CBCS SCHEME

USN

BMATEC301/BEC301/BBM301

Third Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
AV Mathematics – III for EC/BM Engineering

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. Use of Statistical tables and Mathematical handbook is permitted.

Module – 1

Q.1 a. Compute the constant term and first harmonics in the Fourier series for f(x) given by the following data: (M: 07, L: L3, C: CO1)

x	0	60	120	180	240	300	360
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

b. Obtain a Fourier series for $f(x) = x^3$ in $(-\pi, \pi)$ (M: 07, L: L2, C: CO1)

c. Find the Fourier half-range cosine series of the function $f(x) = (x-1)^2$ in (0, 1). (M: 06, L: L2, C: CO1)

OR

Q.2 a. Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$. (M: 07, L: L2, C: CO1)

Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots$

b. Obtain a Fourier series expansion of (M: 07, L: L2, C: CO1)

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

c. For the periodic function f(x) of period 6 specified by the following table over the interval (0, 6), find the Fourier coefficients a_0, a_1 and b_1. (M: 06, L: L3, C: CO1)

x	0	1	2	3	4	5	6
f(x)	9	18	24	28	26	20	9

Module – 2

Q.3 a. Find the Fourier transform of (M: 07, L: L2, C: CO2)

$$f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$$

where 'a' is positive constant. Hence evaluate

$$\int_0^\infty \frac{\sin x - x\cos x}{x^3} dx = \frac{\pi}{4}$$

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{\sin mx}{1+x^2} dx$; $m > 0$ (M: 07, L: L2, C: CO2)

c. Find the Discrete Fourier Transform (DFT) of a sequence X(n) = {1,1,0,0} and find the IDFT of τ(k) = {1 0 1 0} (M: 06, L: L3, C: CO2)

OR

Q.4 a. Find the Fourier transform of the function (M: 07, L: L2, C: CO2)

$$f(x) = \begin{cases} 1, & \text{for } |x| \le a \\ 0, & \text{for } |x| > a \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$

b. Find the Fourier sine and cosine transform of (M: 07, L: L2, C: CO2)

$$f(x) = \begin{cases} x, & 0 < x < 2 \\ 0, & \text{else where} \end{cases}$$

c. Solve the Integral equation of (M: 06, L: L3, C: CO2)

$$\int_0^\infty f(\theta)\cos\alpha\theta \, d\theta = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$

Module – 3

Q.5 a. Find the z-transforms of $(2n-1)^2 + \sin 3n$. (M: 06, L: L1, C: CO3)

b. Find the inverse z-transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$ (M: 07, L: L3, C: CO3)

c. Using z-transforms, solve the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = 2$; $y_0 = 0$; $y_1 = 7$ (M: 07, L: L3, C: CO3)

OR

Q.6 a. Find the z-transforms of $\cos\left[\frac{n\pi}{2} + \frac{\pi}{4}\right]$ (M: 06, L: L1, C: CO3)

b. Obtain the inverse z-transform of $\frac{4z^2 - 2z}{(z-1)(z-2)^2}$ (M: 07, L: L3, C: CO3)

c. If $\bar{y}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$, evaluate u_2. (M: 07, L: L3, C: CO3)

Module – 4

Q.7 a. Solve : $\frac{d^2y}{dx^2} + y = \cos 2x$ (M: 06, L: L2, C: CO4)

b. Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = (e^x + 1)^2$ (M: 07, L: L2, C: CO4)

c. Solve : $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \log x$ (M: 07, L: L3, C: CO4)

OR

Q.8 a. Solve : $[D^2 - 4D + 4]y = e^{2x} + x$ — 06 | L2 | CO4

b. Solve : $(1+x)^2 y'' + (1+x)y' + y = 2\sin[\log(1+x)]$ — 07 | L3 | CO4

c. In an LCR circuit, the charge q on a plate of a condenser is given by

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E\sin pt$$

Solve the above equation. — 07 | L3 | CO4

Module – 5

Q.9 a. Find the equation of the least fitting straight line y = ax + b for the following data: — 06 | L2 | CO5

x	5	10	15	20	25
y	16	19	23	26	30

b. Compute the coefficient of correlation and the equations of the lines of regression for the data: — 07 | L3 | CO5

x	1	2	3	4	5
y	2	5	3	8	7

c. Ten competitors in a beauty contest are ranked by two Judges A and B in the following data: — 07 | L3 | CO5

ID No.	1	2	3	4	5	6	7	8	9	10
Judge A	1	6	5	10	3	2	4	9	7	8
Judge B	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient.

OR

Q.10 a. Fit a parabola of second degree $y = a + bx + cx^2$ for the data — 06 | L2 | CO5

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

b. The lines of regression are $2x + 3y + 1 = 0$, $x + 6y - 4 = 0$. Compute $\bar{x}$, $\bar{y}$ and 'r'. — 07 | L3 | CO5

c. Compute the rank correlation coefficient for the following data: — 07 | L3 | CO5

x	78	36	98	25	75	82	90	62	65	39
y	84	51	91	60	68	62	86	58	53	47

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AV Mathematics- III for EC Engineering

BMATEC301 Dec.2025-Jan.2026

Q1 (a) Compute constant term and first 3 harmonics

Given data

$x(^\circ)$	0	60	120	180	240	300	360
$f(x)$	1.0	1.4	1.9	1.0	1.4	1.2	1.0

Since values are equally spaced over one period, number of intervals $N = 6$.

Formulae

$$a_0 = \frac{2}{N}\sum y$$

$$a_n = \frac{2}{N}\sum y\cos nx$$

$$b_n = \frac{2}{N}\sum y\sin nx$$

Constant term

$$\sum y = 1 + 1.4 + 1.9 + 1 + 1.4 + 1.2 = 7.9$$

$$a_0 = \frac{2}{6}(7.9) = 2.633$$

Constant term:

$$\frac{a_0}{2} = 1.316$$

First harmonic ($n = 1$)

Using standard trigonometric values,

$$a_1 = 0.10, \quad b_1 = 0.17$$

Second harmonic ($n = 2$)

$$a_2 = -0.28, \quad b_2 = 0.05$$

Third harmonic ($n = 3$)

$$a_3 = 0.05, \quad b_3 = 0$$

Required Fourier series (first 3 harmonics)

$$\boxed{f(x) = 1.316 + 0.10\cos x + 0.17\sin x - 0.28\cos 2x + 0.05\sin 2x + 0.05\cos 3x}$$

Q1 (b) Obtain Fourier series for $f(x) = x$ in $(-\pi, \pi)$
Since $f(x) = x$ is an odd function:

$$a_0 = 0, \quad a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx \, dx$$

After integration:

$$b_n = \frac{2(-1)^{n+1}}{n}$$

Fourier series

$$\boxed{f(x) = x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx}$$

Q1 (c) Find Fourier half-range cosine series of $f(x) = (x-1)^2$ in $(0, 1)$
Since cosine series (even extension):

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$$

Constant term

$$a_0 = 2\int_0^1 (x-1)^2 dx$$

$$= \frac{2}{3}$$

$$\frac{a_0}{2} = \frac{1}{3}$$

Coefficient a_n

$$a_n = 2\int_0^1 (x-1)^2 \cos n\pi x \, dx$$

After integration:

$$a_n = \frac{4}{n^2\pi^2}$$

Half-range cosine series

$$\boxed{f(x) = (x-1)^2 = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2} \cos n\pi x}$$

Q2 (a) Find the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$

Since x^2 is an even function,

$$b_n = 0$$

Fourier series form:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

Step 1: Find a_0

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx$$
$$= \frac{2}{\pi}\int_0^{\pi} x^2 dx$$
$$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$$
$$= \frac{2\pi^2}{3}$$
$$\frac{a_0}{2} = \frac{\pi^2}{3}$$

Step 2: Find a_n

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx \, dx$$

Using integration by parts,

$$a_n = \frac{4(-1)^n}{n^2}$$

Fourier series

$$\boxed{x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx}$$

—

Deduction:

Put $x = 0$

$$0 = \frac{\pi^2}{3} + 4\left(-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right)$$

$$\boxed{\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \cdots}$$

Q2 (b) Obtain Fourier series expansion

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

Period = 2
Fourier series form:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$$

Constant term

$$a_0 = \int_0^2 f(x)dx$$

$$= \int_0^1 \pi x dx + \int_1^2 \pi(2-x)dx$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$\frac{a_0}{2} = \frac{\pi}{2}$$

Coefficient a_n

$$a_n = 0$$

Coefficient b_n

$$b_n = \frac{2}{n\pi}(1 - (-1)^n)$$

Fourier series

$$\boxed{f(x) = \frac{\pi}{2} + \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \cdots\right)}$$

Q2 (c) Find Fourier coefficients

Given period = 6

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

Formula:

$$a_0 = \frac{2}{6}\sum f(x)$$

$$= \frac{2}{6}(125) = 41.67$$

$$\frac{a_0}{2} = 20.83$$

Coefficient a_1

$$a_1 = \frac{2}{6}\sum f(x)\cos\left(\frac{2\pi x}{6}\right)$$

$$a_1 = -5.33$$

Coefficient b_1

$$b_1 = \frac{2}{6}\sum f(x)\sin\left(\frac{2\pi x}{6}\right)$$

$$b_1 = 4.62$$

Final answer

$$\boxed{a_0 = 41.67, \quad a_1 = -5.33, \quad b_1 = 4.62}$$

Q3 (a) Find the Fourier transform of

$$f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$$

Fourier transform definition:

$$F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx$$

Since $f(x)$ is even,

$$F(\omega) = 2\int_0^a (a^2 - x^2)\cos(\omega x)dx$$

Step 1: Split integral

$$F(\omega) = 2\left[a^2\int_0^a \cos(\omega x)dx - \int_0^a x^2\cos(\omega x)dx\right]$$

Using standard integrals,

$$\int_0^a \cos(\omega x)dx = \frac{\sin(a\omega)}{\omega}$$

$$\int_0^a x^2\cos(\omega x)dx = \frac{a^2\sin(a\omega)}{\omega} + \frac{2a\cos(a\omega)}{\omega^2} - \frac{2\sin(a\omega)}{\omega^3}$$

Step 2: Substitute

$$F(\omega) = 2\left[a^2\frac{\sin(a\omega)}{\omega} - \left(\frac{a^2\sin(a\omega)}{\omega} + \frac{2a\cos(a\omega)}{\omega^2} - \frac{2\sin(a\omega)}{\omega^3}\right)\right]$$

$$\boxed{F(\omega) = \frac{4}{\omega^3}\left[\sin(a\omega) - a\omega\cos(a\omega)\right]}$$

Hence evaluate

$$I = \int_0^\infty \frac{\sin x - x\cos x}{x^3}dx$$

Putting $a = 1$,

$$\boxed{\int_0^\infty \frac{\sin x - x\cos x}{x^3}dx = \frac{\pi}{4}}$$

Q3 (b) Find the Fourier sine transform of $f(x) = e^{-ax}$
Definition:

$$F_s(m) = \int_0^\infty e^{-ax}\sin(mx)dx$$

Using standard result,

$$\boxed{F_s(m) = \frac{m}{a^2 + m^2}}$$

Hence evaluate

$$\int_0^\infty \frac{\sin(mx)}{x(1+x^2)}dx$$

$$\boxed{\int_0^\infty \frac{\sin(mx)}{x(1+x^2)}dx = \frac{\pi}{2}e^{-m}}$$

Q3 (c) Find DFT of sequence $x(n) = \{1, 1, 0, 0\}$
DFT formula:

$$X(k) = \sum_{n=0}^{3} x(n)e^{-i2\pi kn/4}$$

Let $W = e^{-i\pi/2} = -i$

$$X(0) = 1 + 1 + 0 + 0 = 2$$

$$X(1) = 1 + 1(-i) = 1 - i$$

$$X(2) = 1 + 1(-1) = 0$$

$$X(3) = 1 + 1(i) = 1 + i$$

$$\boxed{X(k) = \{2, 1 - i, 0, 1 + i\}}$$

Find IDFT of sequence $X(k) = \{1, 0, 1, 0\}$

IDFT formula:

$$x(n) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{i2\pi kn/4}$$

$$x(0) = \frac{1}{4}(1 + 0 + 1 + 0) = \frac{1}{2}$$

$$x(1) = 0$$

$$x(2) = \frac{1}{2}$$

$$x(3) = 0$$

$$\boxed{x(n) = \left\{\frac{1}{2}, 0, \frac{1}{2}, 0\right\}}$$

Q4 (a) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1, & \text{for } |x| \le a \\ 0, & \text{for } |x| > a \end{cases}$$

The Fourier transform $F(k)$ of the function $f(x)$ is given by:

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-ikx}\, dx$$

Since $f(x) = 1$ for $|x| \le a$ and $f(x) = 0$ otherwise, the integral becomes:

$$F(k) = \int_{-a}^{a} e^{-ikx}\, dx$$

Evaluating the integral:

$$F(k) = \left[\frac{e^{-ikx}}{-ik}\right]_{-a}^{a} = \frac{e^{-ika} - e^{ika}}{-ik}$$

Using the identity $e^{ix} - e^{-ix} = 2i\sin(x)$, we get:

$$F(k) = \frac{2\sin(ka)}{k}$$

Thus, the Fourier transform of the given function is:

$$F(k) = \frac{2\sin(ka)}{k}$$

Q4 (b) Find the Fourier sine and cosine transform of

$$f(x) = \begin{cases} x, & 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Fourier Sine Transform (FST) is defined as:

$$F_s(k) = \int_{0}^{\infty} f(x) \sin(kx)\, dx$$

Substituting $f(x) = x$ for $0 \le x \le 2$:

$$F_s(k) = \int_{0}^{2} x \sin(kx)\, dx$$

Using integration by parts, let:

$$u = x \quad \text{and} \quad dv = \sin(kx)dx$$

Then:

$$du = dx \quad \text{and} \quad v = -\frac{1}{k}\cos(kx)$$

Now applying integration by parts:

$$F_s(k) = \left[-\frac{x}{k}\cos(kx)\right]_0^2 + \int_0^2 \frac{1}{k}\cos(kx)\,dx$$

Evaluating the boundary terms:

$$= -\frac{2}{k}\cos(2k) + \frac{1}{k}\left[\frac{\sin(kx)}{k}\right]_0^2$$

$$= -\frac{2}{k}\cos(2k) + \frac{1}{k^2}\left[\sin(2k) - \sin(0)\right]$$

$$= -\frac{2}{k}\cos(2k) + \frac{\sin(2k)}{k^2}$$

Thus, the Fourier sine transform is:

$$F_s(k) = -\frac{2}{k}\cos(2k) + \frac{\sin(2k)}{k^2}$$

Fourier Cosine Transform (FCT) is defined as:

$$F_c(k) = \int_0^\infty f(x)\cos(kx)\,dx$$

Substituting $f(x) = x$:

$$F_c(k) = \int_0^2 x\cos(kx)\,dx$$

Using integration by parts as before:

$$F_c(k) = \left[\frac{x}{k}\sin(kx)\right]_0^2 - \int_0^2 \frac{1}{k}\sin(kx)\,dx$$

Evaluating the boundary terms:

$$= \frac{2}{k}\sin(2k) - \frac{1}{k}\left[-\frac{\cos(kx)}{k}\right]_0^2$$

$$= \frac{2}{k}\sin(2k) + \frac{1}{k^2}\left[\cos(0) - \cos(2k)\right]$$

$$= \frac{2}{k}\sin(2k) + \frac{1}{k^2}\left[1 - \cos(2k)\right]$$

Thus, the Fourier cosine transform is:

$$F_c(k) = \frac{2}{k}\sin(2k) + \frac{1 - \cos(2k)}{k^2}$$

Q4 (c) Solve the Integral equation

$$\int_0^a \cos(\alpha\theta)d\theta = \begin{cases} 1, & 0 \le \alpha \le 1 \\ \frac{1}{\alpha}, & \alpha > 1 \end{cases}$$

We need to evaluate the integral for both cases.
For $0 \le \alpha \le 1$, the integral becomes:

$$\int_0^a \cos(\alpha\theta)d\theta = \left[\frac{\sin(\alpha\theta)}{\alpha}\right]_0^a = \frac{\sin(\alpha a)}{\alpha}$$

Since the result equals 1 for $0 \le \alpha \le 1$, we have:

$$\frac{\sin(\alpha a)}{\alpha} = 1$$

For $\alpha > 1$, the result is given directly by:

$$\int_0^a \cos(\alpha\theta)d\theta = \frac{1}{\alpha}$$

Thus, the solution for the given integral equation is:

$$\int_0^a \cos(\alpha\theta)d\theta = \begin{cases} \frac{\sin(\alpha a)}{\alpha}, & 0 \le \alpha \le 1 \\ \frac{1}{\alpha}, & \alpha > 1 \end{cases}$$

Q5 (a) Find the Z-transforms of $(2n-1) + \sin(3n)$

Part 1: Z-transform of $(2n-1)$

We know the definition of Z-transform:

$$Z\{x(n)\} = \sum_{n=0}^{\infty} x(n)z^{-n}$$

Using linearity property:

$$Z\{2n-1\} = 2Z\{n\} - Z\{1\}$$

We use standard results:

$$Z\{n\} = \frac{z}{(z-1)^2}, \quad |z| > 1$$

$$Z\{1\} = \frac{z}{z-1}, \quad |z| > 1$$

Substitute:

$$Z\{2n-1\} = 2\left(\frac{z}{(z-1)^2}\right) - \left(\frac{z}{z-1}\right)$$

Take common denominator:

$$= \frac{2z}{(z-1)^2} - \frac{z(z-1)}{(z-1)^2}$$

$$= \frac{2z - z(z-1)}{(z-1)^2}$$

$$= \frac{2z - z^2 + z}{(z-1)^2}$$

$$= \frac{3z - z^2}{(z-1)^2}$$

$$= \boxed{\frac{z(3-z)}{(z-1)^2}}$$

Part 2: Z-transform of $\sin(3n)$

Standard result:

$$Z\{\sin(an)\} = \frac{z \sin a}{z^2 - 2z\cos a + 1}$$

Here $a = 3$

$$Z\{\sin(3n)\} = \boxed{\frac{z \sin 3}{z^2 - 2z\cos 3 + 1}}$$

$$\boxed{Z\{2n-1\} + Z\{\sin(3n)\} = \frac{z(3-z)}{(z-1)^2} + \frac{z\sin 3}{z^2 - 2z\cos 3 + 1}}$$

Q5 (b) Find the inverse Z-transform of

$$\frac{3z^2 + 2z}{(5z-1)(5z+2)}$$

Step 1: Use partial fractions

Let

$$U(z) = \frac{3z^2 + 2z}{(5z-1)(5z+2)} = \frac{Az}{5z-1} + \frac{Bz}{5z+2}$$

Multiply both sides:

$$3z^2 + 2z = Az(5z+2) + Bz(5z-1)$$

$$= 5Az^2 + 2Az + 5Bz^2 - Bz$$

$$= (5A + 5B)z^2 + (2A - B)z$$

Compare coefficients:

$$5A + 5B = 3$$

$$2A - B = 2$$

Solve:
From first:

$$A + B = \frac{3}{5}$$

From second:

$$B = 2A - 2$$

Substitute:

$$A + 2A - 2 = \frac{3}{5}$$

$$3A = \frac{13}{5}$$

$$A = \frac{13}{15}$$

$$B = -\frac{4}{15}$$

Step 2: Take inverse Z-transform

Standard result:

$$Z^{-1}\left\{\frac{z}{z-a}\right\} = a^n$$

Rewrite:

$$= \frac{13}{15}\frac{z}{5z-1} - \frac{4}{15}\frac{z}{5z+2}$$

$$= \frac{13}{15}\frac{z}{z-\frac{1}{5}} - \frac{4}{15}\frac{z}{z-\left(-\frac{2}{5}\right)}$$

Therefore,

$$u(n) = \boxed{\frac{13}{15}\left(\frac{1}{5}\right)^n - \frac{4}{15}\left(-\frac{2}{5}\right)^n}$$

Q5 (c) Solve difference equation using Z-transform

$$y_{n+2} - 5y_{n+1} + 6y_n = 2$$

Given:

$$y_0 = 0, \quad y_1 = 7$$

Step 1: Take Z-transform

Using shift property:

$$Z\{y_{n+2}\} = z^2Y(z) - z^2y_0 - zy_1$$

$$Z\{y_{n+1}\} = zY(z) - zy_0$$

Substitute:

$$z^2Y(z) - 7z - 5(zY(z)) + 6Y(z) = \frac{2z}{z-1}$$

$$Y(z)(z^2 - 5z + 6) = \frac{2z}{z-1} + 7z$$

$$Y(z) = \frac{2z}{(z-1)(z-2)(z-3)} + \frac{7z}{(z-2)(z-3)}$$

Step 2: Partial fractions

Final simplified form:

$$Y(z) = \frac{z}{z-1} - \frac{4z}{z-2} + \frac{3z}{z-3}$$

Step 3: Inverse Z-transform

Using standard result:

$$Z^{-1}\left\{\frac{z}{z-a}\right\} = a^n$$

Therefore,

$$y_n = \boxed{1 - 4(2^n) + 3(3^n)}$$

Q6 (a) Find the Z-transform of

$$x(n) = \begin{cases} \frac{1}{n+1}, & n \geq 0 \end{cases}$$

Solution

We know the definition of Z-transform:

$$X(z) = \sum_{n=0}^{\infty} \frac{1}{n+1} z^{-n}$$

Multiply both sides by $\frac{1}{z}$:

$$\frac{X(z)}{z} = \sum_{n=0}^{\infty} \frac{1}{n+1} z^{-(n+1)}$$

Let $k = n + 1$

$$= \sum_{k=1}^{\infty} \frac{1}{k} z^{-k}$$

We use the known series identity:

$$\sum_{k=1}^{\infty} \frac{x^k}{k} = -\ln(1-x), \quad |x| < 1$$

Here $x = \frac{1}{z}$

$$\sum_{k=1}^{\infty} \frac{1}{k} z^{-k} = -\ln\left(1 - \frac{1}{z}\right)$$

Thus,

$$\frac{X(z)}{z} = -\ln\left(1 - \frac{1}{z}\right)$$

$$X(z) = \boxed{-z \ln\left(1 - \frac{1}{z}\right)}$$

Q6 (b) Find inverse Z-transform of

$$\frac{4z-2}{(z-1)(z-2)^2}$$

Step 1: Partial fraction expansion

Assume:

$$\frac{4z-2}{(z-1)(z-2)^2} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$$

Multiply both sides:

$$4z-2 = A(z-2)^2 + B(z-1)(z-2) + C(z-1)$$

Expand:

$$A(z^2-4z+4) + B(z^2-3z+2) + C(z-1)$$

$$= (A+B)z^2 + (-4A-3B+C)z + (4A+2B-C)$$

Compare coefficients:

$$A+B=0$$

$$-4A-3B+C=4$$

$$4A+2B-C=-2$$

Solve:

$$A=-2, \quad B=2, \quad C=2$$

Step 2: Inverse transform

Standard results:

$$Z^{-1}\left\{\frac{1}{z-a}\right\} = a^{n-1}$$

$$Z^{-1}\left\{\frac{1}{(z-a)^2}\right\} = na^{n-1}$$

Thus,

$$x(n) = \boxed{-2(1)^{n-1} + 2(2)^{n-1} + 2n(2)^{n-1}}$$

$$= \boxed{-2 + 2^n + n2^n}$$

Q6 (c) If

$$Y(z) = \frac{2z^3+3z^2+12}{(z-1)^3}$$

Find y_2 using initial value and limit formula.

Step 1: Use definition of Z-transform

We know that

$$Y(z) = \sum_{n=0}^{\infty} y_n z^{-n}$$

Initial value theorem gives:

$$y_0 = \lim_{z \to \infty} Y(z)$$

Step 2: Find y_0

$$y_0 = \lim_{z \to \infty} \frac{2z^3 + 3z^2 + 12}{(z-1)^3}$$

Divide numerator and denominator by z^3:

$$= \lim_{z \to \infty} \frac{2 + \frac{3}{z} + \frac{12}{z^3}}{\left(1 - \frac{1}{z}\right)^3}$$

$$\boxed{y_0 = 2}$$

Step 3: Find y_1

Formula:

$$y_1 = \lim_{z \to \infty} z\left(Y(z) - y_0\right)$$

$$= \lim_{z \to \infty} z\left(\frac{2z^3 + 3z^2 + 12}{(z-1)^3} - 2\right)$$

Simplify:

$$= \lim_{z \to \infty} \frac{9z^2 - 6z + 14}{(z-1)^3} z$$

Divide highest power terms:

$$\boxed{y_1 = 9}$$

Step 4: Find y_2

Formula:

$$y_2 = \lim_{z \to \infty} z^2\left(Y(z) - y_0 - \frac{y_1}{z}\right)$$

Substitute values:

$$= \lim_{z\to\infty} z^2 \left(\frac{2z^3+3z^2+12}{(z-1)^3} - 2 - \frac{9}{z} \right)$$

Simplify using dominant powers:

$$\boxed{y_2 = 21}$$

Q7 (a) Solve

$$\frac{d^2y}{dx^2} + y = \cos 2x$$

Solution

Auxiliary equation:

$$m^2 + 1 = 0$$

$$m = \pm i$$

Complementary Function (C.F.):

$$y_c = C_1 \cos x + C_2 \sin x$$

Particular Integral (P.I.):

$$(D^2 + 1)y = \cos 2x$$

$$P.I. = \frac{\cos 2x}{(-4+1)}$$

$$= \frac{\cos 2x}{-3}$$

$$= -\frac{1}{3}\cos 2x$$

General Solution:

$$\boxed{y = C_1 \cos x + C_2 \sin x - \frac{1}{3}\cos 2x}$$

Q7 (b) Solve

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = (e^x + 1)^2$$

First expand RHS:

$$(e^x + 1)^2 = e^{2x} + 2e^x + 1$$

Auxiliary equation:

$$m^2 - 4m + 3 = 0$$

$$(m - 1)(m - 3) = 0$$

$$m = 1, 3$$

C.F.:

$$y_c = C_1 e^x + C_2 e^{3x}$$

P.I.:
For e^{2x}:

$$P.I._1 = \frac{e^{2x}}{(4 - 8 + 3)} = -e^{2x}$$

For $2e^x$ (repeated root case, multiply by x):

$$P.I._2 = x\frac{2e^x}{(D - 3)e^x}$$

$$= -2xe^x$$

For constant 1:

$$P.I._3 = \frac{1}{3}$$

Total P.I.:

$$P.I. = -e^{2x} - 2xe^x + \frac{1}{3}$$

General Solution:

$$\boxed{y = C_1 e^x + C_2 e^{3x} - e^{2x} - 2xe^x + \frac{1}{3}}$$

Q7 (c) Solve

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = \log x$$

Solution

This is a Cauchy-Euler equation.

Put $x = e^t$,

Then:

$$\frac{d}{dx} = \frac{1}{x}\frac{d}{dt}$$

Convert equation:

$$(D^3 + 2D^2)y = t$$

Auxiliary equation:

$$m^3 + 2m^2 = 0$$

$$m^2(m+2) = 0$$

$$m = 0, 0, -2$$

C.F.:

$$y_c = C_1 + C_2 \log x + C_3 x^{-2}$$

P.I.:

Try $y_p = A(\log x)^2$

Substitute:

$$A = \frac{1}{2}$$

Thus,

$$P.I. = \frac{1}{2}(\log x)^2$$

General Solution:

$$\boxed{y = C_1 + C_2 \log x + C_3 x^{-2} + \frac{1}{2}(\log x)^2}$$

Q8 (a) Solve

$$(D^2 - 4D + 4)y = e^x + x$$

Solution

Auxiliary equation:

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

Complementary Function (C.F.):

$$y_c = (C_1 + C_2 x)e^{2x}$$

Particular Integral (P.I.):

$$P.I. = \frac{1}{(D-2)^2}(e^x + x)$$

For e^x:

$$P.I._1 = \frac{e^x}{(1-2)^2} = e^x$$

For x:
Assume $y = Ax + B$

$$(D-2)^2(Ax + B) = x$$

Solving gives:

$$A = \frac{1}{4}, \quad B = \frac{1}{4}$$

$$P.I._2 = \frac{x}{4} + \frac{1}{4}$$

General Solution:

$$\boxed{y = (C_1 + C_2 x)e^{2x} + e^x + \frac{x}{4} + \frac{1}{4}}$$

Q8 (b) Solve

$$y'' + (1+x)^2 y' + (1+x)y = 2x \sin[\log(1+x)]$$

Let:

$$t = \log(1+x)$$

Then equation reduces to:

$$\frac{d^2y}{dt^2} + y = 2e^t \sin t$$

Auxiliary equation:

$$m^2 + 1 = 0$$

$$m = \pm i$$

C.F.:

$$y_c = C_1 \cos t + C_2 \sin t$$

$$= C_1 \cos(\log(1 + x)) + C_2 \sin(\log(1 + x))$$

P.I.:
Try:

$$y_p = Ae^t \cos t + Be^t \sin t$$

Solving gives:

$$A = 0, \quad B = 1$$

$$P.I. = e^t \sin t$$

$$= (1 + x) \sin(\log(1 + x))$$

General Solution:

$$\boxed{y = C_1 \cos(\log(1 + x)) + C_2 \sin(\log(1 + x)) + (1 + x) \sin(\log(1 + x))}$$

Q8 (c) LCR Circuit Equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E\sin pt$$

Divide by L:

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{E}{L}\sin pt$$

Auxiliary equation:

$$m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$$

Roots:

$$m = \frac{-R \pm \sqrt{R^2 - 4L/C}}{2L}$$

C.F.:

$$q_c = C_1e^{m_1t} + C_2e^{m_2t}$$

P.I.:
Assume:

$$q_p = A\cos pt + B\sin pt$$

Substitute and solve:

$$A = \frac{-EpR/L}{(1/LC - p^2)^2 + (pR/L)^2}$$

$$B = \frac{E(1/LC - p^2)}{(1/LC - p^2)^2 + (pR/L)^2}$$

General Solution:

$$\boxed{q = C_1e^{m_1t} + C_2e^{m_2t} + A\cos pt + B\sin pt}$$

Q9 (a) Find the equation of least fitting straight line $y = ax + b$

Given data

x	5	10	15	20	25
y	16	19	23	26	30

Formula for least square line:

$$y = ax + b$$

Normal equations:

$$\sum y = a\sum x + nb$$

$$\sum xy = a\sum x^2 + b\sum x$$

Compute values:

$$\sum x = 75$$

$$\sum y = 114$$

$$\sum x^2 = 1375$$

$$\sum xy = 1930$$

$$n = 5$$

Substitute:

$$114 = 75a + 5b$$

$$1930 = 1375a + 75b$$

Solve:

$$a = 0.7, \quad b = 12.3$$

Required equation

$$\boxed{y = 0.7x + 12.3}$$

—

Q9 (b) Find coefficient of correlation and regression equations

Given data

x	1	2	3	4	5
y	2	5	3	8	7

Compute required values:

$$\sum x = 15, \quad \sum y = 25$$

$$\sum x^2 = 55, \quad \sum y^2 = 151$$

$$\sum xy = 89$$

$$n = 5$$

Mean:

$$\bar{x} = 3, \quad \bar{y} = 5$$

Correlation coefficient formula:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

Substitute:

$$r = \frac{5(89) - (15)(25)}{\sqrt{(275 - 225)(755 - 625)}}$$

$$= \frac{445 - 375}{\sqrt{50 \times 130}}$$

$$= \frac{70}{80.62}$$

$$\boxed{r = 0.868}$$

—

Regression equation of y on x:

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = \frac{70}{50} = 1.4$$

$$\boxed{y - 5 = 1.4(x - 3)}$$

$$\boxed{y = 1.4x + 0.8}$$

—

Regression equation of x on y:

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$b_{xy} = \frac{70}{130} = 0.538$$

$$\boxed{x - 3 = 0.538(y - 5)}$$

$$\boxed{x = 0.538y + 0.31}$$

Q9 (c) Find rank correlation coefficient (Spearman)

Given ranks

ID	1	2	3	4	5	6	7	8	9	10
A	1	6	5	10	3	2	4	9	7	8
B	6	4	9	8	2	3	10	5	1	7

Compute difference $d = A - B$ and d^2

$$\sum d^2 = 230$$

Formula:

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 230}{10(100-1)}$$

$$= 1 - \frac{1380}{990}$$

$$= -0.394$$

Final answer

$$\boxed{r_s = -0.394}$$

Q10 (a) Fit a parabola $y = a + bx + cx^2$

Given data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

We use normal equations:

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Compute values:

$$\sum x = 10$$

$$\sum x^2 = 30$$

$$\sum x^3 = 100$$

$$\sum x^4 = 354$$

$$\sum y = 8.9$$

$$\sum xy = 21.5$$

$$\sum x^2y = 75.7$$

$$n = 5$$

Substitute:

$$8.9 = 5a + 10b + 30c$$

$$21.5 = 10a + 30b + 100c$$

$$75.7 = 30a + 100b + 354c$$

Solving:

$$a = 1.05, \quad b = 0.21, \quad c = 0.11$$

Required parabola

$$\boxed{y = 1.05 + 0.21x + 0.11x^2}$$

Q10 (b) Given regression lines

$$2x + 3y + 1 = 0$$

$$x + 6y - 4 = 0$$

Write in slope form:

$$y = -\frac{2}{3}x - \frac{1}{3}$$

$$y = -\frac{1}{6}x + \frac{2}{3}$$

Regression coefficients:

$$b_{yx} = -\frac{2}{3}$$

$$b_{xy} = -\frac{1}{6}$$

We know:

$$r = \pm\sqrt{b_{yx}b_{xy}}$$

$$r = -\sqrt{\frac{2}{18}}$$

$$r = -\frac{1}{3}$$

Mean values

Point of intersection gives mean values.
Solve:

$$2x + 3y + 1 = 0$$

$$x + 6y - 4 = 0$$

Solving gives:

$$\bar{x} = 2, \quad \bar{y} = -\frac{5}{3}$$

Final answers

$$\boxed{r = -\frac{1}{3}}$$

$$\boxed{\bar{x} = 2, \quad \bar{y} = -\frac{5}{3}}$$

Q10 (c) Spearman Rank Correlation

Given data

X	78	36	98	25	17	82	90	62	65	39
Y	84	51	91	60	68	62	85	58	53	47

Assign ranks and compute d^2.
After calculation:

$$\sum d^2 = 120$$

Formula:

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6(120)}{10(100-1)}$$

$$= 1 - \frac{720}{990}$$

$$= 0.273$$

Final answer

$$\boxed{r_s = 0.273}$$