CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐ **BCS/BAD/BAI/BDS301**

Third Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026

Mathematics for Computer Science

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

<table>
<tr><th colspan="3">Module – 1</th><th>M</th><th>L</th><th>C</th></tr>
<tr><td>Q.1</td><td>a.</td><td>A random variable X has the following probability function for various values of X.
<table>
<tr><td>X :</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P(x) :</td><td>0</td><td>K</td><td>2k</td><td>2k</td><td>3k</td><td>k^2</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr>
</table>
i) Find the value of k.
ii) Evaluate $P[x < 6]$, $P[0 < x < 5]$, $P[x \ge 6]$.</td><td>6</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>b.</td><td>Find the mean and standard deviation of Binomial distribution.</td><td>7</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>If the probability of a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals more than two will get a bad reaction.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><th colspan="6">OR</th></tr>
<tr><td>Q.2</td><td>a.</td><td>Find K such that
$$F(x) = \begin{cases} k e^{-x} &, \quad 0 < x < 1 \\ 0 &, \quad \text{otherwise} \end{cases}$$
Represents a valid pdf and hence find mean of the distribution.</td><td>6</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>b.</td><td>In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for
i) 10 minutes or more ii) less than 10 minutes.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>The marks of 1000 students in an examination follows a normal distribution with $\mu = 70$ and S.D = 5. Find the number of students whose marks will be
i) less than 65 ii) more than 75 iii) between 65 & 75.
Given $\phi(1) = 0.3413$.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><th colspan="6">Module – 2</th></tr>
<tr><td>Q.3</td><td>a.</td><td>The joint probability distribution of two random variables x and y is
<table>
<tr><td>x \ y</td><td>- 4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr>
<tr><td>5</td><td>$\frac{1}{4}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td></tr>
</table>
i) Find the marginal distribution of x and y.
ii) Obtain the covariance of x and y.</td><td>6</td><td>L2</td><td>CO2</td></tr>
</table>

	b.	Find the unique fixed probability vector of $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$	7	L2	CO3
	c.	A student study habit are as follows : If he studies one night , he is 70% sure of not studying next night , on the other hand if he does not study one night , he is 60% sure not to study the next night. In the long run how often does he study?	7	L3	CO3
		OR			
Q.4	**a.**	Define the following : i) Probability vector ii) Regular stochastic matrix iii) Absorbing state.	6	L2	CO3
	b.	If X and Y are two independent random variables with the following distribution. Find the joint probability distribution of X and Y and hence find the covariance. $\begin{array}{\|c\|c\|c\|} \hline x & 1 & 2 \\ \hline f(x) & 0.7 & 0.3 \\ \hline \end{array}$ $\begin{array}{\|c\|c\|c\|c\|} \hline y & -2 & 5 & 3 \\ \hline g(y) & 0.3 & 0.5 & 0.2 \\ \hline \end{array}$	7	L2	CO2
	c.	Three girls A, B, C are throwing the ball to each other. A always throws the ball to B, B always throws the ball to C. C is just as likely to throw the ball to B as to C. If C was the first person to throw the ball, find the probability that after 3 throws A, B, C has the ball.	7	L3	CO3
		Module – 3			
Q.5	**a.**	Explain the following terms : i) Null Hypothesis ii) Type 1 and 2 error iii) Test of significance	6	L1	CO5
	b.	A die is thrown 9000 times and throw of 3 or 4 was observed 3240 times. Do the data indicate that an unbiased dice at 5% level of significance $Z_{0.05} = 1.96$.	7	L3	CO4
	c.	A random sample for 1000 workers in company has mean wage of Rs 50 per day and S.D of Rs 15. Another sample of 1500 workers from another company has mean wage of Rs 45 per day and S.D of Rs 20. Does the mean rate of wages varies between two companies at 1% level of significance.	7	L3	CO4
		OR			
Q.6	**a.**	Certain tubes manufactured by a company have mean life time of 800 hours and S.D of 60 hrs. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time of : i) Between 790 hrs and 810 hrs ii) Less than 785 hrs iii) More than 820 hrs Given $\phi(0.67) = 0.2486$; $\phi(1) = 0.3413$; $\phi(1.33) = 0.4082$	6	L3	CO4

<table>
<tr><td></td><td>b.</td><td>It has been found from experience that the mean breaking strength of a particular brand of thread is 275.6 gms with standard deviation of 39.7 gms. Recently a sample of 36 pieces of thread showed a mean braking strength of 253.2 gms. Can one conclude at a significance level of 5% that the thread has become inferior?</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>In an elementary school examination of mean grade of 32 boys was 72 and S.D 8, while the mean grade of 36 girls was 75 and S.D 6. Test the hypothesis that the performance of girls is better than boys at 1% l.O.S.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>An unknown distribution has mean 635 and S.D 1.36 samples of size 36 are drawn from this population. Find the probability that the sample mean is between 634.76 and 635.24 given $\phi(1.06) = 0.3554$.</td><td>6</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find 95% C.I for mean of the maximum loads of all cables produced by the company.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>A certain stimulus administrated to each of the 12 patients resulted in the following change in blood pressure, 5, 2, 8, -1, 3, 0, 6 -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure given $t_{0.05} = 2.201$ for 11 d.o.f.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches ($t_{0.05} = 2.262$ for 9 d.o.f).</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>A sample of 11 rats from 9 population had an average blood viscosity of 3.92 with a S.D of 0.61. On the basis of the sample establish 95% C.I for the mean blood viscosity of the population. ($Z_{0.05} = 1.96$).</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>A die is thrown 264 times and the number appearing on the face (x) follows the following frequency distribution :
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>40</td><td>30</td><td>26</td><td>56</td><td>52</td><td>60</td></tr>
</table>
Calculate the value of χ^2 at 5% of level of significance.</td><td>7</td><td>L3</td><td>CO4</td></tr>
</table>

Module – 5

Q.9 a. A manufacturing company has purchased three new machines of different makes and wishes to determine whether one of them is faster than the others in producing a certain output. Five hourly production figures are observed at random from each other machine and the results are given below. Use Anova and determine whether the machines are significantly different in their mean speed ($F_{2,12} = 3.89$). **10 L3 CO5**

Observation	A_1	A_2	A_3
1	25	31	24
2	30	39	30
3	36	38	28
4	38	42	25
5	31	35	28

b. Set up on two way Anova analysis for the following two way design results. **10 L3 CO5**

Varieties of fertilizers	A	B	C
W	6	5	5
X	7	5	4
Y	3	3	3
Z	8	7	4

State whether variety differences are significant at 5% level given that $F_{2,6} = 5.14$ and $F_{3,6} = 4.76$.

OR

Q.10 **a.** Set up analysis of variance table for the following per acre production data for 3 varieties of wheat each grown on 4 plots and state if the variety differences are significant given $F_{2,9} = 4.26$. **10 L3 CO6**

Plot of land	Per acre production variety of wheat		
	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

b. Analyse and interpret the following statistics concerning output of wheat per field obtained as a result of experiment conducted to test four varieties of wheat A, B, C, D under Latin square design given $F_{3,6} = 4.76$. **10 L3 CO6**

C_{25}	B_{23}	A_{20}	D_{20}
A_{19}	D_{19}	C_{21}	B_{18}
B_{19}	A_{14}	D_{17}	C_{20}
D_{17}	C_{20}	B_{21}	A_{15}

* * * * *

[7] *A random variable X has the following probability function for various values of x.*

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k **(ii)** *Evaluate* $P(x<6)$, $P(x\geq 6)$ *and* $P(3<x\leq 6)$. **[June 2019]**
Also find the probability distribution and the distribution function of X.

☞ We must have, $P(x)\geq 0$ and $\sum P(x)=1$.

The first condition is satisfied for $k\geq 0$ and we have to find k such that

$\sum P(x)=1$.

ie., $0+k+2k+2k+3k+k^2+2k^2+(7k^2+k)=1$

ie., $10k^2 + 9k - 1 = 0$ or $(10k - 1)(k + 1) = 0$ or $k = 1/10$ and $k = -1$

If $k = -1$ the first condition fails and hence $k \neq -1$ $\therefore$ $\boxed{k = 1/10}$

Hence we have the following table.

x	0	1	2	3	4	5	6	7
$P(x)$	0	1/10	1/5	1/5	3/10	1/100	1/50	17/100

Now, $P(x < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

ie., $P(x<6) = 0 + 1/10 + 1/5 + + 1/5 + 3/10 + 1/100 = 81/100 = \boxed{0.81}$

$P(x \geq 6) = P(6) + P(7)$

ie., $P(x \geq 6) = 1/50 + 17/100 = 19/100 = \boxed{0.19}$

$P(3 < x \leq 6) = P(4) + P(5) + P(6)$

ie., $P(3 < x \leq 6) = 3/10 + 1/100 + 1/50 = 33/100 = \boxed{0.33}$

The probability distribution is as follows.

x	0	1	2	3	4	5	6	7
$P(x)$	**0**	**0.1**	**0.2**	**0.2**	**0.3**	**0.01**	**0.02**	**0.17**

The distribution function of X is $f(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$ is also called cumulative distribution function and the same is as follows.

x	0	1	2	3	4	5	6	7
$f(x)$	0	0 + 0.1 = 0.1	0.1 + 0.2 = 0.3	0.3 + 0.2 = 0.5	0.5 + 0.3 = 0.8	0.8 + 0.01 = 0.81	0.81 + 0.02 = 0.83	0.83 + 0.17 = 1

Henece $P(x)$

3.41 Mean and Standard Deviation of the Binomial Distribution

$$\text{Mean } (\mu) = \sum_{x=0}^{n} x\, P(x)$$

$$\mu = \sum_{x=0}^{n} x \cdot {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n \cdot (n-1)!}{(x-1)!(n-x)!} p \cdot p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]!}\, p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = np \sum_{x=1}^{n} {}^{(n-1)}C_{(x-1)}\; p^{x-1}\, q^{(n-1)-(x-1)}$$

$$\mu = np\,(q+p)^{n-1} = np$$

Mean (μ) = np

$$\text{Variance } (V) = \sum_{x=0}^{n} x^2 P(x) - \mu^2 \qquad \ldots (1)$$

$$\text{Now, } \sum_{x=0}^{n} x^2 P(x) = \sum_{x=0}^{n} [x(x-1)+x] P(x)$$

$$= \sum_{x=0}^{n} x(x-1) P(x) + \sum_{x=0}^{n} x P(x)$$

$$= \sum_{x=0}^{n} x(x-1)\; {}^{n}C_{x}\, p^{x} q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1)\, \frac{n!}{x!(n-x)!}\, p^{x} q^{n-x} + np$$

$$= \sum_{x=0}^{n} \frac{n\cdot(n-1)\cdot(n-2)!}{(x-2)!(n-x)!}\, p^{2} p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)![(n-2)-(x-2)]!}\, p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} {}^{(n-2)}C_{(x-2)}\; p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 (q+p)^{n-2} + np$$

$$\therefore \quad \sum x^2 P(x) = n(n-1)p^2 + np$$

Using this result in (1) along with $\mu = np$ we have,

$$\text{Variance } (V) = \{n(n-1)p^2 + np\} - (np)^2$$

$$= n^2p^2 - np^2 + np - n^2p^2 = np(1-p) = npq$$

Hence, variance $(V) = npq$

$$S.D\ (\sigma) = \sqrt{V} = \sqrt{npq}$$

Thus we have for the Binomial Distribution,

Mean $(\mu) = np$ and $S.D\ (\sigma) = \sqrt{npq}$

4

[29] *If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction.*

[Dec 2017, June 18,19]

☞ As the probability of occurence (*bad reaction*) is very small, this follows Poisson distribution and we have,

$$P(x) = \frac{m^x e^{-m}}{x!}$$

Mean $= m = np = 2000 \times 0.001 = 2$

We have to find $P(x > 2)$

$$P(x > 2) = 1 - P(x \leq 2)$$

ie., $P(x > 2) = 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$

$$= 1 - e^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!}\right]$$

$$P(x > 2) = 1 - e^{-2}\,[1 + 2 + 2] = 1 - 5e^{-2} = \boxed{0.3233}$$

2(a)

$$F(x) = \begin{cases} K\,e^{-x} & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Sol

$$\int_0^1 F(x)\,dx = \int_0^1 K\,e^{-x}\,dx = 1.$$

$$K\left(\frac{e^{-x}}{-1}\right)_0^1 = 1$$

$$\Rightarrow K\left(\frac{e^{-1}}{-1} + e^0\right) = 1.$$

$$\cancel{=K}\cdot\left(1 - \frac{1}{e}\right)K = 1.$$

$$\left(\frac{e-1}{e}\right)K = 1.$$

$$K = \frac{e}{e-1}.$$

$$\text{Mean} = \int_0^1 x\,F(x)\,dx.$$

$$= \int_0^1 x\;K\,e^{-x}\,dx$$

$$= K\left[\left(x\,\frac{e^{-x}}{-1}\right)_0^1 - \int 1\,\frac{e^{-x}}{-1}\,dx\right]$$

$$= K\left[\cancel{x}\,\frac{e^{-1}}{-1} + 0 + \left(\frac{e^{-x}}{-1}\right)_0^1\right]$$

$$K\left[-\frac{1}{e} + \left(\frac{e^{-1}}{-1} + e^{0} \right) \right]$$

$$= K\left[-\frac{1}{e} - \frac{1}{e} + 1 \right]$$

$$K\left[-\frac{2}{e} + 1 \right]$$

$$= K\left[e \; \frac{-2+e}{e} \right]$$

$$= K\left[\frac{e-2}{e} \right]$$

$$= \frac{\cancel{e}}{e-1}\left(\frac{e-2}{\cancel{e}} \right) = \frac{e-2}{e-1}.$$

[43] *In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for :*

(i) *10 minutes or more* ***(ii)*** *less than 10 minutes* ***(iii)*** *between 10 and 12 minutes*

[June 2019]

☞ The p.d.f of the exponential distribution is given by

$$f(x) = \alpha e^{-\alpha x}, \; x > 0 \quad \text{and the mean} = 1/\alpha$$

By data, $1/\alpha = 5$ $\therefore$ $\alpha = 1/5$ and hence $f(x) = \frac{1}{5} e^{-x/5}$

(i) $$P(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} \cdot e^{-x/5}\, dx = -\left[e^{-x/5}\right]_{10}^{\infty}$$

$$\therefore \quad P(x \geq 10) = -(0 - e^{-2}) = e^{-2} = \boxed{0.1353}$$

(ii) $P(x < 10) = \int_0^{10} \frac{1}{5} \cdot e^{-x/5}\, dx = -\left[e^{-x/5}\right]_0^{10}$

$\therefore \quad P(x < 10) = -(e^{-2} - 1) = 1 - e^{-2} = \boxed{0.8647}$

(iii) $P(10 < x < 12) = \int_{10}^{12} \frac{1}{5} \cdot e^{-x/5}\, dx = -\left[e^{-x/5}\right]_{10}^{12}$

$\therefore \quad P(10 < x < 12) = -(e^{-12/5} - e^{-2}) = \boxed{0.0446}$

Illustrations on Normal Distrib

[47] *The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) 65 to 75* **[June 2019]**

☞ Let x represent the marks of students.

By data, $\mu = 70, \sigma = 5$. Hence, s.n.v $z = \dfrac{x-\mu}{\sigma} = \dfrac{x-70}{5}$

Scanned with OKEN Scanner

(i) If $x = 65$, $z = -1$ and we have to find $P(z<-1)$

$$P(z<-1) = P(z>1)$$

$$= P(z \geq 0) - P(0 \leq z \leq 1)$$

$$= 0.5 - \phi(1) = 0.5 - 0.3413 = 0.1587$$

$\therefore$ **number of students scoring less than 65 marks**

$$= 1000 \times 0.1587 = 158.7 \approx \boxed{159}$$

(ii) If $x = 75$, $z = 1$ and we have to find $P(z > 1)$

$$P(z > 1) = P(z \geq 0) - P(0 \leq z \leq 1)$$

$$= 0.5 - \phi(1) = 0.5 - 0.3413 = 0.1587$$

$\therefore$ **number of students scoring more than 75 marks**

$$= 1000 \times 0.1587 = 158.7 \approx \boxed{159}$$

(iii) We have to find $P(-1<z<1)$

$$P(-1<z<1) = 2P(0<z<1)$$

$$= 2\phi(1) = 2(0.3413) = 0.6826$$

$\therefore$ **number of student scoring marks between 65 and 75**

$$= 1000 \times 0.6826 = 682.6 \approx \boxed{683}$$

Solutions to Q.3 and Q.4

Q.3(a)

Given Joint Probability Distribution

$X\backslash Y$	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(i) Marginal Distributions

Marginal of X

$$P(X=1) = \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{1}{2}$$
$$P(X=5) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$$

$$\boxed{P_X(1) = \frac{1}{2}, \quad P_X(5) = \frac{1}{2}}$$

Marginal of Y

$$P(Y=-4) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$
$$P(Y=2) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$
$$P(Y=7) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

$$\boxed{P_Y(-4) = \frac{3}{8},\ P_Y(2) = \frac{3}{8},\ P_Y(7) = \frac{1}{4}}$$

(ii) Covariance of X and Y

$$E(X) = 1 \cdot \frac{1}{2} + 5 \cdot \frac{1}{2} = 3$$

$$E(Y) = (-4)\frac{3}{8} + 2\frac{3}{8} + 7\frac{1}{4} = -\frac{12}{8} + \frac{6}{8} + \frac{14}{8} = 1$$

$$E(XY) = \sum xyp(x,y) = \frac{3}{2}$$

$$\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y) = \frac{3}{2} - (3)(1) = -\frac{3}{2}$$

$$\boxed{\text{Cov}(X,Y) = -\frac{3}{2}}$$

Q.3(b)

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

Let the stationary vector be

$$\mathbf{x} = (x_1, x_2, x_3)$$

such that $\mathbf{x}P = \mathbf{x}$ and

$$x_1 + x_2 + x_3 = 1$$

From $\mathbf{x}P = \mathbf{x}$,

$$x_1 = \frac{1}{6}x_2$$

$$x_3 = \frac{1}{2}x_2$$

Using normalization:

$$\frac{1}{6}x_2 + x_2 + \frac{1}{2}x_2 = 1$$

$$\frac{5}{3}x_2 = 1$$

$$x_2 = \frac{3}{5}$$

$$x_1 = \frac{1}{10}, \quad x_3 = \frac{3}{10}$$

$$\boxed{\mathbf{x} = \left(\frac{1}{10}, \frac{3}{5}, \frac{3}{10}\right)}$$

Q.3(c)

States:

$S =$ Study, $\quad N =$ Not Study

Transition matrix:

$$P = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$$

Let stationary distribution be

$$\mathbf{x} = (x_1, x_2)$$

where

$$x_1 + x_2 = 1$$

From $\mathbf{x}P = \mathbf{x}$,

$$x_1 = 0.3x_1 + 0.4x_2$$

$$0.7x_1 = 0.4x_2$$

$$x_1 = \frac{4}{7}x_2$$

Using normalization:

$$\frac{4}{7}x_2 + x_2 = 1$$

$$\frac{11}{7}x_2 = 1$$

$$x_2 = \frac{7}{11}, \quad x_1 = \frac{4}{11}$$

$$\boxed{\text{Long run probability of study} = \frac{4}{11}}$$

Q.4(a)

(i) Probability Vector:

A vector whose components are non-negative and whose sum is equal to 1.

(ii) Regular Stochastic Matrix:

A stochastic matrix P is said to be regular if some power P^k has all positive entries.

(iii) Absorbing State:

A state i is absorbing if $p_{ii} = 1$.

Q.4(b)

Since X and Y are independent,

$$P(X = x, Y = y) = f(x)g(y)$$

Joint distribution:

	-2	5	3
1	0.21	0.35	0.14
2	0.09	0.15	0.06

$$E(X) = 1(0.7) + 2(0.3) = 1.3$$

$$E(Y) = (-2)(0.3) + 5(0.5) + 3(0.2) = 2.5$$

Since independent,

$$\mathrm{Cov}(X, Y) = 0$$

Q.4(c)

Transition matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Initial state vector:

$$(0, 0, 1)$$

After 3 throws:

$$(0, 0, 1)P^3 = \left(0, \frac{1}{4}, \frac{3}{4}\right)$$

$$\boxed{P(A) = 0,\ P(B) = \frac{1}{4},\ P(C) = \frac{3}{4}}$$

Module - 03

Q5 a.

i) **Null Hypothesis:** Null Hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true.

ii) **Type-I error:** Rejecting the null hypothesis when it is actually true. Also called as (False Positive).

Type-II error: Failing to reject the null hypothesis when it is actually false. Also called as False Negative.

iii) **Test of significance:** A method to determine if the observed data provide enough evidence to reject a null hypothesis.

Q5 b. Given $n = 9000$

$x = 3240$

Let us suppose that the die is unbiased. The probability of throwing 5 or 6 with one die is $p = \frac{1}{3}$

Now, since $p + q = 1 \Rightarrow q = 1 - p \Rightarrow q = 1 - \frac{1}{3} = \frac{2}{3}$

Expected number of 5 or 6 in throwing a die 9000 times, $\mu = np$

$$\Rightarrow \mu = 9000 \times \frac{1}{3} = 3000$$

S.D. (σ) of sampling $= \sqrt{npq} = \sqrt{9000 \times \frac{1}{3} \times \frac{2}{3}} = 44.72$

Hence, $z = \frac{x - \mu}{\sigma} = \frac{3240 - 3000}{44.72} = 5.36$

At 5% level of significance, $5.36 > 1.96$ ($Z_{0.05}$)

∴ the hypothesis has to be rejected at 5% level of significance and we conclude that the die is biased.

Q5 c. Given, $n_1 = 1000$, $\overline{x_1} = 50$, $\sigma_1 = 15$
$n_2 = 1500$, $\overline{x_2} = 45$, $\sigma_2 = 20$.

Ho: Suppose there is no change in the mean rate of wages between the two companies.

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma^2}{n_2}}} = 7.1307$$

Hence, we conclude

$|Z| = 7.1307 > 2.58(Z_{0.01})$ (two-tailed test)

Ho is rejected.

∴ The null hypothesis is rejected at 1% level of significance.

Q6 a. Given $\mu = 800$, $\sigma = 60$, $n = 16$

i) $P(790 < \bar{x} < 810)$

The standard error $= \frac{\sigma}{\sqrt{n}} = \frac{60}{\sqrt{16}} = 15$.

For 790, $Z_1 = \frac{790 - 800}{15} = -0.6667$

For 810, $Z_2 = \frac{810 - 800}{15} = 0.6667$

∴ $P(790 < \bar{x} < 810) = P(Z_1 < Z < Z_2) = P(Z < 0.6667) - P(Z < -0.6667)$

$= 0.7475 - 0.2525$

$= 0.4950.$

Hence, the probability is 0.4950 (or 49.5%)

ii) $P(\bar{x} < 785)$

For 785, $z = \frac{785-800}{15} = -1$

$\Rightarrow P(\bar{x} < 785) = P(Z<-1) = P(-\infty<Z<0) - P(-1<Z<0)$

$= P(0<Z<\infty) - P(0<Z<1)$

$= 0.5 - \phi(1)$

$= 0.5 - 0.3413$

$= 0.1587$ (15.87%)

Hence, the probability is 0.1587 (or 15.87%)

iii) $P(\bar{x} > 820) = P(Z > 1.3333)$

For 820, $z = \frac{820-800}{15} = \frac{20}{15} = 1.3333$

$P(Z > 1.33) = 1 - 0.9082$

$= 0.0918$

Hence, the probability is 0.0918 (or 9.18%)

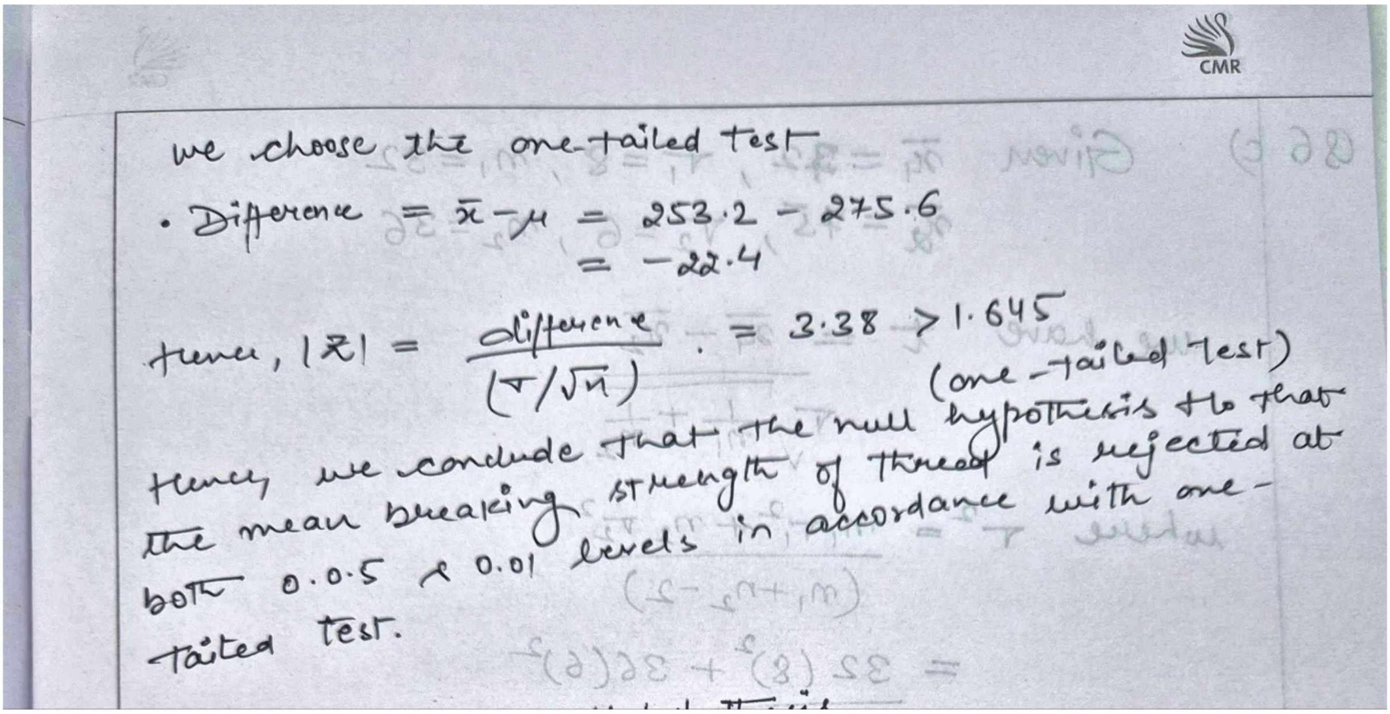
Q6 b. Given $\mu = 275.6$, $\sigma = 39.7$, $n = 36$,

$\bar{x} = 253.2$

Define the hypothesis,

$H_0: \mu = 275.6$, No change in mean breaking strengths

$H_1: \mu < 275.6$, inferior in breaking strength



we choose the one-tailed test

• Difference $= \bar{x} - \mu = 253.2 - 275.6$

$$= -22.4$$

Hence, $|z| = \dfrac{\text{difference}}{(\sigma/\sqrt{n})} = 3.38 > 1.645$

(one-tailed test)

Hence, we conclude that the null hypothesis H_0 that the mean breaking strength of thread is rejected at both 0.0.5 & 0.01 levels in accordance with one-tailed test.

Q6 c) Given $\overline{x_1} = 72$, $\sigma_1 = 8$, $n_1 = 32$

$\overline{x_2} = 75$, $\sigma_2 = 6$, $n_2 = 36$

we have $$t = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{(n_1+n_2-2)}$$

$$= \frac{32(8)^2 + 36(6)^2}{32+36-2}$$

$$= \frac{2048+1296}{66}$$

$$= 50.667$$

$$\Rightarrow \sigma = 7.118$$

$$\Rightarrow t = \left| \frac{72-75}{7.118\sqrt{\frac{1}{32}+\frac{1}{36}}} \right|$$

$$= 1.7345 < t_{0.01} = 2.390$$

Hence, there is no significant difference in the performance of boys and girls at 1% level of significance.

Module - 4

(Q7) (a) Given, $\mu = 635$, $\sigma = 1.36$, $n = 36$

we have to find $P(634.76 < \bar{x} < 635.24)$

By central limit theorem, we have

$$Z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

· If $\bar{x} = 634.76$, $Z_1 = \dfrac{634.76 - 635}{\left(\frac{1.36}{36}\right)} = -6.35$

· If $\bar{x} = 635.24$, $Z_1 = \dfrac{635.24 - 635}{\left(\frac{1.36}{36}\right)} = 6.35$

It is required to find.

$$P(-6.35 < Z < 6.35) = P(-6.35 < Z < 0) + P(0 < Z < 6.35)$$
$$= P(6.35 > Z > 0) + P(0 < Z < 6.35)$$
$$= P(0 < Z < 6.35) + P(0 < Z < 6.35)$$
$$= \phi(6.35) + \phi(6.35)$$
$$= 2\phi(6.35)$$
$$= 2(0.3554)$$
$$= 0.7108$$

Therefore, the probability that the sample mean is between 634.76 and 635.24 is approximately 0.7108

Q7

(b) By data $\overline{x} = 11.09$, $\sigma = 0.73$, $n = 60$

95% Confidence limits for the mean of maximum load are given by

$$\overline{x} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right) = 11.09 \pm 1.96\left(\frac{0.73}{\sqrt{60}}\right)$$

$$= 11.09 \pm 0.18$$

$$= 10.91 \text{ and } 11.27$$

Thus 10.91 tonnes to 11.27 tonnes are the 95% Confidence limits for the mean of maximum loads

Q7 (c)

Let $\bar{x} = \frac{\Sigma x}{n} = \frac{\{5+2+8-1+3+0+6-2+1+5+0+4\}}{12}$

$$\bar{x} = \frac{31}{12} = 2.5833$$

$$s^2 = \frac{1}{(n-1)} \Sigma (x-\bar{x})^2 = \frac{1}{(n-1)} \left\{ \Sigma x^2 - \frac{1}{n} (\Sigma x)^2 \right\}$$

$$s^2 = \frac{1}{11} \left\{ 185 - \frac{1}{12} (31)^2 \right\} = 9.538$$

$$\therefore S = \sqrt{9.538} = 3.088$$

$$t = \frac{\bar{x} - \mu}{s} (\sqrt{n})$$

Let us suppose that the stimulus administration is not accompanied with increas in blood pressure, we can take $\mu = 0$

$$\therefore t = \frac{2.5833 - 0}{3.088} (\sqrt{12}) = 2.8979 \approx 2.9 > 2.201$$

Hence the hypothesis is rejected at 5% level of significance.

2) Ten individuals are chosen at random from population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches.

=> given:- $n = 10, \mu = 66$

H_0 : There is no significance difference between the mean height.

$$\bar{x} = \frac{\sum_{i=1}^{10} x_i}{n} = \frac{1}{10}\left\{63+63+66+67+68+69+70+71+71 + 70\right\}$$

$$\bar{x} = \underline{67.8}$$

$$\sigma^2 = \frac{1}{(n-1)} \sum_{i=1}^{10} (x_i - \bar{x})^2$$

$$\sigma^2 = \frac{1}{89}\left\{23.04 + 23.04 + 3.24 + 0.64 + 0.04 + 1.44 + 4.84 + 10.24 + 10.24 + 4.84\right\}$$

$$\sigma^2 = 9.06$$

$$\sigma = \sqrt{9.06} = \underline{\underline{3.01}}$$

$$\therefore t = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{67.8 - 66}{\left(\frac{3.01}{\sqrt{10}}\right)} = \frac{1.8 \times \sqrt{10}}{3.01} = \underline{\underline{1.89}}$$

$df = n - 1 = 10 - 1 = 9$ //

$t_{0.01} = 3.250$ for d.f 9 (two-tailed)

Since $t = 1.89 < t_{0.01} = 3.250$ for d.f 9.

The hypothesis H_0 is accepted at 1% level of significance.

Q8 b

By data $\bar{x} = 3.92$, $\sigma = 0.61$, $n = 11$

95% Confidence limits for μ are

$$\bar{x} \pm t_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right)$$

i.e, $$= 3.92 \pm (2.228)\left(\frac{0.61}{\sqrt{11}}\right)$$

$$= 3.92 \pm 0.41$$

$$= 3.51 \text{ and } 4.33$$

Thus 95% Confidence limits for μ are 3.51 and 4.33.

Q8

ⓒ

Given, $N = 264$
$n = 6$

The frequencies in the given data are the observed frequencies. Assuming that the dice is unbiased, the expected number of frequencies for the numbers 1, 2, 3, 4, 5, 6 to appear on the face is $\frac{264}{6} = 44$ each

$$\therefore \chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(40-44)^2}{44} + \frac{(30-44)^2}{44} + \frac{(26-44)^2}{44} + \frac{(56-44)^2}{44}$$

$$+ \frac{(52-44)^2}{44} + \frac{(60-44)^2}{44}$$

$$= \frac{968}{44}$$

$$\boxed{\chi^2 = 22}$$

CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐

BCS/BAD/BAI/BDS301

Third Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026

Mathematics for Computer Science

Time: 3 hrs. Max Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

Module – 1

Q.1 a. A random variable X has the following probability function for various values of X. — M: 6, L: L2, C: CO1

X :	0	1	2	3	4	5	6	7
P(x) :	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

i) Find the value of k

ii) Evaluate $P[x < 6]$, $P[0 < x < 5]$, $P[x \geq 6]$.

b. Find the mean and standard deviation of Binomial distribution. — M: 7, L: L2, C: CO2

c. If the probability of a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals more than two will get a bad reaction. — M: 7, L: L3, C: CO2

OR

Q.2 a. Find K such that — M: 6, L: L2, C: CO1

$$F(x) = \begin{cases} k\,e^{-x} & , \quad 0 < x < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Represents a valid pdf and hence find mean of the distribution.

b. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for — M: 7, L: L3, C: CO2

i) 10 minutes or more ii) less than 10 minutes.

c. The marks of 1000 students in an examination follows a normal distribution with $\mu = 70$ and S.D = 5. Find the number of students whose marks will be — M: 7, L: L3, C: CO2

i) less than 65 ii) more than 75 iii) between 65 & 75.

Given $\phi(1) = 0.3413$.

Module – 2

Q.3 a. The joint probability distribution of two random variables x and y is — M: 6, L: L2, C: CO2

x \ y	- 4	2	7
1	1/8	1/4	1/8
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

i) Find the marginal distribution of x and y.

ii) Obtain the covariance of x and y.

	b.	Find the unique fixed probability vector of $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$	**7**	**L2**	**CO3**
	c.	A student study habit are as follows : If he studies one night , he is 70% sure of not studying next night , on the other hand if he does not study one night , he is 60% sure not to study the next night. In the long run how often does he study?	**7**	**L3**	**CO3**
		OR			
Q.4	**a.**	Define the following : i) Probability vector ii) Regular stochastic matrix iii) Absorbing state.	**6**	**L2**	**CO3**
	b.	If X and Y are two independent random variables with the following distribution. Find the joint probability distribution of X and Y and hence find the covariance. $\begin{array}{\|c\|c\|c\|}\hline x & 1 & 2 \\ \hline f(x) & 0.7 & 0.3 \\ \hline\end{array}$ $\begin{array}{\|c\|c\|c\|c\|}\hline y & -2 & 5 & 3 \\ \hline g(y) & 0.3 & 0.5 & 0.2 \\ \hline\end{array}$	**7**	**L2**	**CO2**
	c.	Three girls A, B, C are throwing the ball to each other. A always throws the ball to B, B always throws the ball to C. C is just as likely to throw the ball to B as to C. If C was the first person to throw the ball, find the probability that after 3 throws A, B, C has the ball.	**7**	**L3**	**CO3**
		Module – 3			
Q.5	**a.**	Explain the following terms : i) Null Hypothesis ii) Type 1 and 2 error iii) Test of significance	**6**	**L1**	**CO5**
	b.	A die is thrown 9000 times and throw of 3 or 4 was observed 3240 times. Do the data indicate that an unbiased dice at 5% level of significance $Z_{0.05} = 1.96$.	**7**	**L3**	**CO4**
	c.	A random sample for 1000 workers in company has mean wage of Rs 50 per day and S.D of Rs 15. Another sample of 1500 workers from another company has mean wage of Rs 45 per day and S.D of Rs 20. Does the mean rate of wages varies between two companies at 1% level of significance.	**7**	**L3**	**CO4**
		OR			
Q.6	**a.**	Certain tubes manufactured by a company have mean life time of 800 hours and S.D of 60 hrs. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time of : i) Between 790 hrs and 810 hrs ii) Less than 785 hrs iii) More than 820 hrs Given $\phi(0.67) = 0.2486$; $\phi(1) = 0.3413$; $\phi(1.33) = 0.4082$	**6**	**L3**	**CO4**

<table>
<tr><td></td><td>b.</td><td>It has been found from experience that the mean breaking strength of a particular brand of thread is 275.6 gms with standard deviation of 39.7 gms. Recently a sample of 36 pieces of thread showed a mean braking strength of 253.2 gms. Can one conclude at a significance level of 5% that the thread has become inferior?</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>In an elementary school examination of mean grade of 32 boys was 72 and S.D 8, while the mean grade of 36 girls was 75 and S.D 6. Test the hypothesis that the performance of girls is better than boys at 1% l.O.S.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>An unknown distribution has mean 635 and S.D 1.36 samples of size 36 are drawn from this population. Find the probability that the sample mean is between 634.76 and 635.24 given $\phi(1.06) = 0.3554$.</td><td>6</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find 95% C.I for mean of the maximum loads of all cables produced by the company.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>A certain stimulus administrated to each of the 12 patients resulted in the following change in blood pressure, 5, 2, 8, -1, 3, 0, 6 -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure given $t_{0.05} = 2.201$ for 11 d.o.f.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches.($t_{0.05} = 2.262$ for 9 d.o.f).</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>A sample of 11 rats from 9 population had an average blood viscosity of 3.92 with a S.D of 0.61. On the basis of the sample establish 95% C.I for the mean blood viscosity of the population. ($Z_{0.05} = 1.96$).</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>A die is thrown 264 times and the number appearing on the face (x) follows the following frequency distribution :<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>40</td><td>30</td><td>26</td><td>56</td><td>52</td><td>60</td></tr></table>Calculate the value of χ^2 at 5% of level of significance.</td><td>7</td><td>L3</td><td>CO4</td></tr>
</table>

Module – 5

Q.9 a. A manufacturing company has purchased three new machines of different makes and wishes to determine whether one of them is faster than the others in producing a certain output. Five hourly production figures are observed at random from each other machine and the results are given below. Use Anova and determine whether the machines are significantly different in their mean speed ($F_{2,12} = 3.89$).

10 L3 CO5

Observation	A_1	A_2	A_3
1	25	31	24
2	30	39	30
3	36	38	28
4	38	42	25
5	31	35	28

b. Set up on two way Anova analysis for the following two way design results.

10 L3 CO5

Varieties of fertilizers	A	B	C
W	6	5	5
X	7	5	4
Y	3	3	3
Z	8	7	4

State whether variety differences are significant at 5% level given that $F_{2,6} = 5.14$ and $F_{3,6} = 4.76$.

OR

Q.10 a. Set up analysis of variance table for the following per acre production data for 3 varieties of wheat each grown on 4 plots and state if the variety differences are significant given $F_{2,9} = 4.26$.

10 L3 CO6

Plot of land	Per acre production variety of wheat		
	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

b. Analyse and interpret the following statistics concerning output of wheat per field obtained as a result of experiment conducted to test four varieties of wheat A, B, C, D under Latin square design given $F_{3,6} = 4.76$.

10 L3 CO6

C_{25}	B_{23}	A_{20}	D_{20}
A_{19}	D_{19}	C_{21}	B_{18}
B_{19}	A_{14}	D_{17}	C_{20}
D_{17}	C_{20}	B_{21}	A_{15}

* * * * *

9(a)

Given that

	A_1	A_2	A_3
1	25	31	24
2	30	39	30
3	36	38	28
4	38	42	25
5	31	35	28

we apply coding method to the above ∴ By subtracting 30 from each of the $n = 15$ items, the above table reduces to

Variety / observation	A_1	A_2	A_3	Row totals
1	-5	1	-6	-10
2	0	9	0	9
3	6	8	-2	12
4	8	12	-5	15
5	1	5	-2	4
Column totals	$T_1 = 10$	$T_2 = 35$	$T_3 = -15$	30 = T = Total

$n_1 = n_2 = n_3 = 5$

G = Total number of items $= n = 15$

c = no of columns = 3

r = no of rows = 5

X_{ij} = item value

$$T = \sum \sum X_{ij} = 30$$

$$\sum X_{ij}^2 = \left\{ \begin{array}{l} 25 + 1 + 36 \\ + 0 + 81 + 0 \\ + 36 + 64 + 4 + 64 + 144 + 25 \\ + 1 + 25 + 4 \end{array} \right\} = 485, \; + 25 = 510$$

CF = Correction factor $= \frac{T^2}{n} = \frac{30^2}{15} = 60$

TSS = Total SS $= \Sigma X_{ij}^2 - \frac{T^2}{n}$

$= 510 - 60 = 450$

SSB = SS Between $= \Sigma \frac{T_J^2}{n_J} - CF$

$= \left(\frac{10^2}{5} + \frac{35^2}{5} + \frac{(-15)^2}{5} \right) - 60$

$= \frac{100 + 1225 + 225}{5} - 60$

$= \frac{1550}{5} - 60$

$= \frac{1250}{5} = 250$

SSW = SS within

$= TSS - SSB = 450 - 250 = 200$

ANOVA table for this problem

Source of variation	SS	df	MS	F-ratio	F-table value
Between Samples	250	$c-1$ $=3-1=2$	$MSB = \frac{250}{2} = 125$	F_{obs} $= \frac{MSB}{MSW}$ $= \frac{125}{16.7}$ $= 7.49$	F_{tab} $= F_{(2,12)}$ $= 3.89$
Within Samples	200	$c(r-1)$ $= 3\times4=12$	$MSW = \frac{200}{12} = 16.7$		

H_0 : Variety differences are not significant i.e. variety differences are insignificant

Condition for rejection of H_0 is

given by $F_{obs} > F_{tab}$

observe that $7.49 = F_{obs} > F_{tab} = 3.89$

$\therefore$ we reject H_0

$\therefore$ variety differences are significant

9(b)

No of columns = $c = 3$

No of rows = $r = 4$

n = No of total data items = $cr = 3 \times 4 = 12$

X_{ij} = value of the data item

$n_{.1} = n_{.2} = n_{.3} = 4$; $n_{1.} = n_{2.} = n_{3.} = n_{4.} = 3$

Given that

		← J →			
		A	B	C	row totals
↑ i ↓	W	6	5	5	$16 = T_{1.}$
	X	7	5	4	$16 = T_{2.}$
	Y	3	3	3	$9 = T_{3.}$
	Z	8	7	4	$19 = T_{4.}$
	Column totals	$24 = T_{.1}$	$20 = T_{.2}$	$16 = T_{.3}$	T = total $= 60 = \sum X_{ij}$

$$\sum X_{ij}^2 = 6^2 + 5^2 + 5^2 + 7^2 + 5^2 + 5^2 + 3^2 + 3^2 + 3^2 + 8^2 + 7^2 + 4^2 = 332$$

correction factor = CF

$$= \frac{T^2}{n} = \frac{60^2}{12} = 300$$

SS total = SST = $\sum X_{ij}^2 - CF$

$$= 332 - 300 = 32$$

SS between columns = SSBC = $\sum \frac{T_{.J}^2}{n_{.J}} - CF$

$$= \left(\frac{24^2}{4} + \frac{20^2}{4} + \frac{16^2}{4}\right) - 300$$

$$= 144 + 100 + 64 - 300 = 8$$

SS between raws = SSBR = $\sum \frac{T_{i.}^2}{n_{i.}} - CF$

$$= \left(\frac{16^2}{3} + \frac{16^2}{3} + \frac{9^2}{3} + \frac{19^2}{3}\right) - 300$$

$$= 85.33 + 85.33 + 27 + 120.33 - 300$$

$$= 18$$

SS residual = SSE = SST − (SSBC + SSBR)

$$= 32 - (8 + 18) = 6$$

ANOVA Table

Source of variation	SS	df	MS	F ratio	$F_{table value}$
Between columns	SSBC $= 8$	$c-1=3-1$ $=2$	MSC $= \frac{8}{2}=4$	F_{obc} $\frac{MSC}{MSE}$ $=\frac{4}{1}=4$	F_{tabc} $=F(2,6)$ $=5.14$
Between rows	SSBR $=18$	$r-1=4-1$ $=3$	MSR $=\frac{18}{3}=6$	F_{obr} $=\frac{MSR}{MSE}$ $=\frac{6}{1}=6$	F_{tabr} $=F(3,6)$ $=4.76$
Residual	SSE $=6$	$(r-1)(c-1)$ $=3\times2=6$	MSE $=\frac{6}{6}=1$		

H_{oc} : column variety differences are insignificant

H_{or} : row variety differences are insignificant

Condition for rejection of H_{ok} is given by $F_{obk} > F_{tabk}$, $k=c,r$

It follows that ~~row~~ column variety differences are insignificant while ~~and~~ variety differences are significant.

10(a)

Given that

Variety / Plots of Land	A	B	C	
1	6	5	5	16
2	7	5	4	16
3	3	3	3	9
4	8	7	4	19
Column total	$T_1 = 24$	$T_2 = 20$	$T_3 = 16$	$T = 60$

$n_1 = n_2 = n_3 = 4$

cr = Total number of items $= n = 3 \times 4$
$= 12$

$c =$ no of columns $= 3$

$r =$ no of rows $= 4$

X_{ij} = item value

$T = \sum\sum X_{ij} = 60$

$$\sum x_{ij}^2 = 6^2 + 5^2 + 5^2 + 7^2 + 5^2 + 4^2 + 3^2 + 3^2 + 3^2 + 8^2 + 7^2 + 4^2$$

$= 332$

CF = correction factor $= \frac{T^2}{n}$

$= \frac{60^2}{12} = 300$

SS total $= SST = \sum X_{ij}^2 - CF$

$= 332 - 300 = 32$

SS Between $= SSB = \sum \frac{T_J^2}{n_J} - CF$

$= \left(\frac{24^2}{4} + \frac{20^2}{4} + \frac{16^2}{4} \right) - 300$

$= 144 + 100 + 64 - 300$

$= 8$

SS within $= SSW = SST - SSB$

$= 32 - 8 = 24$

ANOVA table for this problem

Source of variation	SS	df	MS	F-ratio	F-table value
Between samples	SSB $= 8$	$c-1$ $= 3-1$ $= 2$	MSB $= \frac{8}{2} = 4$	F_{obs} $\frac{MSB}{MSW}$ $= \frac{4}{2.67}$ $= 1.5$	F_{tab} $= F(2,9)$ $= 4.26$
Within samples	SSW $= 24$	$c(r-1)$ $= 3(4-1)$ $= 9$	MSW $= \frac{24}{9} = 2.67$		

H_0: variety differences are are insignificant

---

condition for rejection of H_0 is given by $F_{obs} > F_{tab}$

---

observe that $1.5 = F_{obs} < F_{tab} = 4.26$

$\therefore$ we fail to reject

$\therefore$ variety differences are insignificant

10(b).

Using the coding method, we subtract 20 from the figures given in each of the small squares and obtain the coded figures as under

		Columns				Row totals
		1	2	3	4	
Rows	1	C 5	B 3	A 0	D 0	8
	2	A -1	D -1	C 1	B -2	-2
	3	B -1	A -6	D -3	C 0	-10
	4	D -3	C 0	B 1	A -5	-7
Column totals		0	-4	-1	-7	T = -12

Squaring these coded figures in various columns and rows, we have columns

row

	1	2	3	4	Sum of Squares
1	C 25	B 9	A 0	D 0	34
2	A 1	D 1	C 1	B 4	7
3	B 1	A 36	D 9	C 0	46
4	D 9	C 0	B 1	A 25	35
Sum of Squares	36	46	11	29	~~X~~122 $=\sum X_{ij}^2$

$$CF = \text{correction factor} = \frac{T^2}{n}$$

$$= \frac{(-12)^2}{16} = 9$$

$$SST = \sum (X_{ij})^2 - CF = 122 - 9$$

$$= 113$$

$$SSBC = \sum \frac{T_J^2}{n_J} - CF$$

$$= \left(\frac{0^2}{4} + \frac{(-4)^2}{4} + \frac{(-1)^2}{4} + \frac{(-7)^2}{4}\right) - 9$$

$$= \frac{222}{4} - 9 = 46.5$$

$$SSBR = \sum \frac{T_i^2}{n_i} - CF$$

$$= \left(\frac{8^2}{4} + \frac{(-3)^2}{4} + \frac{(-10)^2}{4} + \frac{(-7)^2}{4}\right)$$

$$-9$$

$$= \frac{222}{4} - 9 = 46.5$$

For finding SS for variance between varieties, we would first rearrange the coded data in the following form

Varieties of Wheat	yield in different parts of field				Total (T)
	I	II	III	IV	
A	-1	-6	0	-5	-12
B	-1	3	1	-2	1
C	5	0	1	0	6
D	-3	-1	-3	0	-7

$$SSV = \sum \frac{T_v^2}{n_v} - CF$$

$$= \left(\frac{(-12)^2}{4} + \frac{1^2}{4} + \frac{6^2}{4} + \frac{(-7)^2}{4} \right) - 9$$

$$= \frac{230}{4} - 9 = 48.5$$

$$SSE = SST - (SSBC + SSBR + SSV)$$

$$= 113 - (7.5 + 46.5 + 48.5) = 10.5$$

df for columns $= c - 1 = 4 - 1 = 3$

df for rows $= r - 1 = 4 - 1 = 3$

df for varieties $= v - 1 = 4 - 1 = 3$

df for error $= (c-1)(c-2) = (4-1)(4-2)$
$= 6$

ANOVA table.

Sources of variation	SS	df	MS	F-ratio	F-table F_{tab}
columns	SSBC $=7.5$	3	MSC $=\frac{7.5}{3}=2.5$	$F_{obc}=\frac{MSC}{MSE}=1.43$	$F(3,6)=4.76$
rows	SSBR $=46.5$	3	MSR $=\frac{46.5}{3}=15.5$	$F_{obr}=\frac{MSR}{MSE}=8.85$	$F(3,6)=4.76$
varieties	SSV $=48.5$	3	MSV $=\frac{48.5}{3}=16.17$	$F_{obv}=\frac{MSV}{MSE}=9.24$	$F(3,6)=4.76$
residuals	SSE $=10.5$	6	MSE $=\frac{10.5}{6}=1.75$		

The above table shows that variance between rows and variance between varieties are significant but variance between columns is insignificant.

Note that

H_{0K}: K differences are insignificant

where K = columns, rows, varieties.

Condition for rejection for H_{0K} is

$F_{obK} > F_{tab}$.