SOLUTION TO VTU PHYSICS QUESTION PAPER CSE STREAM (2025 MODEL QUESTION PAPER)

1A

Time independent Schrödinger equation

Differential equation for a wave is

$$\frac{d^2\Psi}{dx^2} = \frac{1}{v^2}\frac{d^2\Psi}{dt^2} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

A matter wave can be represented in complex form as

$$\Psi = A \sin k\,x(\cos w\,t + i \sin w\,t)$$

$$\Psi = A \sin k\,x e^{iwt}$$

Differentiating wrt t

$$\frac{d\Psi}{dt} = iw\ A.sinkxe^{iwt}$$

Differentiating wrt t

$$\frac{d^2\Psi}{dt^2} = iw\ iw\ \ A.sinkxe^{iwt} = -w^2\ \ \psi$$

$$w = 2\pi f \qquad \omega = 4\pi^2 f^2 \qquad v\ = c = f^2\lambda^2$$

Substitute in (1)

$$\frac{d^2\Psi}{dx^2} = \frac{1}{v^2}\frac{d^2\Psi}{dt^2} = \frac{1}{c^2}(-)4\pi^2 f^2\ \ \Psi = -\frac{4\pi^2 f^2}{f^2\lambda^2}\ \psi$$

From Debroglie's relation

$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$

$$\frac{d^2\Psi}{dx^2} = -\frac{4\pi^2 p^2}{h^2}\ \psi$$

Total energy of a particle

E = Kinetic energy + Potential Energy

$$E = \frac{p^2}{2m} + V$$

$$p^2 = (E - V)2m$$

$$\frac{d^2\Psi}{dx^2} = -\frac{4\pi^2(E - V)2m}{h^2}\ \psi$$

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E - V)\Psi}{h^2} = 0$$

$For\ \ 3D$

$$\frac{d^2\Psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dx^2} + \frac{8\Pi^2 m(E - V)\Psi}{h^2} = 0$$

1B

Wave function: It represents the matter wave of a particle.It is obtained as solution to Schrodinger equation

Significance of Ψ^2:

According to Max Born,$|\psi|^2$ represents probability of finding a particle per unit volume.

Normalisation: Total probability of finding a particle in a closed volume is unity.

$$\int \Psi^2 dv = 1$$

A function is said to be a normalisable wave function if it is

1. Single valued
2. Continuous
3. Finite

Complementarity principle : The inability to observe the wave nature and the particle nature of the matter simultaneously is known as the complementarity principle.

Correspondence principle states that **the behavior of systems described by the theory of quantum mechanics (or by the old quantum theory) reproduces classical physics in the limit of large quantum numbers**.

Expectation value is the average value of the parameter being measure through large number of measurements.

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* \, x\psi . dx$$

Associated with each measurable parameter in a physical system is a quantum mechanical operator.

Ex: Position expectation value

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^* \, x\psi . dx$$

Momentum expectation value $\hat{p} = \int_{-\infty}^{\infty} \frac{h}{4\pi i} . \frac{\partial}{\partial x}$

$$< p > = \int_0^L |\psi|^2 . \hat{p} dx$$

Quantum Tunneling

Quantum tunneling is a quantum mechanical phenomenon where a particle can pass through a potential energy barrier, even if its energy is less than the barrier's height. This is impossible in classical physics, where a particle needs sufficient energy to overcome a barrier. It's a fundamental aspect of quantum mechanics and has important implications in various phenomena and technologies.

- **Probability:**

  The probability of a particle tunneling through the barrier depends on the barrier's width and height, as well as the particle's mass and energy.

- **Examples:**

  Quantum tunneling is essential for various phenomena like:

  - **Radioactive Decay:** The decay of atomic nuclei, where alpha particles tunnel out of the nucleus.
  - **Nuclear Fusion in Stars:** At the core of stars, nuclear fusion is facilitated by quantum tunneling, allowing lighter nuclei to fuse into heavier ones despite the high potential barrier.
  - **Scanning Tunneling Microscope (STM):** This microscope uses the tunneling current between a sharp tip and a surface to image atomic structures.
  - **Josephsons junction**

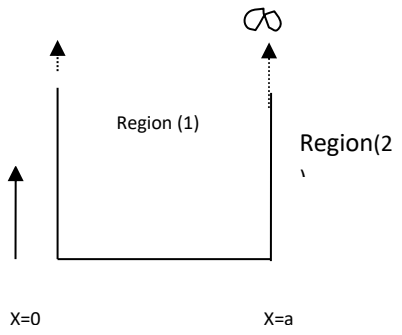
1C

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}} = \frac{6.62x10^{-34}}{\sqrt{2x9.1x10^{-31}x100x1.6x10^{-19}}} = 3.87\ x10^{-9}m$$

2A

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X-axis in the region from X=0 to X=a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \because V = 0$$

$$\text{But } k^2 = \frac{8\Pi^2 mE}{h^2}$$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Auxiliary equation is $\left(D^2 + k^2\right)x = 0$

Roots are D = +ik and D = -ik

The general solution is

$$x = Ae^{ikx} + Be^{-ikx}$$
$$= A(\cos kx + i\sin kx) + B(\cos kx - i\sin kx)$$
$$= (A+B)\cos kx + i(A-B)\sin kx$$
$$= C\cos kx + D\sin kx$$

The boundary conditions are

1. At x=0, $\Psi = 0 \therefore C = 0$

2. At x=a, $\Psi = 0$

D sin ka = 0 $\Rightarrow$ ka = nΠ ……(2)

where n = 1, 2 3…

$\therefore \Psi = D\sin\left(n\frac{\Pi}{a}\right)x$

From (1) and (2) Eigen value E = $\frac{n^2h^2}{8ma^2}$

To evaluate the constant D:

Normalisation: For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2(\frac{n\Pi}{a})xdx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}(1-\cos 2(\frac{n\Pi}{a})x)dx = 1$$

$$\int_0^a \frac{D^2}{2}dx - \int_0^a \frac{1}{2}\cos 2(\frac{n\Pi}{a})x)dx = 1$$

$$\frac{D^2a}{2} - \quad [\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2 \frac{a}{2} - 0 = 1$$

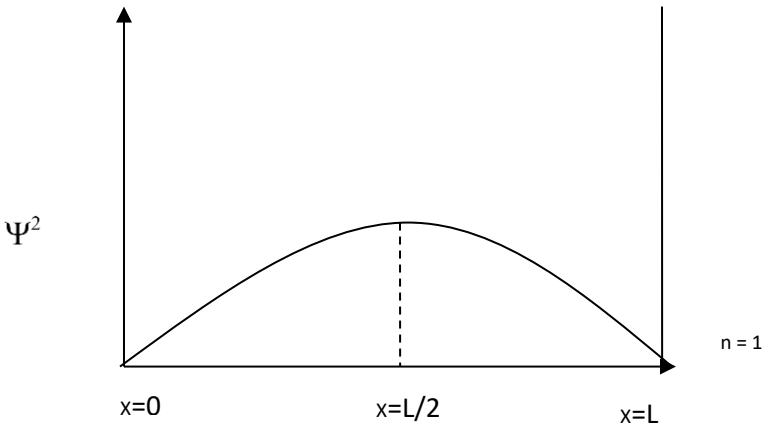
$$D = \sqrt{\frac{2}{a}}$$

Eigen function $\therefore \Psi_n = \sqrt{\frac{2}{a}} \sin\left(n\frac{\Pi}{a}\right)x$

For n = 1, First state

$$\therefore \Psi_1 = \sqrt{\frac{2}{a}} sin\left(1.\frac{\Pi}{a}\right)x$$

The graph of Ψ^2 $versus$x is shown below.

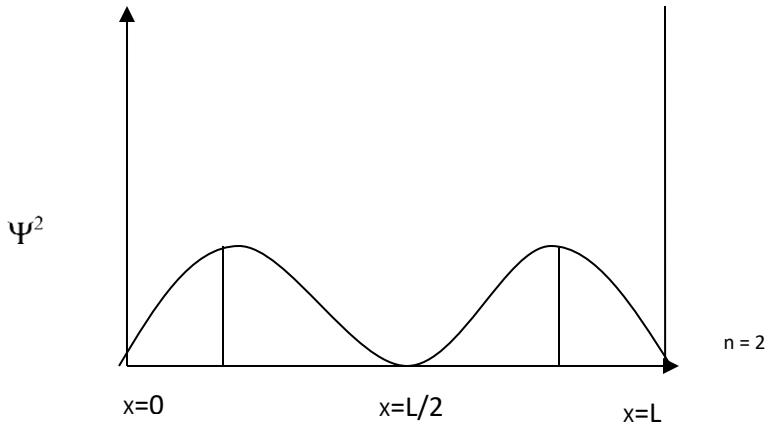


It is seen from the graph that probability density is maximum at the centre for the particle in the first state.

For n = 2, Second state

$$\therefore \Psi_2 = \sqrt{\frac{2}{a}} sin\left(2.\frac{\Pi}{a}\right)x$$

The graph of Ψ^2 $versus$x is shown below.



It is seen from the graph that probability density is maximum at x = L / 4 and x = 3L/4 for the particle in the second state.

2B

HEISENBERG'S UNCERTAINTY PRINCIPLE:

The position and momentum of a particle cannot be determined accurately and simultaneously.The product of uncertainty in the measurement of position (Δx) and momentum (Δp) is always greater than or equal to$\frac{h}{2\Pi}$.

$$(\Delta x).(\Delta p) \geq \frac{h}{4\Pi}$$

This uncertainty is not due to discrepancy with the apparatus or with the method of measurement, but because of the very wave nature of the object. This uncertainty persists as long as matter possesses wave nature.

Different forms of Heisenberg's Principle:

$$(\Delta x).(\Delta p) \geq \frac{h}{4\Pi}$$

$$(\Delta L).(\Delta \theta) \geq \frac{h}{4\pi}$$

$$(\Delta E).(\Delta t) \geq \frac{h}{4\pi}$$

Here ΔL is the uncertainty in angular momentum

Δθ is the uncertainty in the measurement of angular displacement

ΔE is the uncertainty in the measurement of energy

Δt is the uncertainty in the measurement of time interval during which the particle exists in the state E

Application of - Broadening of Spectral Lines

Whenever an atom is excited to a higher energy state E_2, it remains in that state only for a very short duration (lifetime τ). Soon after, it returns to the lower energy state E_1, emitting energy in the form of a spectral line. The wavelength of the emitted spectral line is given by:

$E_2 - E_1 = h\vartheta = h\frac{c}{\lambda}$

However, in reality, the atoms do not emit radiation at precise wavelengths. Each spectral line is broadened due to various causes (Fig.1).

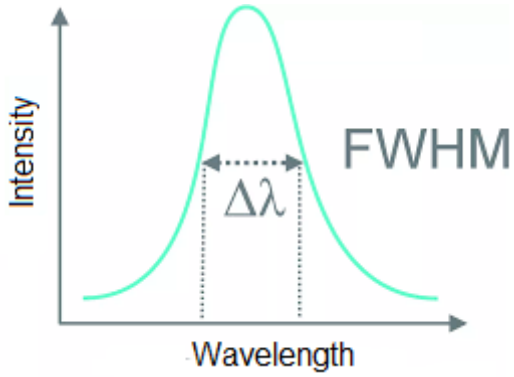


Fig.1 Intensity Vs. wavelength plot

The natural line broadening of a spectral line is a result of Heisenberg's uncertainty principle .According to this principle, there is an uncertainty in the energy state (ΔE) of a system due to the uncertainty in its life time (Δt). The lifetime of the excited states will result in natural broadening of the spectral line.

According to uncertainty principle, energy level above the ground state with energy E and lifetime Δt, has uncertainty in the measurement of energy ΔE given by

$\Delta E \times \Delta t \geq \frac{h}{4\pi}$ ------- (1)

For an excited state with mean lifetime we take $\Delta t \sim \tau$. Thus , the energy uncertainty is

$\Delta E \geq \frac{h}{4\pi \times \tau}$ ------- (2)

Relation between energy and wavelength is given by

$$E = \frac{hc}{\lambda}$$

Differentiating (taking only magnitudes and ignoring the minus sign, since we're concerned with widths),

$\Delta E = \frac{hc}{\lambda^2} \times \Delta\lambda$ -------- (3)

Substituting eq. (3) in eq. (2) $\frac{hc}{\lambda^2} \times \Delta\lambda \geq \frac{h}{4\pi \times \tau}$

Or $\Delta\lambda \geq \frac{\lambda^2}{4\pi \times \tau \times c}$

When measuring spectral lines, the lifetime of the excited state determines the width of these lines. If a state has a long lifetime, the uncertainty in energy is small, resulting in a narrow spectral line. Conversely, a short lifetime leads to greater energy uncertainty, causing a wider spectral line.

2C

$$E = \frac{n^2 h^2}{8mL^2}$$

GROUND STATE n = 1

$$E_1 = \frac{1^2 h^2}{8mL^2} = \frac{(6.62x10^{-34})^2}{8x9.1x10^{-31}x1x10^{-10}} = 6x10^{-18}J = 37.5eV$$

FIRST EXCITED STATE n = 2

$$E_2 = \frac{2^2 h^2}{8mL^2} = \frac{(6.62x10^{-34})^2}{8x9.1x10^{-31}x1x10^{-10}} = 24x10^{-18}J = 150eV$$

FIRST EXCITED STATE n = 3

$$E_3 = \frac{3^2 h^2}{8mL^2} = \frac{(6.62x10^{-34})^2}{8x9.1x10^{-31}x1x10^{-10}} = 54x10^{-18}J = 337.5eV$$

3A

Fermi Energy: It is the highest energy state fully occupied by free electrons at 0K

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\Pi}\right)^{\frac{2}{3}}$$

Fermi probability factor: It represents the probability of occupation of an energy level.

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1}$$

Density of energy of states:

It represents the number of energy levels per unit energy range per unit volume.

$$g(E)=8\Pi\sqrt{2}m^{\frac{3}{2}}\frac{E^{\frac{1}{2}}}{h^3}$$

To show that energy levels below Fermi energy are completely occupied:

For E **<** E_F, at T = 0,

$$f(E) = \frac{1}{e^{(\frac{E-E_F}{kT})}+1} = 1$$

To show that energy levels above Fermi energy are empty:

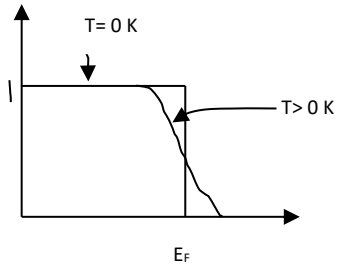
For E >E_F, at T=0

$$f(E) = \frac{1}{e^{(\frac{E-E_F}{kT})}+1} = 0$$

At ordinary temperatures, for E = E_F,

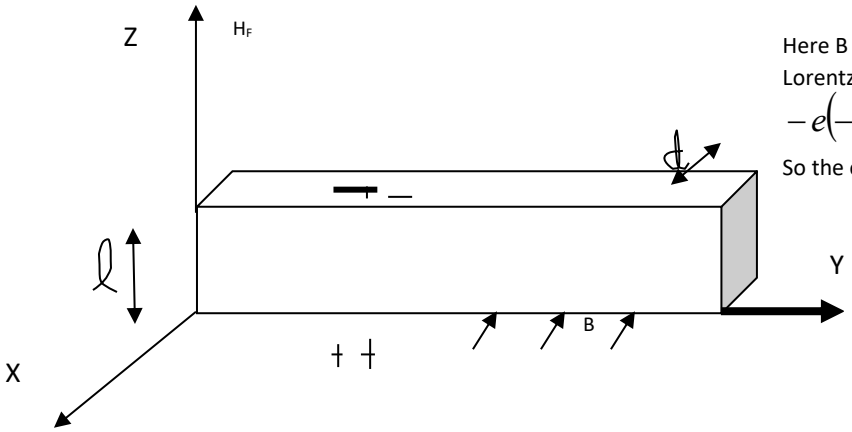
$$f(E) = \frac{1}{e^0+1}$$

$$f(E) = \mathbf{\frac{1}{2}}$$



3B

Hall effect: When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.



Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

Expression for electron concentration:

At equilibrium, Lorentz force is equal to force due to applied electric field

$$Bev = -eE_H$$

Hall Field $E_H = Bv$

$$Current\ \ density\ \ J = I/A = neAV/A = n_e ev$$

$$v = \frac{J}{n_e e}$$

Lorentz force (F_L) ↑

Hence

Electric force (eE) ↓

$$\frac{E_H}{JB} = \frac{1}{n_e e} = R_H$$

$E_H = JBR_H$

R_H is known as Hall coefficient. It is negative for n type and p type for positive semiconductors.

Hall voltage $V_H = E_H . l = Bvl = \frac{JB\,l}{ne} = JBR_H\ \ l$

3C

Given f(E) = 0.1

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

$$E - E_F = +0.5eV = 0.5x1.6x10^{-19}J$$

$$e^{\frac{E-E_F}{kT}} + 1 = \frac{1}{f(E)}$$

$$e^{\frac{E-E_f}{kT}} = \frac{1}{f(E)} - 1$$

$$\frac{E - E_F}{kT} ln_e\, e = ln\left(\frac{1}{f(E)} - 1\right)$$

$$T = \frac{E - E_F}{k.\, ln\left(\frac{1}{f(E)} - 1\right)} \quad = \frac{0.5x1.6x10^{-19}}{1.38x10^{-23}\ ln\left(\frac{1}{0.01} - 1\right)} = 1261K$$

4A

Failures of Classical free electron theory:

1. Prediction of low specific heats for metals:

Classical free electron theory assumes that conduction electrons are classical particles similar to gas molecules. Hence,they are free to absorb energy in a continuous manner. Hence metals possessing more electrons must have higher heat content. This resulted in high specific heat given by the expression $C_V = 10^{-4}R$.

This was contradicted by experimental results which showed low specific heat for metals.

2. Temperature dependence of electrical conductivity:

From the assumption of kinetic theory of gases

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$

$$\therefore v \propto \sqrt{T}$$

Also mean collision time τ is inversely proportional to velocity,

$$\tau \propto \frac{1}{v}$$

$$\tau \propto \frac{1}{\sqrt{T}}$$

$$\therefore \sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

However experimental studies show that $\sigma \propto \frac{1}{T}$

3. Dependence of electrical conductivity on electron concentration:

As per free electron theory, $\sigma \propto n$

The electrical conductivity of Zinc and Cadmium are 1.09 x 10^7 /ohm m and .15 x 10^7 /ohm m respectively which are very much less than that for Copper and Silver for which the values are 5.88x10^7 /ohm m and 6.2 x 10^7 /ohm m. On the contrary, the electron concentration for zinc and cadmium are 13.1x10^{28} /m^3 and 9.28 x10^{28} /m^3 which are much higher than that for Copper and Silver which are 8.45x10^{28} /m^3 and 5.85 x10^{28} /m^3.

These examples indicate that $\sigma \propto n$ does not hold good.

4. Mean free path, mean collision time found from classical theory are incorrect.

Quantum free electron theory:

Assumptions:

1. The energy of conduction electrons in a metal is quantized.

2. The distribution of electrons amongst various energy levels is according to Pauli's exclusion principle and Fermi – Dirac statistical theory.

3. The average kinetic energy of an electron is equal to $\frac{3}{5}E_F$

4. The attraction between the electrons and ions, the repulsion between electrons are ignored.

4B

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2} ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4} kT\, ln\left(\frac{m_e^*}{m_h^*}\right)$$

At T = 0 K

$E_f = \frac{E_v + E_c}{2} = \frac{E_g}{2}$

4C

$$E_H = B\frac{J}{n_e e} = BJ\ R_H$$

$$J = \frac{I}{A} = \frac{0.005}{0.0005x0.0005} = 20000\frac{A}{m^2}$$

$$Hall\ voltage\ V_H = BJ\ R_H X\ t$$

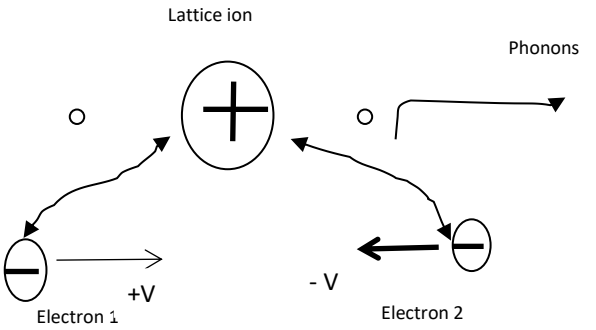
$$R_H = \frac{V_H}{BJt} = \frac{0.001}{0.2x20000x0.0005} = 5x10^{-4}C^{-1}m^3$$

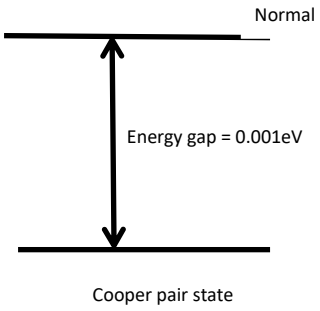
5A

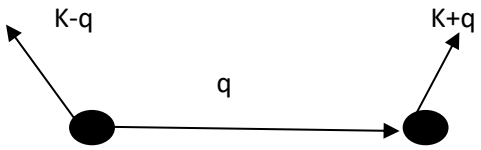
Phonons are the quanta of lattice vibrational energy.

BCS Theory :[Bardeen , Cooper, Schrieffer]

1. When the temperature of the material is reduced below critical temperature, electrons attain lower energy state than the normal energy creating an energy gap of few milli electron volt.
2. **Positively charged lattice ion attracts a pair of electrons with equal and opposite spin and momentum , lattice vibrations known as phonons are created**. The two electrons and the lattice atom interact through a feeble attractive interaction known as electron-lattice-electron interaction constituting cooper pairs.
3. Cooper pairs interact through exchanging Phonons.
4. All the cooper pairs are in same energy state known as **Phase quantum state** and possess common wavefunction and Energy.
5. When a potential differnce applied, the current is constituted by flow of cooper pairs and are not scattered as the energy required to break it up is large enough. This reduces the resistance.







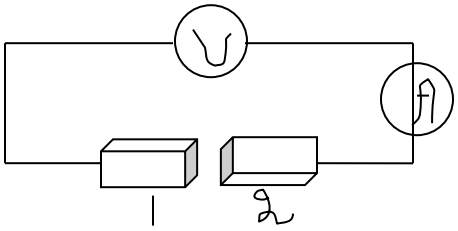
6. When the temperature / magnetic field is increased beyond critical limit, cooper pairs breakup and normal state is restored.

5B

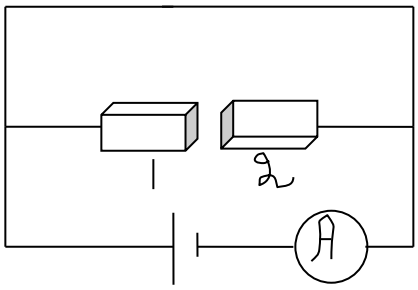
Josephson junction

It consists of an an insulator, sandwiched between two superconducting layers of which becomes a superconductor when cooled below critical temeperature. The insulating layer is so thin (a few nanometres) that Cooper pairs can tunnel through it and couple the superconducting wavefunctions on either side of the barrier. Most of the circuits for superconducting qubits consist of Josephson junctions and other components like capacitors connected by superconducting leads .

DC Josphsons effect: When the distance between the two superconducting bars is reduced below 1nm, the voltmeter suddenly due to the passage of electric current in the energy gap.



AC Josephsons effect:



When the superconducting bars are separated by thin insulator, on application of a voltage between the superconducting bars, a electromagnetic radiation emanates from the gap.

$$I = I_o \sin \varphi$$

I_o is critical current without voltage

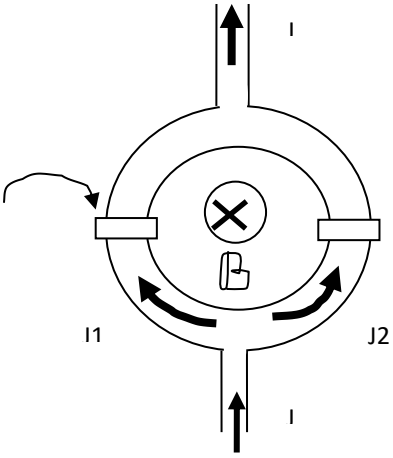
Φ is phase difference

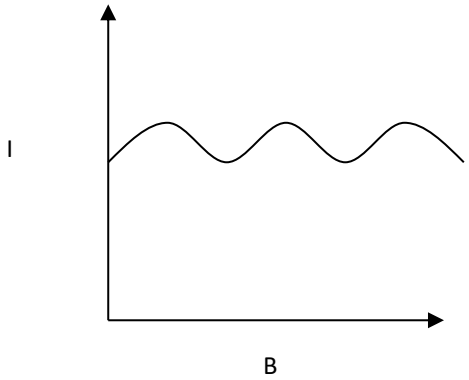
Frequency =$\frac{2eV}{h}$

FLUX QUANTIZATION

When magnetic field is applied normal to the plane of Josephson junction, current is induced in the superconductor which opposes the external magnetic field. Magnetic Flux through the superconductor is quantized .The current through the superconductor varies periodically with the external magnetic field.

$$\varphi = \frac{nhc}{2e}$$





5C

$$H_c = H_o[1 - \left(\frac{T}{T_c}\right)^2]$$
$$= 6.5x10^4\left(1 - \left(\frac{4}{7.2}\right)^2\right)$$
$$= 44850A/m$$

6A

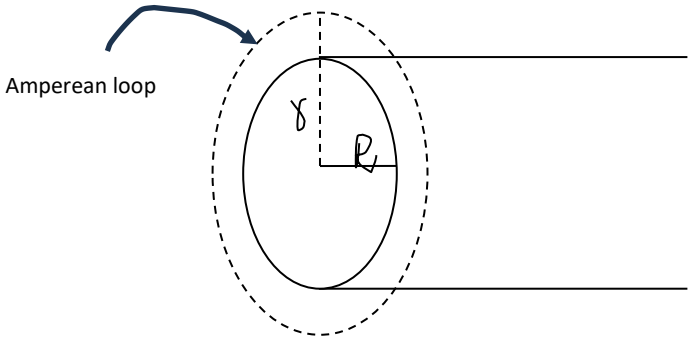
Critical Current (Silsbee effect)

It is the maximum current that a superconductor can carry beyond which superconductivity is lost.

Critical current $I_C = 2\Pi RH$

Magnetic field due to straight solid cylindrical conductor:

Case 1 : B at at a distance r > R



From Amperes law

$$\oint \vec{B}.\vec{dl} = \mu I$$
$$passing\ throgh\ amperean\ loop$$

$$B = \frac{\mu I}{2\pi r} \quad \because \oint dl = 2\pi r$$

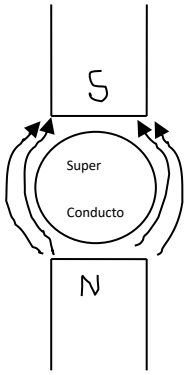
$$H = \frac{I_C}{2\pi r}$$

6B

MESISSENER EFFECT

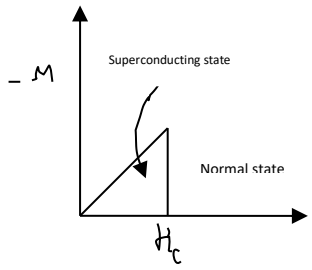
When a superconductor is placed in a magnetic field, it prevents entry of magnetic flux lines . This effect is known as **Meisseners effect.**

When a superconductor is kept in magnetic field, the surface current induced by the operation of Lenz's law. This generates opposite magnetic field and external field lines are prevented from entering the material. This effect is reversible. This is the principle of MAGLEV.



<u>Types of superconductors</u>

Type 1 Superconductors:



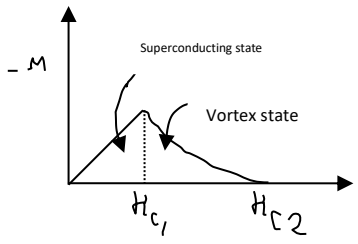
These are pure superconductors.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. At critical magnetic field there is a sharp transition to normal state due to the penetration of magnetic flux lines. The transition is sharp.

These possess low critical magnetic fields. Their critical temperatures also low.They are generally pure metals.

Ex: Al, Pb

Type 2 superconductor:



These are generally alloys.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. At lower critical magnetic field H_{C1}, the flux lines start penetrating .As the

magnetic field is increased, the super conductivity coexists with magnetic field and this phase is known as mixed state(vortex state). At higher critical magnetic field H_{C2}, the penetration is complete and the material transforms to normal state. They possess higher critical magnetic fields. Their critical temperatures are high.

Ex: Nb_3Ge, $YBa_2Cu_3O_7$

6C

Critical current $I = 2\Pi RH$

$I = 2\ x\ 3.14\ x\ 0.0005\ x\ 8000 = 25.12A$

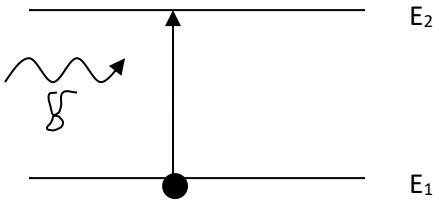
7A

Expression for energy density:

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N_1 and N_2 be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident.



$$\gamma = \frac{E_2 - E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_{γ}.

$$\text{Rate of absorption} = B_{12}\ N_1 U_{\gamma}$$

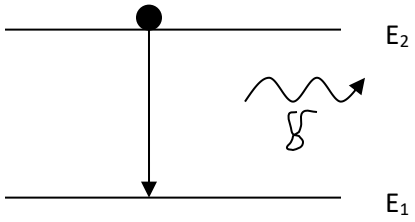
Here B_{12} is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

$$\text{Rate of spontaneous emission} = A_{21}\ N_2$$

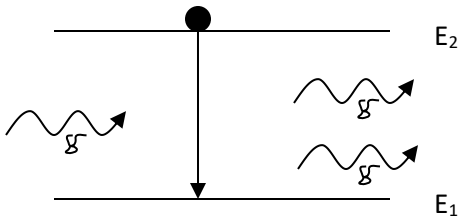
Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



$$\gamma = \frac{E_2 - E_1}{h}$$

Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_{γ}.

$$\text{Rate of stimulated emission} = B_{21}\ N_2 U_{\gamma}$$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12}\ N_1 U_{\gamma} = \quad A_{21}\ N_2 \quad + \quad B_{21}\ N_2\ U_{\gamma}$$

$$U_{\gamma} = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Rearranging this, we get

$$U_{\gamma} = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right]$$

From Boltzmans law , $\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\gamma}{kT}}-1}\right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h\gamma^3}{c^3}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}-1}\right]$$

Comparing these expressions, we get

$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3}$ and $\frac{B_{12}}{B_{21}} = 1$

$$\therefore U_\gamma = \frac{A}{B}\left[\frac{1}{e^{\frac{h\gamma}{kT}}-1}\right]$$

Conclusions

1.Rate of stimulated emission is directly proportional to wavelength
2. Rate of Induced absorption is equal to rate of Stimulated emission

7B

Superconducting Nanowire Single Photon Detector (SNSPD)

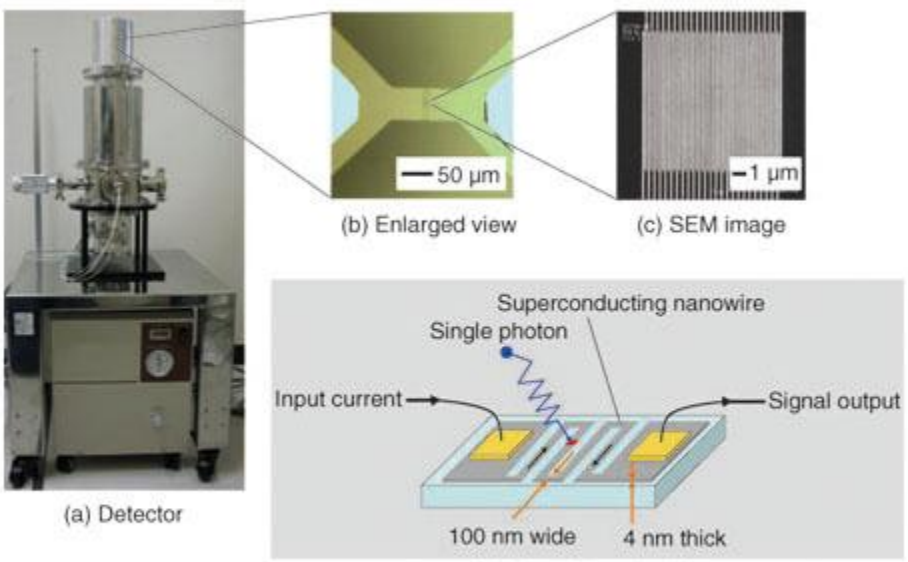
 An **ultra-sensitive photodetector** capable of detecting **individual photons**.

 Uses a **superconducting nanowire** cooled below its critical temperature (Tc).

 Based on the principle that photon absorption **locally disrupts superconductivity**, creating a measurable voltage pulse.

Principle

1. **Nanowire biased with a current** slightly below its critical current (Ic).
2. When a **single photon** is absorbed:
    - It deposits energy → creates a local "hotspot".
    - This hotspot temporarily drives a segment of the nanowire into a **normal (resistive) state**.
3. The resistive segment diverts current → produces a voltage pulse.
4. System resets as the hotspot cools, and superconductivity is restored.



Working:

The mechanism of single-photon detection in the device is illustrated in Fig. When electrical current flows in the superconducting nanowire, no voltage appears on the nanowire because the resistance is completely zero. When a single photon is absorbed at some point in the nanowire, the temperature at that point increases, superconductivity is locally destroyed, and the state at the point becomes normal (non-superconducting). If the nanowire is very narrow, the flowing current is completely protected by the normal state, and finite resistivity appears in the nanowire. Since the nanowire is being cooled by the cryocooler, its finite resistance rapidly disappears and it becomes superconducting again. These resistance changes create a voltage pulse, which is used to detect the arrival of a single photon at the superconducting nanowire. It may sound strange that only one photon can destroy superconductivity. However, while the superconducting energy gap of the superconductor is a few millielectron volts, the energy of a single photon at a wavelength of 1.5 µm is 0.83 eV, which is high enough to destroy superconductivity locally. From the above device operating principle, it is clear that the nanowire must be narrow enough to protect the superconducting current completely. This is achieved by using superconductor nanofabrication as fine as 100 nm and growth of an ultrathin superconducting film with a thickness of 4 nm.

7C

$$\alpha = \frac{10}{L} log_{10}\left(\frac{P_{in}}{P_{Out}}\right)$$

$$\left(\frac{P_{in}}{P_{out}}\right) = 10^{\frac{\alpha L}{10}}$$

CASE 1 L = 2 Km

$$\left(\frac{P_{in}}{P_{out}}\right) = 10^{\frac{\alpha L}{10}} = 2.75$$

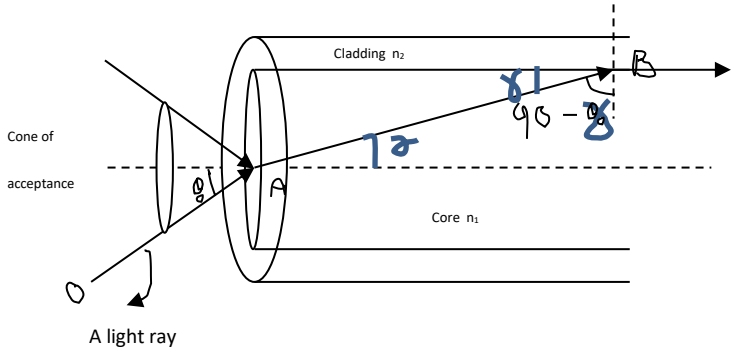
CASE 1 L = 6 Km

$$\left(\frac{P_{in}}{P_{out}}\right) = 10^{\frac{\alpha L}{10}} = 20.89$$

8A

<u>Numerical aperture:</u> Numerical aperture: It is defined as sine of angle of acceptance.

Acceptance angle: It is the maximum angle of Incidence within which if light enters undergoes total internal reflection.



<u>Expression for condition for propagation :</u>

Consider a light ray falling in to the optical fibre at an angle of incidence θ_0 equal to acceptance angle. Let n_0 be the refractive index of the surrounding medium .

Let n_1 be the refractive index of the core.

Let n_2 be the refractive index of the cladding.

From Snell's Law:

For the ray OA $n_0 \sin\theta_0 = n_1 \sin r = n_1\left(\sqrt{1-\cos^2 r}\right)$ ..... (1)

For the ray AB
$$n_1 \sin(90-r) = n_2 \sin 90$$
$$n_1 \cos r = n_2$$
$$\cos r = \frac{n_2}{n_1}$$

[here the angle of incidence is (90 - θ_1) for which angle of refraction is 90^0].

Substituting for cosr in equation (1)

$$n_o Sin\vartheta_0 = \sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$Sin\theta_o = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If the medium surrounding the fiber is air then $n_0 = 1$,

Numerical aperture $Sin\theta_o = \sqrt{n_1^2 - n_2^2}$

The total internal reflection will take place only if the angle of incidence $\theta_i < \theta_0$

$$\therefore \sin\theta_i < \sin\theta_0$$

$\sin\theta_i < \sqrt{n_1^2 - n_2^2}$

This is the condition for propagation.

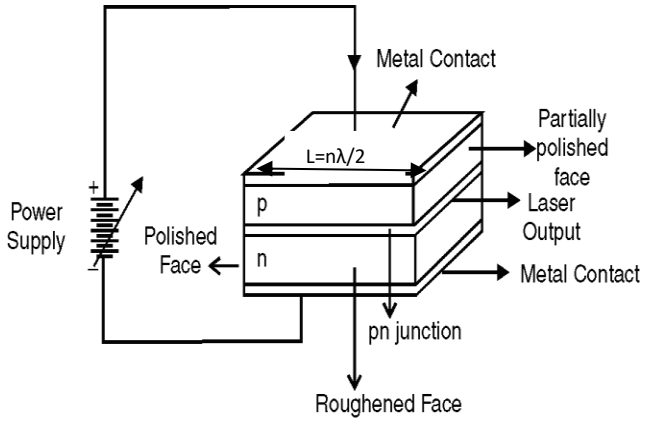
8B

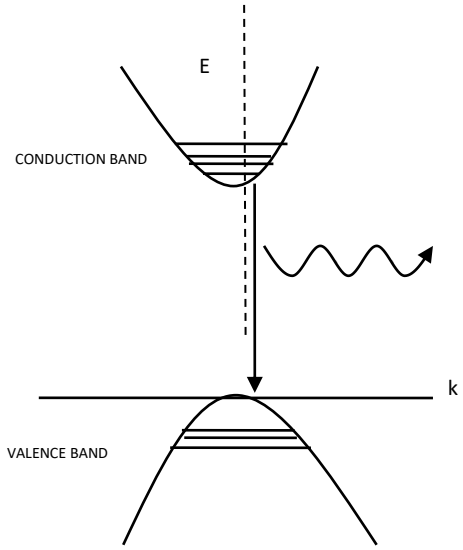
Gallium – Arsenide Semiconductor laser :

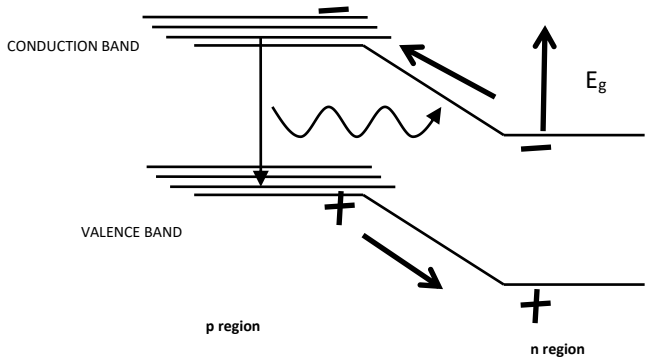
It is the only device which can be used for amplification in the infrared and optical ranges.

CONSTRUCTION

Gallium Arsenide is heavily doped with Tellurium (n side) and Zinc (P side) to a concentration of 10^{19} atoms /cm^3. Resonant cavity is formed by polishing the end faces of Junction diode. Amplification is possible if the population of the valence and conduction bands could be inverted as shown in the diagram.







WORKING

The first laser action was observed in a GaAs junction(8400Å) which is a direct gap semiconductor.

When a heavily doped junction is forward biased, electrons from n side are injected into p side causing population inversion. They combine with holes on the P side releasing photons. The junction region is the active region .The optical cavity is formed by the faces of the crystal itself which are taken on the cleavage plane perpendicular to junction and are then polished. The wavelength of the radiation depends on temperature. The wavelength of laser increases as the energy gap decreases. The frequency can be increased to the optical region by alloying with phosphor according to the relation $Ga(As)_{1-x}P_x$.

If E_g is the energy gap, then $E_g = eV_{forward} = \frac{hc}{\lambda}$

Ex: ZnS, CdS,InSb,PbS

Note : If the current density is less, the diode emits ordinary light. At sufficiently high densities, LASER is emitted.

Ex	Energy Gap
AlAs	2.16eV
AlP	2.45eV
GaN	3.36eV
InSb	0.17eV

8C

$$NA = n_o \sin\theta_A = \sqrt{n_{core}^2 - n_{clad}^2} = \sqrt{1.48^2 - 1.46^2} = 0.24$$

$$Angle\ of\ \ acceptance\ \ \theta_A = sin^{-1}\left[\sqrt{n_{core}^2 - n_{clad}^2}\right] = 13.98^0$$

9A

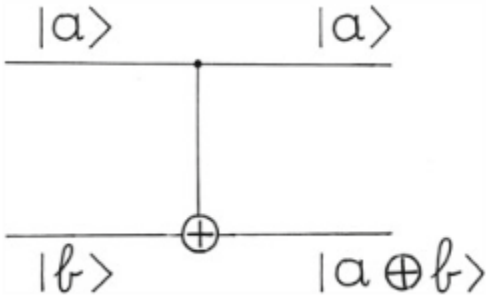
Moores Law: Gordon Moore, cofounder of Intel, indicated that - due to the shrinking size of transistors to the nano scale (allowing integrated circuits to be composed of more transistors, resulting in more powerful computer systems) - every year, twice as many transistors would be able to fit onto computer chips. Moore's Law states that the number of transistors on a microchip doubles every two years

Classical computer	Quantum computer
1.Data is stored 0's and 1's represented by capacitor / transistor/etc at LOW / HIGH voltage	1.Data is stored as 0's (lower energy state), 1's (upper energy state) and linear combination of these states occupied by photons/electrons/ atoms/ nuclei / ions
2.Processing performed through logic gates	2. superposition allows for exponentially many quantum states at once
3. A bit is either in 0 or 1 state	QUBIT can 0 or 1 or superposed state
4. Quantum entanglement not possible	3. Quantum entanglement applicable Two particles that are too far apart can be strongly correlated.
5.Classical computer conduct sequential operations	5. Quantum computers can do 2^n operations at a time
6 Classical gates are irreversible	6. Quantum gates are reversible
7. Number of BITS needed for memory is linear function of number of numbers	7 Number of QUBITS needed for memory is logarithmic function of number of numbers

9B

CONTROLLED NOT GATE

a and b are two inputs. a is called control qubit and b the target qubit. Target qubit flips if and only if a= 1. If a = 0, the second qubit remains unchanged.



INPUT CONTROL BIT	INPUT TARGET BIT	OUTPUT CONTROL BIT	OUTPUT TARGET BIT
X_1	X_o	Y_1	y_o
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Matrix $\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$

9C

$$\langle\psi|\phi\rangle \equiv [\alpha_1^* \quad \alpha_2^*]\begin{bmatrix}\beta_1 \\ \beta_2\end{bmatrix} = [\alpha_1^*\beta_1 + \alpha_2^*\beta_2]$$

$$\langle\varphi|\psi\rangle \quad =\equiv [\beta_1^* \quad \beta_2^*]\begin{bmatrix}\alpha_1 \\ \alpha_2\end{bmatrix} = [\alpha_1\beta_1^* + \alpha_1\beta_2^*]$$

$$\langle\varphi|\psi\rangle^* \quad =\equiv \begin{bmatrix}\alpha_1^*\beta_1 + \alpha_2^* & \beta_2\end{bmatrix} = \langle\psi|\varphi\rangle$$

10A

BIT : A light bulb can be either in the ON and OFF state and thus serves as a storage device for a single bit of information. O and 1 denote the value of a bit.

QUBIT: An atom / ion / photon / Nuclei/ etc in a lower energy level / upper energy level/ superposed state.

Properties of QUBIT

1. Superposed states are possible

A qubit in a superposed state is represented as

$$|\psi\rangle = c|0\rangle + d|1\rangle \equiv \begin{bmatrix} c \\ d \end{bmatrix}$$

2.

Transpose of a matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 3 & 0 & 4 \end{bmatrix}^T = \begin{bmatrix} 5 & 3 \\ -2 & 0 \\ 1 & 4 \end{bmatrix}$

3

Symmetric matrix : $A = A^T$

4

Identity matrix $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5

HERMITIAN (Self Adjoint)

$$\boldsymbol{A = (A^T)^*}$$

$A = \begin{bmatrix} a & b - ic \\ b + ic & d \end{bmatrix}$

$Transpose$

$\begin{bmatrix} a & b + ic \\ b - ic & d \end{bmatrix}$

$Complex \;\; conjugate$

$\begin{bmatrix} a & b - ic \\ b + ic & d \end{bmatrix}$

If $A = A^{+}$, then it is said to be self adjoint or HERMITIAN

6

Unitary Inverse matrix is equal to conjugate transpose

$AA^{+} = 1$

7. INNER PRODUCT: It is a scalar product and is a measure of overlap between the vectors.

Let

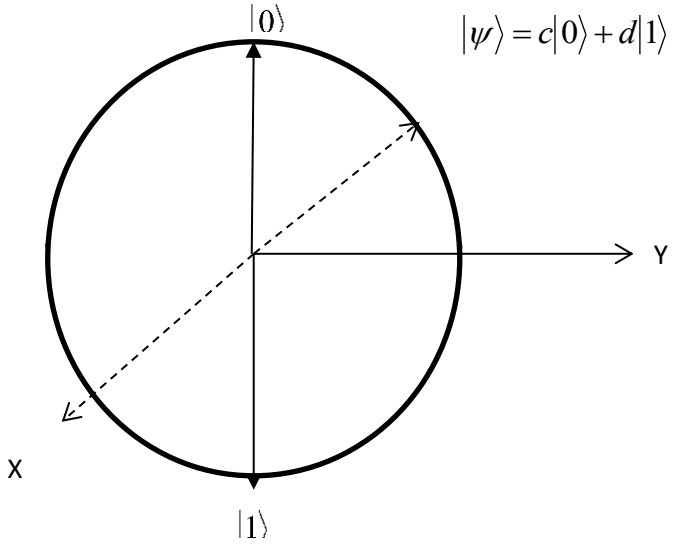
$$|\psi\rangle = c_1|\alpha\rangle + c_2|\beta\rangle$$
$$|\phi\rangle = d_1|\alpha\rangle + d_2|\beta\rangle$$

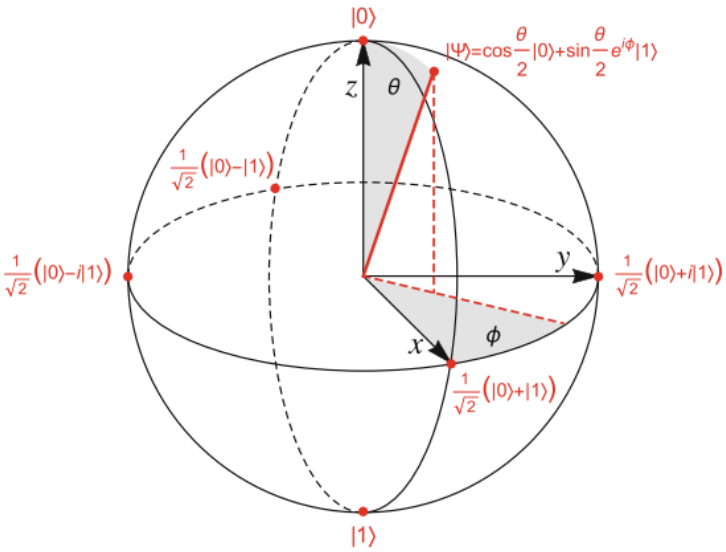
Inner product $\langle\psi|\varphi\rangle \equiv c_1^* d_1\langle\alpha|\alpha\rangle + c_1^* d_2\langle\alpha|\beta\rangle + c_2^* d_1\langle\beta|\alpha\rangle + c_2^* d_2\langle\beta|\beta\rangle$

$$\langle\alpha|\beta\rangle \equiv \langle\beta|\alpha\rangle = 0(Orthogonal \quad condition)$$
$$\langle\alpha|\alpha\rangle = \langle\beta|\beta\rangle = 1\ (\ Normalized\ condition)$$

BLOCK SPHERE

It represents a sphere with all the points on its surface correspond to state vectors in Hilbert space. The vector drawn to any point on the surface from the centre represents a state. In the diagram, $|\psi\rangle = c|0\rangle + d|1\rangle$ is a superposed state. $|0\rangle$ and $|1\rangle$are represented along + Z and – Z axes.





10B

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$
$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

σ_x is a classical not gate. When operated on a state vector say $|0\rangle$, it flips to $|1\rangle$

$$\sigma_x|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + 1x0 \\ 1x1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\sigma_x|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + 1x1 \\ 1x0 + 0x1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + (-ix0) \\ ix1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix}$$

$$\sigma_y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + (-ix1) \\ ix0 + 0x1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix}$$

$$\sigma_z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1x1 + 0x0 \\ 0x1 + -1x0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1x0 + 0x1 \\ 0x0 + -1x1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

10 C

To Show that $T^2 = S$

$$T^2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} \end{bmatrix} X \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} (1X1) + (0) & (1X0) + 0 \\ 0 + 0 & 0 + i \end{bmatrix} = S$$