.$$

Radius of curvature:

$$\rho = \frac{\left(1 + y'^2\right)^{3/2}}{|y''|}.$$

$$\rho = \frac{(1+1)^{3/2}}{83a} = \frac{(2)^{3/2} \cdot 3a}{8}.$$

$$(2)^{3/2} = 2\sqrt{2}.$$

$$\rho = \frac{3a\sqrt{2}}{4}.$$

Q2(b) Question

Show that the curves

$$r = a(1 + \sin\theta), \qquad r = a(1 - \sin\theta)$$

cut orthogonally.

Solution

Angle between curves in polar form:

$$\tan\phi = r\frac{d\theta}{dr}.$$

Curve 1: $r = a(1 + \sin\theta)$

$$\frac{dr}{d\theta} = a\cos\theta.$$

$$\frac{d\theta}{dr} = \frac{1}{a\cos\theta}.$$

$$\tan\phi_1 = r\frac{d\theta}{dr} = \frac{a(1+\sin\theta)}{a\cos\theta} = \frac{1+\sin\theta}{\cos\theta}.$$

Curve 2: $r = a(1-\sin\theta)$

$$\frac{dr}{d\theta} = -a\cos\theta, \quad \frac{d\theta}{dr} = -\frac{1}{a\cos\theta}.$$

$$\tan\phi_2 = \frac{-a(1-\sin\theta)}{a\cos\theta} = -\frac{1-\sin\theta}{\cos\theta}.$$

Angle between curves:

$$\tan\alpha = \left|\frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}\right|.$$

Substitute:

$$\tan\alpha = \frac{1+\sin\theta\cos\theta + 1-\sin\theta\cos\theta}{1-(1+\sin\theta\cos\theta)\,(1-\sin\theta\cos\theta)}.$$

$$= \frac{2/\cos\theta}{1-1-\sin^2\theta\cos^2\theta} = \frac{2/\cos\theta}{1-1} \to \infty.$$

Hence,

$$\alpha = 90^\circ.$$

$$Curves cut orthogonally.$$

Q2(c) Question

Find the pedal equation of the curve

$$r^2 = a^2\sec 2\theta.$$

Solution

Pedal formula:

$$p = r\sin\phi, \qquad \tan\phi = r\frac{d\theta}{dr}.$$

Hence,

$$p^2 = \frac{r^4}{r^2 + \left(\frac{dr}{d\theta}\right)^2}.$$

Given:

$$r^2 = a^2\sec 2\theta.$$

Differentiate:

$$2r\frac{dr}{d\theta} = a^2\sec 2\theta\tan 2\theta \cdot 2.$$

$$\frac{dr}{d\theta} = \frac{a^2\sec 2\theta\tan 2\theta}{r}.$$

Using $r^2 = a^2\sec 2\theta$:

$$\frac{dr}{d\theta} = r\tan 2\theta.$$

Substitute:

$$p^2 = \frac{r^4}{r^2 + r^2\tan^2 2\theta} = \frac{r^2}{1+\tan^2 2\theta} = r^2\cos^2 2\theta.$$

Using $\sec 2\theta = r^2 a^2$:

$$\cos 2\theta = \frac{a^2}{r^2}.$$

$$p^2 = r^2\left(\frac{a^2}{r^2}\right)^2 = \frac{a^4}{r^2}.$$

$$p^2r^2 = a^4 \quad (Pedalequation).$$

Module 2

Q3(a) Question

Expand $\log(\sec x)$ in powers of x up to and including the term containing x^4.

Solution

We use the standard expansion:

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \cdots$$

Let

$$y = \sec x - 1 = \frac{x^2}{2} + \frac{5x^4}{24} + \cdots$$

Using

$$\log(1+y) = y - \frac{y^2}{2} + \cdots$$

Compute y^2 up to x^4:

$$y^2 = \left(\frac{x^2}{2}\right)^2 = \frac{x^4}{4}.$$

Hence,

$$\log(\sec x) = \left(\frac{x^2}{2} + \frac{5x^4}{24}\right) - \frac{1}{2} \cdot \frac{x^4}{4}.$$

$$= \frac{x^2}{2} + \frac{5x^4}{24} - \frac{x^4}{8}.$$

Take LCM:

$$\frac{5}{24} - \frac{3}{24} = \frac{2}{24} = \frac{1}{12}.$$

$$\log(\sec x) = \frac{x^2}{2} + \frac{x^4}{12} + \cdots$$

Q3(b) Question

Evaluate:

$$(i) \quad \lim_{x\to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x},$$

$$(ii) \quad \lim_{x\to \pi/2} (\sec x)^{\cot x}.$$

Solution

(i)

Take logarithm:

$$\ln L = \lim_{x\to 0} \frac{1}{x} \ln\left(\frac{a^x + b^x + c^x}{3}\right).$$

Use expansions:

$$a^x = 1 + x\ln a + \cdots$$

$$b^x = 1 + x\ln b + \cdots$$

$$c^x = 1 + x\ln c + \cdots$$

Thus,

$$a^x + b^x + c^x = 3 + x(\ln a + \ln b + \ln c).$$

$$\frac{a^x + b^x + c^x}{3} = 1 + \frac{x}{3}(\ln a + \ln b + \ln c).$$

Now,

$$\ln(1+u) \approx u.$$

Hence,

$$\ln L = \frac{1}{x} \cdot \frac{x}{3}(\ln a + \ln b + \ln c).$$

$$\ln L = \frac{\ln a + \ln b + \ln c}{3}.$$

$$L = \exp\left(\frac{\ln a + \ln b + \ln c}{3}\right).$$

$$L = (abc)^{1/3}.$$

(ii)

Let

$$L = \lim_{x \to \pi/2} (\sec x)^{\cot x}.$$

Take logarithm:

$$\ln L = \lim_{x \to \pi/2} \cot x \, \ln(\sec x).$$

Put

$$x = \frac{\pi}{2} - t, \quad t \to 0.$$

Then

$$\sec x = \csc t \sim \frac{1}{t}, \quad \cot x = \tan t \sim t.$$

Thus,

$$\ln L = t \ln\left(\frac{1}{t}\right) = -t \ln t \to 0.$$

Hence,

$$L = e^0 = 1.$$

Q3(c) Question

Discuss the maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 3x - 27y + 24.$$

Solution

First partial derivatives:

$$f_x = 3x^2 - 3, \qquad f_y = 3y^2 - 27.$$

Set to zero:

$$3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1,$$

$$3y^2 - 27 = 0 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3.$$

Critical points:

$$(1,3),\ (1,-3),\ (-1,3),\ (-1,-3).$$

Second derivatives:

$$f_{xx} = 6x, \quad f_{yy} = 6y, \quad f_{xy} = 0.$$

Determinant:

$$D = f_{xx}f_{yy} - f_{xy}^2 = 36xy.$$

1. At $(1,3)$

$$D = 36(3) = 108 > 0, \quad f_{xx} = 6 > 0 \Rightarrow Minimum.$$

$$f(1,3) = 1 + 27 - 3 - 81 + 24 = -32.$$

2. At $(-1,-3)$

$$D = 36(3) = 108 > 0, \quad f_{xx} = -6 < 0 \Rightarrow Maximum.$$

$$f(-1,-3) = -1 - 27 + 3 + 81 + 24 = 80.$$

3. At $(1,-3)$

$$D = 36(-3) = -108 < 0 \Rightarrow Saddlepoint.$$

4. At $(-1,3)$

$$D = 36(-3) = -108 < 0 \Rightarrow Saddlepoint.$$

$$Maximum = 80 at (-1,-3)$$

$$Minimum = -32 at (1,3)$$

Q4(a) Question

If

$$u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right),$$

prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$$

Solution

Let

$$p = \frac{x}{y}, \quad q = \frac{y}{z}, \quad r = \frac{z}{x}.$$

Then

$$u = f(p, q, r).$$

Using chain rule:

$$\frac{\partial u}{\partial x} = f_p\frac{\partial p}{\partial x} + f_q\frac{\partial q}{\partial x} + f_r\frac{\partial r}{\partial x}.$$

Compute derivatives:

$$\frac{\partial p}{\partial x} = \frac{1}{y}, \quad \frac{\partial q}{\partial x} = 0, \quad \frac{\partial r}{\partial x} = -\frac{z}{x^2}.$$

Hence,

$$x\frac{\partial u}{\partial x} = \frac{x}{y}f_p - \frac{z}{x}f_r.$$

Similarly,

$$y\frac{\partial u}{\partial y} = -\frac{x}{y}f_p + \frac{y}{z}f_q,$$

$$z\frac{\partial u}{\partial z} = -\frac{y}{z}f_q + \frac{z}{x}f_r.$$

Add:

$$xu_x + yu_y + zu_z = 0.$$

0

Q4(b) Question

If

$$u = x^2 + y^2 + z^2, \quad v = xy + yz + zx, \quad w = x + y + z,$$

find the Jacobian $\partial(u, v, w)\partial(x, y, z)$.

Solution

$$J = 2x2y2zy + zx + zx + y111.$$

Expand:

$$J = 2x[(x + z) - (x + y)] - 2y[(y + z) - (x + y)] + 2z[(y + z) - (x + z)].$$

$$= 2x(z - y) - 2y(z - x) + 2z(y - x).$$

$$= 2(x - y)(x - z)(y - z) \cdot (-1).$$

$$J = 2(x - y)(y - z)(z - x).$$

Q4(c) Question

If

$$u = x^2 + y^2 + z^2, \quad x = e^{2t},\ y = \cos 3t,\ z = \sin 3t,$$

find $dudt$.

Solution

$$u = e^{4t} + \cos^2 3t + \sin^2 3t.$$

Using identity:

$$\cos^2 3t + \sin^2 3t = 1.$$

$$u = e^{4t} + 1.$$

Differentiate:

$$\frac{du}{dt} = 4e^{4t}.$$

$$dudt = 4e^{4t}$$

Module 3

Q5(a) Question

Solve the differential equation

$$(2xy + y^2)\,dx + (x^2 + 2xy)\,dy = 0.$$

Solution

Let

$$M = 2xy + y^2, \qquad N = x^2 + 2xy.$$

Check exactness:

$$\frac{\partial M}{\partial y} = 2x + 2y, \qquad \frac{\partial N}{\partial x} = 2x + 2y.$$

Since

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x},$$

the equation is exact.

Find potential function ϕ such that

$$\phi_x = M = 2xy + y^2.$$

Integrate w.r.t. x:

$$\phi = \int (2xy + y^2)\,dx = x^2y + xy^2 + h(y).$$

Differentiate w.r.t. y:

$$\phi_y = x^2 + 2xy + h'(y).$$

But $\phi_y = N = x^2 + 2xy$.
Hence,

$$h'(y) = 0 \Rightarrow h(y) = C.$$

Therefore solution:

$$x^2y + xy^2 = C.$$

Q5(b) Question

Solve

$$\frac{dy}{dx} = \frac{x+y}{x-y}.$$

Solution

This is homogeneous.

Put

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

Substitute:

$$v + x\frac{dv}{dx} = \frac{x + vx}{x - vx} = \frac{1+v}{1-v}.$$

Hence,

$$x\frac{dv}{dx} = \frac{1+v}{1-v} - v.$$

$$= \frac{1 + v - v(1-v)}{1-v} = \frac{1+v^2}{1-v}.$$

Separate variables:

$$\frac{1-v}{1+v^2}\,dv = \frac{dx}{x}.$$

Integrate:

$$\int \frac{1}{1+v^2}dv - \int \frac{v}{1+v^2}dv = \int \frac{dx}{x}.$$

$$\tan^{-1} v - 12\ln(1+v^2) = \ln x + C.$$

Substitute $v = yx$:

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\ln(x^2+y^2) = \ln x + C.$$

Q5(c) Question

Solve the differential equation of Clairaut type

$$y = px + p^2,$$

where $p = dydx$.

Solution

Differentiate:

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + 2p\frac{dp}{dx}.$$

But $dydx = p$.
Hence,

$$p = p + (x + 2p)\frac{dp}{dx}.$$

$$(x + 2p)\frac{dp}{dx} = 0.$$

Two cases:

1. $dpdx = 0 \Rightarrow p = C$.
Substitute:

$$y = Cx + C^2.$$

This is the general solution.

2. $x + 2p = 0 \Rightarrow p = -\frac{x}{2}$.
Substitute into original:

$$y = -\frac{x^2}{2} + \frac{x^2}{4} = -\frac{x^2}{4}.$$

Singular solution:

$$y = -\frac{x^2}{4}.$$

Q6(a) Question

Solve

$$\frac{dy}{dx} + y\tan x = \sin x.$$

Solution

Linear equation:

$$P = \tan x, \quad Q = \sin x.$$

Integrating factor:

$$I.F. = e^{\int \tan x \, dx} = e^{-\ln \cos x} = \sec x.$$

Multiply:

$$\sec x \frac{dy}{dx} + y \sec x \tan x = \sec x \sin x.$$

LHS becomes:

$$\frac{d}{dx}(y \sec x) = \tan x.$$

Integrate:

$$y \sec x = \int \tan x \, dx = -\ln|\cos x| + C.$$

$$y = \cos x \left(C - \ln|\cos x|\right).$$

Q6(b) Question

Solve by variation of parameters

$$y'' + y = \sec x.$$

Solution

Complementary function:

$$m^2 + 1 = 0 \Rightarrow m = \pm i.$$

$$y_c = C_1 \cos x + C_2 \sin x.$$

Let

$$y_p = A(x) \cos x + B(x) \sin x.$$

Auxiliary conditions:

$$A' \cos x + B' \sin x = 0,$$

$$-A' \sin x + B' \cos x = \sec x.$$

Solve:
Multiply first by $\cos x$ and second by $\sin x$ and add:

$$B'(\sin^2 x + \cos^2 x) = \sec x \sin x.$$

$$B' = \tan x.$$

$$B = \int \tan x \, dx = -\ln|\cos x|.$$

Now,

$$A' \cos x + (-\ln|\cos x|)' \sin x = 0.$$

$$A' \cos x + \tan x \sin x = 0.$$

$$A' = -\tan^2 x.$$

$$A = \int -\tan^2 x \, dx = x - \tan x.$$

Particular solution:

$$y_p = (x - \tan x) \cos x - (\ln|\cos x|) \sin x.$$

General solution:

$$y = C_1 \cos x + C_2 \sin x + (x - \tan x) \cos x - (\ln|\cos x|) \sin x.$$

Q6(c) Question

Solve the Cauchy–Euler equation

$$x^2 y'' - x y' + y = 0.$$

Solution

Assume $y = x^m$.

$$y' = mx^{m-1}, \qquad y'' = m(m-1)x^{m-2}.$$

Substitute:

$$x^2 m(m-1)x^{m-2} - xmx^{m-1} + x^m = 0.$$

$$m(m-1)x^m - mx^m + x^m = 0.$$

$$[m(m-1) - m + 1]x^m = 0.$$

$$(m-1)^2 = 0.$$

$$m = 1 (repeated root).$$

Hence solution:

$$y = (C_1 + C_2 \ln x)\, x.$$

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7 a) Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 0$

Sol: → Given → $y''' - y'' + 4y' - 4y = 0$

for the given eq^n Auxiliary Eq^n is as follows:

$$m^3 - m^2 + 4m - 4 = 0$$

$$m^2(m-1) + 4(m-1) = 0$$

$$(m^2+4)(m-1) = 0$$

$$\Longrightarrow \quad m = 1 \; ; \; m^2 = -4$$

$$m = 1 \; ; \Rightarrow m = \pm 2i$$

∴ So, Complementary f^n is

for Complex roots, we know → $\alpha \pm i\beta$

here $\alpha = 0$; $\beta = 2$.

So;

$$\boxed{\text{Our required CF} \Rightarrow y = C_1e^x + C_2\cos 2x + C_3\sin 2x.}$$

7 b) Solve $x^2\frac{d^2y}{dx^2} - 2y = x^2 + \frac{1}{x}$

Sol:. the given eq^n is Cauchy - Euler Eq^n

∴ Put $x = e^z$ then $\Rightarrow z = \ln x$.

$$\frac{dy}{dx} = \frac{1}{x}\frac{dy}{dz} \; ; \; \frac{d^2y}{dx^2} = \frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right.$$

By substituting we get;

$$y_{zz} - y_z - 2y = e^{2z} + e^{-z}$$

Auxiliary eqⁿ → $m^2 - m - 2 = 0$

$$m = 2, -1.$$

$$CF = c_1 e^{2z} + c_2 e^{-z}$$

for PI ⇒ Assume → $y = Aze^{2z} + Bze^{-z}$

After substituting into the eqⁿ :→

we get

for e^{2z} → $3A = 1 \Rightarrow A = \frac{1}{3}$.

for e^{-z} → $-3B = 1 \Rightarrow B = -\frac{1}{3}$

∴ Complete sol in z

$$\Rightarrow y = c_1 e^{2z} + c_2 e^{-z} + \frac{1}{3} z e^{2z} - \frac{1}{3} z e^{-z}$$

Converting back to x.

$$\Rightarrow z = \ln x.$$

$$y = c_1 x^2 + c_2 x^{-1} + \frac{1}{3} x^2 \ln x - \frac{1}{3} x^{-1} \ln x$$

7c) Solve by the method of Variation of Parameters $\frac{d^2y}{dx^2} + y = \tan x$

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Sol:. $y'' + y = \tan x$

* Complementary F^n :→

$$m^2 + 1 = 0$$

$$\Rightarrow m = \pm i$$

$$\therefore C.F. = C_1 \cos x + C_2 \sin x$$

* for P.I. we use V.O.P.

Let $y_1 = \cos x$; $y_2 = \sin x$, then

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$W = \cos^2 x + \sin^2 x = 1$$

Now, we know

$$u_1' = -\frac{y_2 R}{W}; \quad u_2' = \frac{y_1 R}{W}$$

$\therefore$ Here $R = \tan x$

$$\therefore u_1 = -\int \frac{\sin x \tan x \, dx}{1}$$

$$\Rightarrow -\int \frac{\sin^2 x}{\cos x} dx$$

$$= -\int \frac{1-\cos^2 x}{\cos x} dx$$

$$= -\int \sec x - \cos x \, dx$$

$$= -\left[\ln|\sec x + \tan x| - \sin x\right]$$

$$u_1 = -\ln|\sec x + \tan x| + \sin x.$$

for $u_2 = \int \frac{y_1 R}{W}\, dx$.

$$= \int \cos x \tan x\, dx$$

$$= \int \cos x \frac{\sin x}{\cos x}\, dx$$

$$= \int \sin x\, dx$$

$$= -\cos x.$$

∴ Particular Integ ⇒ $y_p = u_1 y_1 + u_2 y_2$

$$= \cos x\left(-\ln|\sec x + \tan x| + \sin x\right) + \sin x(-\cos x)$$

$$= -\cos x \ln|\sec x + \tan x|$$

∴ Sol: $\boxed{y = c_1 \cos x + c_2 \sin x - \ln|\sec x + \tan x| \cos x}$

8 a) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 12x^2$

Sol: ∴ Auxiliary Eqⁿ → $m^2 + 3m + 2 = 0$

$\Rightarrow m^2 + 2m + m + 2 = 0$

$m(m+2) + 1(m+2) = 0$

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$\Rightarrow (m+1)(m+2) = 0$

$\Longrightarrow m = -1, -2.$

$\therefore C.F. = c_1 e^{-x} + c_2 e^{-2x}$

Now, for P.I ;

Assume $y_p = Ax^2 + Bx + C.$

$y'_p = 2Ax + B$

$y''_p = 2A.$

Substituting in eq^n :-

$2A + 3(2Ax + B) + 2(Ax^2 + Bx + C) = 12x^2$

$\therefore \rightarrow 2Ax^2 + (6A + 2B)x + (2A + 3B + 2C)$

$= 12x^2$

Comparing we get

$2A = 12 \Rightarrow A = 6$

$6A + 2B = 0.$

$2B = -36 \Rightarrow B = -18$

$\therefore C = 21$

$y_p = 6x^2 - 18x + 21$

$\therefore \boxed{y = c_1 e^{-x} + c_2 e^{-2x} + 6x^2 - 18x + 21}$

8 b) Solve $y'' - 6y' + 9y = 5e^{3x} - \sin 2x$

Sol:: A.E. $\Rightarrow$ $m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) + 3(m-3) = 0$

$(m-3)(m-3) = 0$

$m = 3, 3.$

Repeated root so, $CF = (C_1 + C_2 x)e^{3x}$

∴ For P.I.;

$$= \frac{5e^{3x}}{D^2 - 6D + 9} + \frac{-\sin 2x}{D^2 - 6D + 9}.$$

For $5e^{3x}$

$$\Rightarrow \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}.$$

∴ $f(D) = (D-3)^2$

∴ By comparing $D = a = 3.$

$f(3) = 0.$ twice

∴ repeated root ; so multiply by x^2

$PI_1 = Ax^2e^{3x}.$

Substituting we get

$\therefore\ 2Ae^{3x} = 5e^{3x}$

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$2A = 5$

$\boxed{A = \frac{5}{2}}$

$PI_1 = \frac{5}{2}x^2e^{3x}$

$\therefore$ for $PI_2 \Rightarrow$ let $y_{p_2} = A\cos 2x + B\sin 2x$

$y' = -2A\sin 2x + 2B\cos 2x$

$y'' = -4A\sin 2x - 4B\sin 2x$

Substituting into LHS;

$y'' - 6y' + 9y$

$\Rightarrow (-4A\cos 2x - 4B\sin 2x) - 6(-2A\sin 2x + 2B\cos 2x) + 9(A\cos 2x + B\sin 2x)$

comparing

$\Rightarrow$ Collecting $\cos 2x$ terms

$-4A - 12B + 9A = 5A - 12B.$

Similarly $\sin 2x$

$\hookrightarrow 12A + 5B$

Comparing

$5A - 12B = 0$

$12A + 5B = -1$

$\Rightarrow A = -\frac{12}{169}$; $B = \frac{-5}{169}$

$PI_2 = -\frac{12}{169}\cos 2x - \frac{5}{169}\sin 2x$

$$\therefore\ y = (C_1 + C_2 x)e^{3x} + \frac{5}{2}x^2e^{3x} - \frac{12}{169}\cos 2x - \frac{5}{169}\sin 2x$$

8 c) Solve $(3x+2)^2 y'' + 3(3x+2)y' - 36y = 8x$.

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Put $3x+2 = e^z$. $\Rightarrow$ ~~$x = \frac{z-2}{3}$~~

$\downarrow$ we know. $x = \frac{e^2-2}{3}$

$3x+2 = e^z$

$\Rightarrow z = \log(3x+2)$

~~$\frac{dz}{dx}$~~ $\therefore (3x+2)D = 3D'$

$(3x+2)^2 D'' = 3^2 D'(D'-1)$

$\therefore$ we get ;

$$\left(9D'(D'-1) + 3\cdot 3D' - 36\right)y = 8\left(\frac{e^2-2}{3}\right)$$

$$\left(D'(D'-1) + D' - \overset{4}{\cancel{36}}\right)y = \frac{8}{27}(e^2-2)$$

$\therefore$ Auxiliary eqⁿ

$m(m-1) + m - 4 = 0$

$m^2 - m + m - 4 = 0$

$m^2 - 4 = 0 \Rightarrow m^2 = 4$

$m = \pm 2$

$CF = c_1 e^{2z} + c_2 e^{-2z}$

for Particular Put

$$PI = \frac{1}{(D'^2-4)}\frac{8}{27}(e^z-2)$$

$$\Rightarrow \frac{8}{27}\left[\left(\frac{e^z}{D'^2-4}\right)-2\times\frac{e^{0z}}{D'^2-4}\right)$$

$$\Rightarrow \frac{8}{27}\left(\frac{e^z}{1-4}-2\times\frac{1}{0-4}\right)$$

$$\Rightarrow \frac{8}{27}\left(\frac{e^z}{-3}+\frac{\cancel{2}}{\cancel{4}2}\right)$$

$$\Rightarrow \frac{8}{27}\left(\frac{1}{2}-\frac{1}{3}e^z\right)$$

$$PI = \frac{4}{27}-\frac{8}{81}e^z$$

$$\therefore y = c_1e^{2z}+c_2e^{-2z}-\frac{8}{81}e^z+\frac{4}{27}$$

$$\because z = \log(3x+2)$$

$$y = c_1(3x+2)^2 + c_2(3x+2)^{-2} - \frac{8}{81}(3x+2) + \frac{4}{27}$$

$$\boxed{y = c_1(3x+2)^2 + \frac{c_2}{(3x+2)^2} - \frac{8}{81}(3x+2) + \frac{4}{27}}$$

9ⓐ Determine the rank of the Matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$.

Soln

Let the given matrix be A. Then applying $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

we get $A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$.

applying $R_3 \to R_3 + R_2$ we get $A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
& then $R_2 \to (-1)R_2$
we get

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ which is in Row echelon form with 2 non-zero rows

So rank of $A = 2$

9ⓑ Solve the following system of equations by Gauss-Seidel method. Carry out Three iterations.
$10x + y + z = 12$; $x + 10y + z = 12$; $x + y + 10z = 12$

Soln: The given system is diagonally dominant & can be written as $x = \frac{1}{10}[12 - y - z]$; $y = \frac{1}{10}[12 - x - z]$ & $z = \frac{1}{10}[12 - x - y]$.

Starting with initial approximation $x_0 = y_0 = z_0 = 0$, we get

$x_1 = \frac{1}{10}[12 - 0 - 0] = 1.2$; $y_1 = \frac{1}{10}[12 - 1.2 - 0] = 1.08$ & $z_1 = \frac{1}{10}[12 - 1.2 - 1.08] = 0.972$

$\Rightarrow x_2 = \frac{1}{10}[12 - 1.08 - 0.972] = 0.9948$; $y_2 = \frac{1}{10}[12 - 0.9948 - 0.972] = 1.00332$; $z_2 = \frac{1}{10}[12 - 0.9948 - 1.0032] = 1.000188$.

$\Rightarrow x_3 = \frac{1}{10}[12 - 1.00332 - 1.000188] = 0.99965$; $y_3 = \frac{1}{10}[12 - 0.99965 - 1.000188] = 1.00002$; $z_3 = \frac{1}{10}[12 - 0.99965 - 1.0000] = 1.00003$

So after 3 iterations $x = 0.99965$; $y = 1.00002$; $z = 1.00003$

9 ⓒ Find the dominant eigen value & the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by Power methods with the initial eigen vector $X = [1 \ 0 \ 0]'$. carry out 4 iterations.

Soln :- For the given A, let $X = [1 \ 0 \ 0]' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$;

we use $AX = \lambda X$ to carry out the iterations.

$$\text{so } AX^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$\Rightarrow AX^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$\Rightarrow AX^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$\Rightarrow AX^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = 2.93 \begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

So after 4 iterations; eigen value $\lambda = 2.93$

corresponding eign vector $X = \begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix}$

0

ⓐ solve by Gauss elimination Method

$x+y+z=9;\ x-2y+3z=8;\ 2x+y-z=3$

Soln:- The augmented matrix for the given system is

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right].$$

applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - 2R_1$, we get

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

applying $R_3 \to (-3R_3) + R_2$, we get.

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{bmatrix}.$$ which is an upper Triangular system.

So we have $x+y+z=9 \to ①$

$-3y+2z=-1 \to ②$

and $11z=44$

$\Rightarrow \boxed{z=4}$; using this in ② we get $-3y=-9 \Rightarrow \boxed{y=3}$

Using y & z in ① we get $x+3+4=9 \Rightarrow x=2$

$\therefore$ The soln is $x=2;\ y=3\ \&\ z=4$

10

ⓑ Investigate the values of λ & μ such that the system of equations $x+y+z=6;\ x+2y+3z=10;\ x+2y+\lambda z=\mu$ may have ⓐ unique solution ⓑ infinite solutions ⓒ No solution

Soln:. Writing the given system of linear eqns as $AX=B$

We have $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}$ $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B=\begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$.

Considering the augmented matrix $[A:B]=\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$

applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we get

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix}.$$

applying $R_3 \to R_3 - R_2$, we get

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}.$$

ⓐ To get Unique solution, we must have $\lambda \neq 3$ So that $\rho(A) = \rho([A:B]) = 3$, the no. of Variables

So we must have $\lambda \neq 3$, irrespective of μ, for Unique soln

ⓑ To obtain infinitely many solutions, we have $\rho(A) = \rho([A:B]) = r < n$, the no of variables. Here $n=3$

$\Rightarrow \lambda - 3 = 0$ & $\mu - 10 = 0$ or $\lambda = 3$ & $\mu = 10$.

So for $\lambda = 3$ & $\mu = 10$, we obtain infinitely many solns

ⓒ For the system to have no solution, we must have $\rho(A) \neq \rho([A:B])$; So if $\lambda = 3$ we get $\rho(A) = 2$

and if $\mu \neq 10$ we get $\rho(A) = 3$

In this case, the System has no solution.

10 ⓒ Find the eigen Values & eigen vectors of matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

Soln:- Consider $|(A - \lambda I)| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0.$

$\Rightarrow (1-\lambda)[(5-\lambda)(1-\lambda) - 1] - 1[1(1-\lambda) - 3] + 3[1 - 3(5-\lambda)] = 0$

$\Rightarrow \lambda^3 - 7\lambda^2 + 36 = 0 \Rightarrow$ ~~$\lambda^2(\lambda+2) - 4(\lambda+2) = 0$~~ $(\lambda+2)(\lambda^2 - 9\lambda + 18) = 0$

$\Rightarrow (\lambda+2)(\lambda^2+6)(\lambda-3) = 0 \Rightarrow \lambda = 3, 6, \cancel{+}2$ are eigen Values.

For $\lambda = 7$ we have $(A - 7I)X = 0 \Rightarrow \begin{bmatrix} -6 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 1 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

10c Find the eigen values & eigen vectors of matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

Soln : The characteristic eqn is $|(A - \lambda I)| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)[(5-\lambda)(1-\lambda)-1] - 1[1(1-\lambda) - 3] + 3[1 - 3(5-\lambda)] = 0$

$\Rightarrow (1-\lambda)[5-6\lambda+\lambda^2-1] - [-\lambda-2] + 3[\lambda-14] = 0 \Rightarrow \lambda^3 - 7\lambda^2 + 36 = 0.$

for $\lambda = -2$, we get $-8-7(4)+36) = 0 \Rightarrow (\lambda+2)$ is a factor

So $\lambda^3 - 7\lambda^2 + 36 = (\lambda+2)(\lambda^2 - 9\lambda + 18) = 0 \Rightarrow (\lambda+2)(\lambda-3)(\lambda-6) = 0$

$\Rightarrow \lambda = -2, 3, 6$ are eigen Vectors;

For $\lambda = -2$, we have $(A - \lambda I)X = 0 \Rightarrow \begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$

For $\lambda = 3$; we have $(A - \lambda I)X = 0 \Rightarrow \begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

For $\lambda = 6$, we have $(A - \lambda I)X = 0 \Rightarrow \begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

∴ The eigen values are -2, 3 and 6;

The corresponding eigen Vectors are $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

respectively.