

CBCS SCHEME

BEC701

USN

Seventh Semester B.E/B.Tech. Degree Examination, Dec.2025/Jan.2026 Microwave Engineering and Antenna Theory

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level, C: Course outcomes.

		Module - 1	M	L	C
1	a.	Explain clearly how GUNN diode is being a negative resistance device.	6	L2	CO1
	b.	A certain transmission line has the characteristics impedance of $(75 + j0.01)\Omega$ and is terminated in load impedance of $(70 + j50)\Omega$. Compute : i. The reflection coefficient ii. Transmission coefficient.	6	L3	CO1
	c.	Derive the equation of transmission line to find voltage and current on the line.	8	L3	CO1
OR					
2	a.	A transmission line is terminated in a resistive load of 1000Ω and has $L = 9\mu\text{H/m}$ and $C = 100\text{ pF/m}$. Calculate reflection co-efficient and standing wave ratio.	6	L3	CO1
	b.	Define reflection coefficient. Derive an expression for reflection co-efficient at load in terms of characteristic impedance and load impedance.	8	L3	CO1
	c.	Explain Microwave System with relevant diagram.	6	L2	CO1
Module - 2					
3	a.	Deduce the relation between incident and reflected waves in terms of S-parameters for a two port network.	6	L3	CO2
	b.	Derive an expression for input reflection co-efficient for two port network with mismatched load.	10	L3	CO2
	c.	Write a note on different losses in microwave network.	4	L4	CO2
OR					
4	a.	Explain the following with necessary sketches : i. Flexible co-axial cable ii. Movable vane attenuator.	10	L2	CO2
	b.	Explain magic tee and write its S-Matrix representation.	10	L2	CO2
1 of 2					

				BEC701		
Module – 3						
5	a.	Explain parallel strip line with relevant diagram.	6	L2	CO3	
	b.	A lossless parallel strip line has a conducting strip of width 'W'. The substrate dielectric separating the 2 conducting strips has a relative dielectric constant ϵ_{rd} of 6 and a thickness 'd' of 4mm. Calculate : i. Value of W so that $Z_0 = 50\Omega$ ii. Strip line capacitance iii. Strip line inductance iv. Phase velocity.	10	L3	CO3	
	c.	Explain the following terms related to antenna systems : i. Directivity ii. Power Density.	4	L2	CO4	
OR						
6	a.	Explain antenna radiation pattern. Prove that maximum effective aperture of short electric dipole is $0.119\lambda^2$.	10	L3	CO4	
	b.	State and prove Frii's Transmission formula.	6	L2	CO4	
	c.	Explain the construction and field pattern of micro strip line.	4	L2	CO3	
Module – 4						
7	a.	Derive the expression for radiation resistance of short electric dipole antenna.	10	L3	CO4	
	b.	Obtain the expression for total electric field for array of n-point sources consider uniform linear array.	10	L2	CO4	
OR						
8	a.	Explain principle of pattern multiplication.	6	L2	CO4	
	b.	Write a note on Thin Linear Antenna.	8	L1	CO4	
	c.	A thin dipole antenna is $\lambda/10$ long. If its loss resistance is 2.5Ω . Find the radiation resistance and efficiency.	6	L3	CO4	
Module – 5						
9	a.	Explain different types of horn antenna with relevant diagrams.	10	L2	CO5	
	b.	The radius of a circular loop antenna is 0.02λ . How many turns of the antenna will give radiation resistance of 35Ω ?	6	L3	CO5	
	c.	Compare the far field components of small loop and short dipole antenna.	4	L1	CO5	
OR						
10	a.	Explain Yagi-Uda antenna and list its applications.	10	L2	CO5	
	b.	Explain Parabolic dish antenna or microwave dish antenna with relevant diagram.	10	L2	CO5	

1.a.

□ **High Field Domain or Gunn Mode of Oscillations** In GaAs (n or p type) and other multivalley compound semiconductors, a decrease in mobility in the upper valley in the conduction band with increase in electric field leads to decrease in drift velocity of electrons. This yields the formation of a high field domain for microwave generation. This will now be explained with the help of Fig. 10.22.

Let us assume that at point A in the device, there exists an excess $-ve$ charge due to random noise fluctuation on biasing or by non-uniformity in the doping as shown in Fig. 10.22 (a). Some electric field will be created by the accumulated charge. The field to the left (cathode side) of point A is lower than that to the right (anode side). If the diode is biased at the field point E_A on the $J-E$ curve, the carriers (or the current) flowing into point A from cathode are larger than those flowing out of A towards anode since the velocity on the left hand side of A is larger than that on the right-hand side of A . This will increase the excess $-ve$ charge at A . When the RF noise cycle reverses, the field in the left of A is lower than that before and the field to the right of A is greater than the original one, resulting in more space charge accumulation. This process continues until the low and high fields both reach values outside the differential $-ve$ resistance region of $J-E$ curve and settle at points a and c in Fig. 10.22(b), where the currents in the two field regions are equal.

Therefore, no further accumulation of charge at A occurs and a stable accumulation layer is formed. The analogous effect occurs in the event of a random deficiency of electrons at some point in the sample resulting in a depletion layer. When the enhancement (or the accumulation) and the depletion layers approach each other, they attract each other and together pass through the diode in the form of a domain.

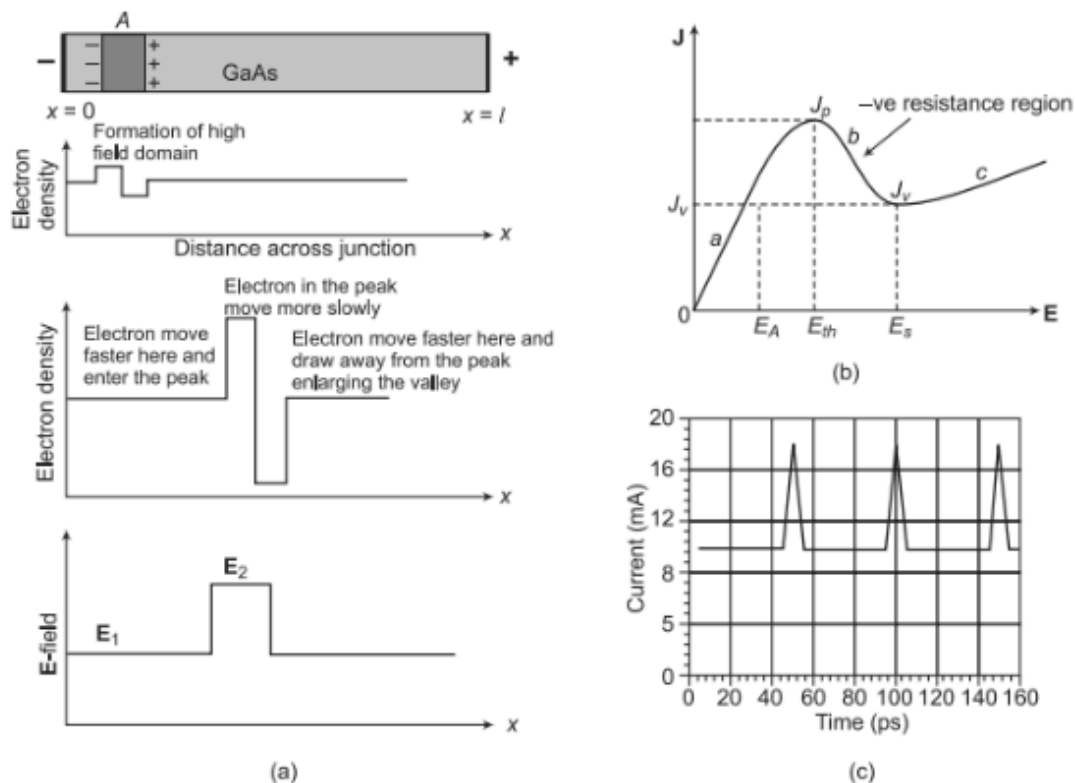


Fig. 10.22 (a) Formation of an electron accumulation layer in GaAs (b) $J-E$ characteristics (c) Gunn current vs time

1.b.

a. From Eq. (3-2-17) the reflection coefficient is

$$\begin{aligned}\Gamma &= \frac{\mathbf{Z}_\ell - \mathbf{Z}_0}{\mathbf{Z}_\ell + \mathbf{Z}_0} = \frac{70 + j50 - (75 + j0.01)}{70 + j50 + (75 + j0.01)} \\ &= \frac{50.24/95.71^\circ}{153.38/19.03^\circ} = 0.33/76.68^\circ = 0.08 + j0.32\end{aligned}$$

b. From Eq. (3-2-18) the transmission coefficient is

$$\begin{aligned}\mathbf{T} &= \frac{2\mathbf{Z}_\ell}{\mathbf{Z}_\ell + \mathbf{Z}_0} = \frac{2(70 + j50)}{70 + j50 + (75 + j0.01)} \\ &= \frac{172.05/35.54^\circ}{153.38/19.03^\circ} = 1.12/16.51^\circ = 1.08 + j0.32\end{aligned}$$

1. C.

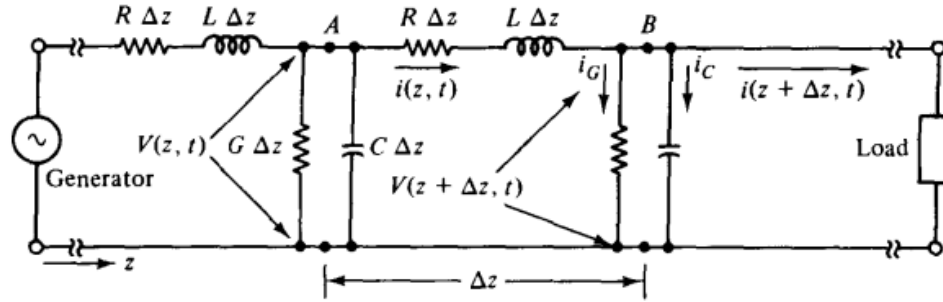


Figure 3-1-1 Elementary section of a transmission line.

By Kirchhoff's voltage law, the summation of the voltage drops around the central loop is given by

$$v(z, t) = i(z, t)R \Delta z + L \Delta z \frac{\partial i(z, t)}{\partial t} + v(z, t) + \frac{\partial v(z, t)}{\partial z} \Delta z \quad (3-1-1)$$

Rearranging this equation, dividing it by Δz , and then omitting the argument (z, t) , which is understood, we obtain

$$-\frac{\partial v}{\partial z} = Ri + L \frac{\partial i}{\partial t} \quad (3-1-2)$$

Using Kirchhoff's current law, the summation of the currents at point B in Fig. 3-1-1 can be expressed as

$$\begin{aligned} i(z, t) &= v(z + \Delta z, t)G \Delta z + C \Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} + i(z + \Delta z, t) \\ &= \left[v(z, t) + \frac{\partial v(z, t)}{\partial z} \Delta z \right] G \Delta z \\ &\quad + C \Delta z \frac{\partial}{\partial t} \left[v(z, t) + \frac{\partial v(z, t)}{\partial z} \Delta z \right] + i(z, t) + \frac{\partial i(z, t)}{\partial z} \Delta z \end{aligned} \quad (3-1-3)$$

By rearranging the preceding equation, dividing it by Δz , omitting (z, t) , and assuming Δz equal to zero, we have

$$-\frac{\partial i}{\partial z} = Gv + C \frac{\partial v}{\partial t} \quad (3-1-4)$$

Then by differentiating Eq. (3-1-2) with respect to z and Eq. (3-1-4) with respect to t and combining the results, the final transmission-line equation in voltage form is

found to be

$$\frac{\partial^2 v}{\partial z^2} = RGv + (RC + LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2} \quad (3-1-5)$$

Also, by differentiating Eq. (3-1-2) with respect to t and Eq. (3-1-4) with respect to z and combining the results, the final transmission-line equation in current form is

$$\frac{\partial^2 i}{\partial z^2} = RGi + (RC + LG)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2} \quad (3-1-6)$$

All these transmission-line equations are applicable to the general transient solution. The voltage and current on the line are the functions of both position z and time t . The instantaneous line voltage and current can be expressed as

$$v(z, t) = \text{Re } \mathbf{V}(z)e^{j\omega t} \quad (3-1-7)$$

$$i(z, t) = \text{Re } \mathbf{I}(z)e^{j\omega t} \quad (3-1-8)$$

where Re stands for “real part of.” The factors $\mathbf{V}(z)$ and $\mathbf{I}(z)$ are complex quantities of the sinusoidal functions of position z on the line and are known as *phasors*. The phasors give the magnitudes and phases of the sinusoidal function at each position of z , and they can be expressed as

$$\mathbf{V}(z) = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{\gamma z} \quad (3-1-9)$$

$$\mathbf{I}(z) = \mathbf{I}_+ e^{-\gamma z} + \mathbf{I}_- e^{\gamma z} \quad (3-1-10)$$

$$\gamma = \alpha + j\beta \quad (\text{propagation constant}) \quad (3-1-11)$$

where \mathbf{V}_+ and \mathbf{I}_+ indicate complex amplitudes in the positive z direction, \mathbf{V}_- and \mathbf{I}_- signify complex amplitudes in the negative z direction, α is the attenuation constant in nepers per unit length, and β is the phase constant in radians per unit length.

If we substitute $j\omega$ for $\partial/\partial t$ in Eqs. (3-1-2), (3-1-4), (3-1-5), and (3-1-6) and divide each equation by $e^{j\omega t}$, the transmission-line equations in phasor form of the frequency domain become

$$\frac{d\mathbf{V}}{dz} = -\mathbf{Z}\mathbf{I} \quad (3-1-12)$$

$$\frac{d\mathbf{I}}{dz} = -\mathbf{Y}\mathbf{V} \quad (3-1-13)$$

$$\frac{d^2\mathbf{V}}{dz^2} = \gamma^2\mathbf{V} \quad (3-1-14)$$

$$\frac{d^2\mathbf{I}}{dz^2} = \gamma^2\mathbf{I} \quad (3-1-15)$$

in which the following substitutions have been made:

$$\mathbf{Z} = R + j\omega L \quad (\text{ohms per unit length}) \quad (3-1-16)$$

$$\mathbf{Y} = G + j\omega C \quad (\text{mhos per unit length}) \quad (3-1-17)$$

$$\gamma = \sqrt{\mathbf{ZY}} = \alpha + j\beta \quad (\text{propagation constant}) \quad (3-1-18)$$

For a lossless line, $R = G = 0$, and the transmission-line equations are expressed as

$$\frac{d\mathbf{V}}{dz} = -j\omega L \mathbf{I} \quad (3-1-19)$$

$$\frac{d\mathbf{I}}{dz} = -j\omega C \mathbf{V} \quad (3-1-20)$$

$$\frac{d^2\mathbf{V}}{dz^2} = -\omega^2 LC \mathbf{V} \quad (3-1-21)$$

$$\frac{d^2\mathbf{I}}{dz^2} = -\omega^2 LC \mathbf{I} \quad (3-1-22)$$

It is interesting to note that Eqs. (3-1-14) and (3-1-15) for a transmission line are similar to equations of the electric and magnetic waves, respectively. The only difference is that the transmission-line equations are one-dimensional.

3-1-2 Solutions of Transmission-Line Equations

The one possible solution for Eq. (3-1-14) is

$$\mathbf{V} = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{\gamma z} = \mathbf{V}_+ e^{-\alpha z} e^{-j\beta z} + \mathbf{V}_- e^{\alpha z} e^{j\beta z} \quad (3-1-23)$$

The factors \mathbf{V}_+ and \mathbf{V}_- represents complex quantities. The term involving $e^{-j\beta z}$ shows a wave traveling in the positive z direction, and the term with the factor $e^{j\beta z}$ is a wave going in the negative z direction. The quantity βz is called the *electrical length of the line* and is measured in radians.

Similarly, the one possible solution for Eq. (3-1-15) is

$$\mathbf{I} = \mathbf{Y}_0(\mathbf{V}_+ e^{-\gamma z} - \mathbf{V}_- e^{\gamma z}) = \mathbf{Y}_0(\mathbf{V}_+ e^{-\alpha z} e^{-j\beta z} - \mathbf{V}_- e^{\alpha z} e^{j\beta z}) \quad (3-1-24)$$

In Eq. (3-1-24) the characteristic impedance of the line is defined as

$$\mathbf{Z}_0 = \frac{1}{\mathbf{Y}_0} \equiv \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 \pm jX_0 \quad (3-1-25)$$

The magnitude of both voltage and current waves on the line is shown in Fig. 3-1-2.

2.a.

$$Z_L = 1000 \Omega$$

$$L = 9 \times 10^{-6} \text{ H/m}$$

$$C = 100 \times 10^{-12} \text{ F/m}$$

1. $Z_0 =$ Characteristic impedance

$$= \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{9 \times 10^{-6}}{100 \times 10^{-12}}}$$

$$= 300$$

2. Reflection Coefficient (Γ)

$$= \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{1000 - 300}{1000 + 300} = 0.5384$$

3. Standing wave ratio (V)

$$= \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$= \frac{1 + 0.5384}{1 - 0.5384}$$

$$= 3.33$$

2.b.

3-2-1 Reflection Coefficient

In the analysis of the solutions of transmission-line equations in Section 3-1, the traveling wave along the line contains two components: one traveling in the positive z direction and the other traveling the negative z direction. If the load impedance is equal to the line characteristic impedance, however, the reflected traveling wave does not exist.

Figure 3-2-1 shows a transmission line terminated in an impedance Z_ℓ . It is usually more convenient to start solving the transmission-line problem from the receiving rather than the sending end, since the voltage-to-current relationship at the load point is fixed by the load impedance. The incident voltage and current waves traveling along the transmission line are given by

$$\mathbf{V} = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{+\gamma z} \quad (3-2-1)$$

$$\mathbf{I} = \mathbf{I}_+ e^{-\gamma z} + \mathbf{I}_- e^{+\gamma z} \quad (3-2-2)$$

in which the current wave can be expressed in terms of the voltage by

$$\mathbf{I} = \frac{\mathbf{V}_+}{Z_0} e^{-\gamma z} - \frac{\mathbf{V}_-}{Z_0} e^{+\gamma z} \quad (3-2-3)$$

If the line has a length of ℓ , the voltage and current at the receiving end become

$$\mathbf{V}_\ell = \mathbf{V}_+ e^{-\gamma \ell} + \mathbf{V}_- e^{+\gamma \ell} \quad (3-2-4)$$

$$\mathbf{V}_\ell = \mathbf{V}_+ e^{-\gamma \ell} + \mathbf{V}_- e^{+\gamma \ell} \quad (3-2-4)$$

$$\mathbf{I}_\ell = \frac{1}{Z_0} (\mathbf{V}_+ e^{-\gamma \ell} - \mathbf{V}_- e^{+\gamma \ell}) \quad (3-2-5)$$

The ratio of the voltage to the current at the receiving end is the load impedance. That is,

$$Z_\ell = \frac{\mathbf{V}_\ell}{\mathbf{I}_\ell} = Z_0 \frac{\mathbf{V}_+ e^{-\gamma \ell} + \mathbf{V}_- e^{+\gamma \ell}}{\mathbf{V}_+ e^{-\gamma \ell} - \mathbf{V}_- e^{+\gamma \ell}} \quad (3-2-6)$$

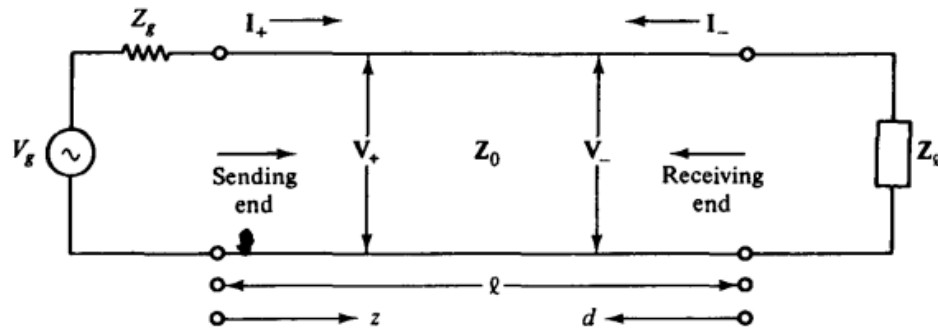


Figure 3-2-1 Transmission line terminated in a load impedance.

The reflection coefficient, which is designated by Γ (gamma), is defined as

Reflection coefficient $\equiv \frac{\text{reflected voltage or current}}{\text{incident voltage or current}}$

$$\Gamma \equiv \frac{V_{\text{ref}}}{V_{\text{inc}}} = \frac{-I_{\text{ref}}}{I_{\text{inc}}} \quad (3-2-7)$$

If Eq. (3-2-6) is solved for the ratio of the reflected voltage at the receiving end, which is $V_- e^{\gamma \ell}$, to the incident voltage at the receiving end, which is $V_+ e^{-\gamma \ell}$, the result is the reflection coefficient at the receiving end:

$$\Gamma_{\ell} = \frac{V_- e^{\gamma \ell}}{V_+ e^{-\gamma \ell}} = \frac{Z_{\ell} - Z_0}{Z_{\ell} + Z_0} \quad (3-2-8)$$

If the load impedance and/or the characteristic impedance are complex quantities, as is usually the case, the reflection coefficient is generally a complex quantity that can be expressed as

$$\Gamma_{\ell} = |\Gamma_{\ell}| e^{j\theta_{\ell}} \quad (3-2-9)$$

2.C.

MICROWAVE SYSTEMS

A microwave system normally consists of a transmitter subsystem, including a microwave oscillator, waveguides, and a transmitting antenna, and a receiver subsystem that includes a receiving antenna, transmission line or waveguide, a microwave amplifier, and a receiver. Figure 0-1 shows a typical microwave system.

In order to design a microwave system and conduct a proper test of it, an adequate knowledge of the components involved is essential. Besides microwave devices, the text therefore describes microwave components, such as resonators, cavities, microstrip lines, hybrids, and microwave integrated circuits.

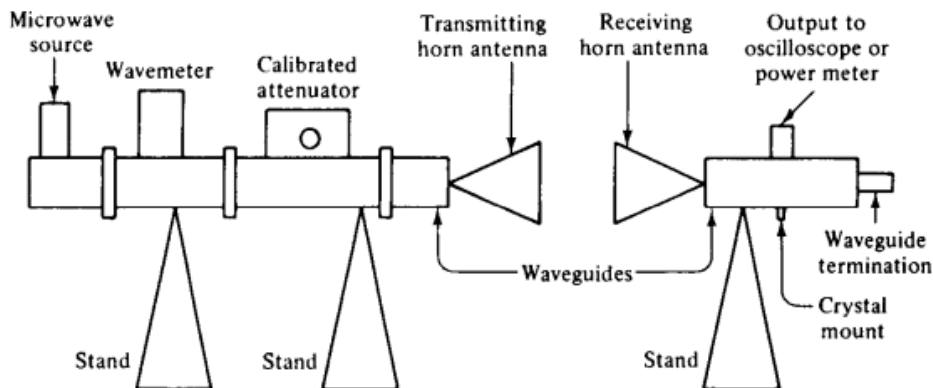


Figure 0-1 Microwave system.

3.a.

SCATTERING OR S-MATRIX REPRESENTATION OF MULTI-PORT NETWORK

6.3

As discussed in Section 6.1, the incident and reflected wave amplitudes of microwaves at any port are used to characterize a microwave circuit. The amplitudes are normalized in such a way that the square of any of these variables gives the average power in that wave in the following manner:

$$\text{Input power at the } n\text{th port, } P_{in} = 1/2 |a_n|^2 \quad (6.11)$$

$$\text{Reflected power at the } n\text{th port } P_{rn} = 1/2 |b_n|^2 \quad (6.12)$$

where a_n and b_n represent the normalized incident wave peak amplitude and normalized reflected wave peak amplitude at the n th port. The concept of scattering(s) parameters comes from the fact that rf and microwave circuit may contain some discontinuity or discontinuities in the signal propagation path. At an discontinuity the wave scattered in different directions as evanescent waves containing infinite number of higher order modes. These modes attenuated very fast after a short distance from the point of discontinuity within about a quarter wavelength. Then only the executed mode comes out from the different ports. All these emerging waves are considered as reflected waves at the corresponding parts. The waves entering the parts are considered the input or incident wave.

In a two-port network, we can express the normalized waves in terms of normalized voltages:

$$a_1 = \frac{V_1^+}{\sqrt{Z_0}} = \frac{V_1 - V_1^-}{\sqrt{Z_0}}, \quad a_2 = \frac{V_2^+}{\sqrt{Z_0}} = \frac{V_2 - V_2^-}{\sqrt{Z_0}} \quad (6.13)$$

$$b_1 = \frac{V_1^-}{\sqrt{Z_0}} = \frac{V_1 - V_1^+}{\sqrt{Z_0}}, \quad b_2 = \frac{V_2^-}{\sqrt{Z_0}} = \frac{V_2 - V_2^+}{\sqrt{Z_0}} \quad (6.14)$$

where a 's represent normalized incident wave amplitude and b 's represent normalized reflected wave amplitude at the corresponding ports. Here, the total voltage wave is the sum of incident and emergent voltage waves V^+ and V^- respectively:

$$V_1 = V_1^+ + V_1^- \quad I_1 = I_1^+ - I_1^- \quad (6.15)$$

$$V_2 = V_2^+ + V_2^- \quad I_2 = I_2^+ - I_2^- \quad (6.16)$$

The numeric suffices represent the port number. The total or net power flow into any port is given by r.m.s. value

$$P = P_i - P_r = \frac{1}{2} (|a|^2 - |b|^2) \quad (6.17)$$

Factor $\frac{1}{2}$ comes from the relation $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$ or $|a|$ and $|b|$ are magnitor of peak values.

Therefore, in this normalization process, the characteristic impedance is normalized to unity. For a two-port network (Fig. 6.1), the relation between incident and reflected waves are expressed in terms of scattering parameters S_{ij} 's:

$$b_1 = S_{11} a_1 + S_{12} a_2 \quad (6.18)$$

$$b_2 = S_{21} a_1 + S_{22} a_2 \quad (6.19)$$

The normalization process leads to a symmetrical scattering matrix for reciprocal structures. The physical significance of S -parameters can be described as follows:

$$S_{11} = (b_1/a_1)_{a_2=0} = \text{Reflection coefficient } \Gamma_1 \text{ at Port 1 when Port 2 is terminated with a matched load } (a_2 = 0)$$

$$S_{22} = (b_2/a_2)_{a_1=0} = \text{Reflection coefficient } \Gamma_2 \text{ at Port 2 when Port 1 is terminated with a matched load } (a_1 = 0)$$

$$S_{12} = (b_1/a_2)_{a_1=0} = \text{Attenuation of wave travelling from Port 2 to Port 1 when } a_1 = 0$$

$$S_{21} = (b_2/a_1)_{a_2=0} = \text{Attenuation of wave travelling from Port 1 to Port 2 when } a_2 = 0$$

In general, since the incident and reflected waves have both amplitude and phase, the S -parameters are complex numbers.

For multiport (N) networks or components, the S -parameter equations are expressed by

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1N} \\ S_{21} & S_{22} & \dots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \dots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \quad (6.20)$$

Figure 6.2 shows a linear multiport network having N ports. We can write for any arbitrary n -th port of this junction the following:

$$a_n = V_n^+ / \sqrt{Z_{0n}} = I_n^+ \sqrt{Z_{0n}} \quad (6.20a)$$

$$b_n = V_n^- / \sqrt{Z_{0n}} = I_n^- \sqrt{Z_{0n}} \quad (6.20b)$$

Net input power at port n

$$P_{ninp} = \frac{1}{2} (|a_n|^2 - |b_n|^2) \quad (6.20c)$$

Power dissipated in a termination at port n is

$$P_{ndissp} = \frac{1}{2} (|b_n|^2 - |a_n|^2) = \frac{1}{2} |b_n|^2 (1 - |\Gamma_n|^2) \quad (6.20d)$$

$$\Gamma_n = \frac{b_n}{a_n} \quad (6.20e)$$

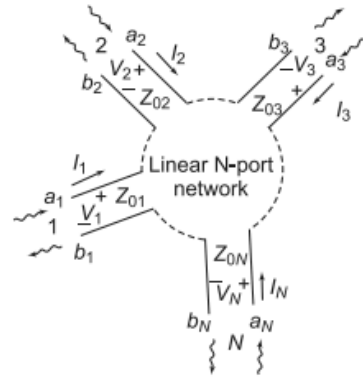


Fig. 6.2 A multiport (N) linear network

A two-port network or junction is formed when there is a discontinuity between the input and output ports of a transmission line. Many configurations of such junctions practically exist, some of which are shown in Fig. 6.5.

During propagation of a microwave through the junction from one port to another, evanescent modes are excited at each discontinuity which contains reactive energy. Evanescent modes decay very fast away from the junction and become negligible after a distance of the order of one wavelength. The terminal reference planes 1 and 2 are chosen beyond this distance so that the equivalent voltage and currents at these positions are proportional to the total transverse electric and magnetic fields, respectively, for the propagating mode only. These circuits are analyzed using *S*-matrix formulation.

Consider a two-port network of Fig. 6.5(c) terminated by normalized load and generator impedances $\frac{Z_L}{Z_0}$ and $\frac{Z_g}{Z_0}$. Then the load reflection coefficient

$$\Gamma_2 = \frac{a_2}{b_2} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} \quad (6.42)$$

Now, $b_1 = S_{11} a_1 + S_{12} a_2 = S_{11} a_1 + S_{12} b_2 \Gamma_2$ (6.43)

$$b_2 = S_{21} a_1 + S_{22} a_2 = S_{21} a_1 + S_{22} b_2 \Gamma_2 \quad (6.44)$$

Solving for the input reflection coefficient,

$$\Gamma_1 = b_1/a_1 = S_{11} + \frac{S_{12} S_{21} \Gamma_2}{1 - S_{22} \Gamma_2} \quad (6.45)$$

Therefore, for a mismatch load, input reflection coefficient $\Gamma_1 \neq S_{11}$. For a reciprocal network, $S_{12} = S_{21}$ so that

$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22} \Gamma_2} \quad (6.46)$$

Further, if the junction is lossless, from Eqs (6.39) and (6.40),

$$S_{11} S_{11}^* + S_{12} S_{12}^* = 1 \quad (6.47)$$

$$S_{22} S_{22}^* + S_{12} S_{12}^* = 1 \quad (6.48)$$

$$S_{11} S_{12}^* + S_{12} S_{22}^* = 0 \quad (6.49)$$

Therefore, for a lossless, reciprocal two-port network, terminated by a mismatch load, Eqs 6.47) and (6.48) give

$$|S_{11}| = |S_{22}| \quad (6.50)$$

From Eqs (6.49) and (6.50),

$$|S_{12}| = \sqrt{(1 - |S_{11}|^2)} \quad (6.51)$$

and the input reflection coefficient

$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22} \Gamma_2} \quad (6.52)$$

The last equation is the working equation for the computation of the *S*-parameters. By measuring Γ_1 for known values of Γ_2 (0, -1 and 1) a set of simultaneous equations are obtained which will give the *S*-parameters of a reciprocal junction.

Losses in a Microwave Network

Microwave networks suffer from various losses that reduce the transmitted signal power and degrade system performance. The major losses are:

- 1. Conductor Loss**
Occurs due to finite conductivity of metallic conductors. At microwave frequencies, current flows only near the surface because of the **skin effect**, increasing resistance and power dissipation.
- 2. Dielectric Loss**
Caused by imperfect insulating materials used as substrates or fillers. Polarization lag in dielectrics converts electromagnetic energy into heat, characterized by the **loss tangent ($\tan \delta$)**.
- 3. Radiation Loss**
Results when part of the guided electromagnetic energy leaks into free space due to discontinuities, bends, open structures, or poor shielding.
- 4. Mismatch (Reflection) Loss**
Occurs due to impedance mismatch between components, causing reflections and reducing the power delivered to the load. It is related to **return loss** and **VSWR**.

These losses collectively limit the efficiency and performance of microwave communication systems.

Coaxial Cables

6.4.1

A length of coaxial cable is used for interconnecting several microwave components. The theory of coaxial lines was described in Chapter 3. In this section some practical aspects of these lines are described. The outer conductor of the coaxial line is used to guide the signal through TEM mode and shields the external or internal signal leakage through it. The standard characteristic impedance of these cables are 50 ohms and 75 ohms. There are three basic types of coaxial cables with increasing order shielding, i.e., flexible, semi-rigid and rigid. Flexible coaxial cables use low-loss solid or foam polyethylene dielectrics. The outer single braid or double braid of the flexible cable is constructed for electromagnetic shielding by using knitted metal wire mesh as shown in Fig. 6.18. Rigid cables have air dielectric and conductors are supported by small dielectric spacers such that they do not produce significant discontinuities to the signal flow. Semi-rigid cables have solid dielectric and use a thin copper outer conductor so that it can be bent for convenient routing. Coaxial cables are used in the frequency range from dc to microwaves. Since the attenuation in these cables increases with frequency, the upper frequency of operation is limited.

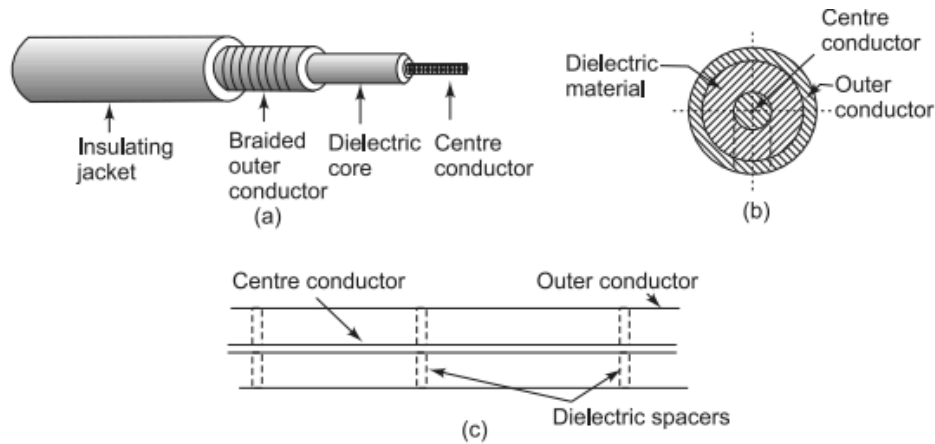


Fig. 6.18 Coaxial cable: (a) flexible (b) semi-rigid (c) rigid

Precision Variable Attenuator

A precision-type variable attenuator makes use of a circular waveguide section (C) containing a very thin tapered resistive card (R_2), to both sides of which are connected axisymmetric sections of circular-to-rectangular waveguide tapered transitions (RC_1 and RC_2) as shown in Fig. 6.40(a). The centre circular section with the resistive card can be precisely rotated by 360° with respect to the two fixed sections of circular to rectangular waveguide transitions. The induced current on the resistive card R_2 due to the incident signal is dissipated as heat and produces attenuation of the transmitted signal. The incident TE_{10} dominant wave in the rectangular waveguide is converted into a dominant TE_{11} mode in the circular waveguide. A very thin tapered resistive card is placed perpendicular to the \mathbf{E} field at the circular end of each transition section

so that it has a negligible effect on the field perpendicular to it but absorbs any component parallel to it. Therefore, a pure TE_{11} mode is excited in the middle section.

With reference to Fig. 6.40(b), if the resistive card in the centre section is kept at an angle θ relative to the \mathbf{E} field direction of the TE_{11} mode, the component $E \cos \theta$ parallel to the card gets absorbed while the component $E \sin \theta$ is transmitted without attenuation. This later component finally appears as electric field component $E \sin^2 \theta$ in the rectangular output guide. Therefore, the attenuation of the incident wave is

$$\alpha = \frac{E}{E \sin^2 \theta} = \frac{1}{\sin^2 \theta} = \frac{1}{|S_{21}|}$$

$$\text{or, } \alpha \text{ (dB)} = -40 \log (\sin \theta) = -20 \log |S_{21}| \quad (6.79)$$

Therefore, the precision rotary attenuator produces attenuation which depends only on the angle of rotation θ of the resistive card with respect to the incident wave polarization. Attenuators are normally matched reciprocal devices, so that

$$|S_{21}| = |S_{12}| \quad (6.80)$$

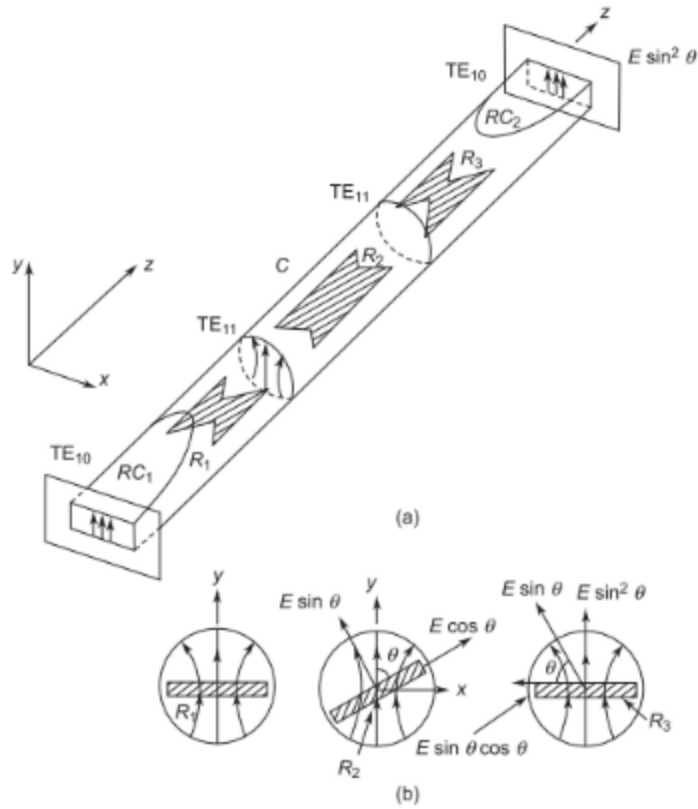


Fig. 6.40 Precision-type variable attenuator
 R_1, R_2, R_3 : Tapered resistive cards
 RC_1 and RC_2 : Rectangular-to-circular waveguide transitions
 C : Circular waveguide section

$$\text{and } |S_{11}| \text{ or } |S_{22}| = \frac{\text{VSWR}-1}{\text{VSWR}+1} \ll 0.1 \quad (6.81)$$

where the VSWR is measured at the input or output port concerned. The S -matrix of an ideal precision rotary attenuator is

$$[S] = \begin{bmatrix} 0 & \sin^2 \theta \\ \sin^2 \theta & 0 \end{bmatrix} \quad (6.82)$$

4.b.

Hybrid or Magic-T

A hybrid tee is formed with the combination of the E-plane and H-plane tees and is called a *magic-T*. It has four ports as shown in Fig. 6.49(a) and 6.49(b).

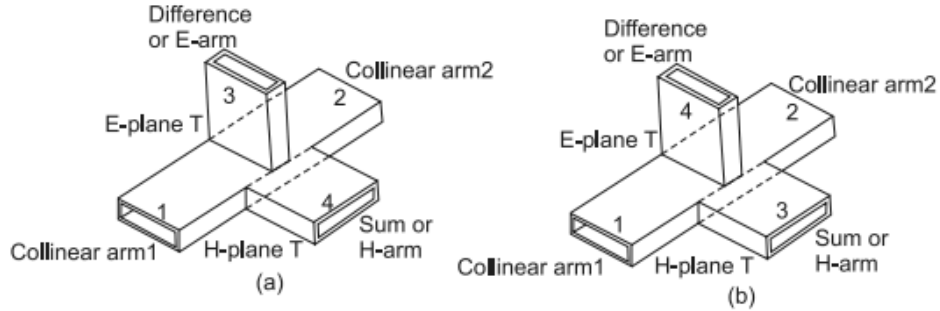


Fig. 6.49 Magic-T s with ports 3 and 4 interchanged

The magic-T has the following characteristics when all the ports are terminated with a matched loads. Let us consider the structure (a) with the ports as indicated.

1. If two waves of equal magnitude and equal phase are fed into ports 1 and 2, the output at Port 3 is subtractive and becomes zero and total output will appear additively at the port 4. Hence, Port 3 is called the difference or E-arm and 4, the sum or H-arm.
2. A wave incident at Port 3 (E-arm) divides equally between ports 1 and 2 but is opposite in phase with no coupling to Port 4 (H-arm). Thus,

$$S_{13} = -S_{23}, S_{43} = 0 \quad (6.100)$$

3. A wave incident at Port 4 (H-arm) divides equally between ports 1 and 2 in phase with no coupling to port 3 (E-arm). Thus,

$$S_{14} = S_{41} = 1/\sqrt{2} = S_{24} = S_{42} \text{ and } S_{34} = 0 \quad (6.101a)$$

4. A wave fed into one collinear port, 1 or 2, will not appear in the other collinear Ports, 2 or 1, respectively. Hence, two collinear ports 1 and 2 are isolated from each other, making

$$S_{12} = S_{21} = 0 \quad (6.101b)$$

A magic-T can be matched by putting tuning screws suitably in the E and H-arms without destroying the symmetry of the junctions. Therefore, for an ideal lossless magic-T matched at ports 3 and 4, $S_{33} = S_{44} = 0$. The procedure of derivation of the S -matrix considers the symmetry property at the junction for which S_{14}

$= S_{41} = S_{24} = S_{42}, S_{31} = S_{13} = -S_{23} = -S_{32}, S_{34} = S_{43} = 0, S_{12} = S_{21} = 0$. Therefore, the S -matrix for a magic- T , matched at ports 3 and 4 given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & -S_{13} & S_{14} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \quad (6.102)$$

From the unitary property applied to rows 1 and 2, we get

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad (6.103)$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad (6.104)$$

Subtracting these two equations:

$$|S_{11}|^2 - |S_{22}|^2 = 0 \quad \text{or,} \quad |S_{11}| = |S_{22}| \quad (6.105)$$

From the unitary property applied to rows 3 and 4,

$$2 |S_{13}|^2 = 1, \quad \text{or} \quad |S_{13}| = 1/\sqrt{2} \quad (6.106)$$

$$2 |S_{14}|^2 = 1, \quad \text{or} \quad |S_{14}| = 1/\sqrt{2} \quad (6.107)$$

Substituting these values in Eq. (6.103),

$$|S_{11}|^2 + |S_{12}|^2 + 1/2 + 1/2 = 1 \quad \text{or,} \quad |S_{11}|^2 + |S_{12}|^2 = 0 \quad (6.108)$$

$$\text{which is valid if} \quad S_{11} = S_{12} = 0 \quad (6.109)$$

$$\text{From Eqs (6.105) and (6.109), } S_{22} = 0 \quad (6.110)$$

$$\text{Therefore, } [S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{13} \\ 0 & 0 & -S_{13} & S_{13} \\ S_{13} & -S_{13} & 0 & 0 \\ S_{13} & S_{13} & 0 & 0 \end{bmatrix} \quad (6.111)$$

$$\text{where} \quad |S_{13}| = 1/\sqrt{2} = |S_{14}|$$

By proper choice of reference planes in arms 3 and 4, it is possible to make both S_{13} and S_{14} real, resulting in the final form of S -matrix of magic- T .

$$[S] = 1/\sqrt{2} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad (6.112a)$$

For the structure (b) where ports 3 and 4 are interchanged, the S -matrix becomes

$$[S] = 1/\sqrt{2} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \quad (6.112b)$$

PARALLEL STRIP LINES

A parallel strip line consists of two perfectly parallel strips separated by a perfect dielectric slab of uniform thickness, as shown in Fig. 11-2-1. The plate width is w , the separation distance is d , and the relative dielectric constant of the slab is ϵ_{rd} .

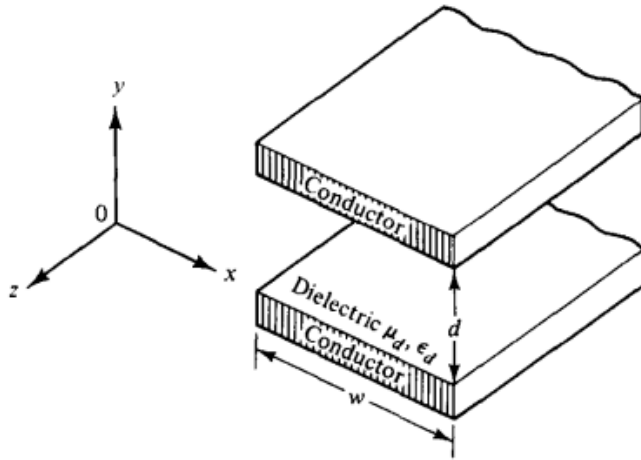


Figure 11-2-1 Schematic diagram of a parallel strip line.

11-2-1 Distributed Parameters

In a microwave integrated circuit a strip line can be easily fabricated on a dielectric substrate by using printed-circuit techniques. A parallel stripline is similar to a two-conductor transmission line, so it can support a quasi-TEM mode. Consider a TEM-mode wave propagating in the positive z direction in a lossless strip line ($R = G = 0$). The electric field is in the y direction, and the magnetic field is in the x direction. If the width w is much larger than the separation distance d , the fringing capacitance is negligible. Thus the equation for the inductance along the two conducting strips can be written as

$$L = \frac{\mu_c d}{w} \quad \text{H/m} \quad (11-2-1)$$

where μ_c is the permeability of the conductor. The capacitance between the two conducting strips can be expressed as

$$C = \frac{\epsilon_d w}{d} \quad \text{F/m} \quad (11-2-2)$$

where ϵ_d is the permittivity of the dielectric slab.

If the two parallel strips have some surface resistance and the dielectric substrate has some shunt conductance, however, the parallel stripline would have some losses. The series resistance for both strips is given by

$$R = \frac{2R_s}{w} = \frac{2}{w} \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \quad \Omega/\text{m} \quad (11-2-3)$$

where $R_s = \sqrt{(\pi f \mu_c)/\sigma_c}$ is the conductor surface resistance in Ω/square and σ_c is the conductor conductivity in U/m . The shunt conductance of the strip line is

$$G = \frac{\sigma_d w}{d} \quad \text{U}/\text{m} \quad (11-2-4)$$

where σ_d is the conductivity of the dielectric substrate.

11-2-2 Characteristic Impedance

The characteristic impedance of a lossless parallel strip line is

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{w} \sqrt{\frac{\mu_d}{\epsilon_d}} = \frac{377}{\sqrt{\epsilon_{rd}}} \frac{d}{w} \quad \text{for } w \gg d \quad (11-2-5)$$

The phase velocity along a parallel strip line is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_d \epsilon_d}} = \frac{c}{\sqrt{\epsilon_{rd}}} \quad \text{m/s} \quad \text{for } \mu_c = \mu_0 \quad (11-2-6)$$

The characteristic impedance of a lossy parallel strip line at microwave frequencies ($R \ll \omega L$ and $G \ll \omega C$) can be approximated as

$$Z_0 \approx \sqrt{\frac{L}{C}} = \frac{377}{\sqrt{\epsilon_{rd}}} \frac{d}{w} \quad \text{for } w \gg d \quad (11-2-7)$$

11-2-3 Attenuation Losses

The propagation constant of a parallel strip line at microwave frequencies can be expressed by

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \quad \text{for } R \ll \omega L \quad \text{and} \quad G \ll \omega C \\ &\approx \frac{1}{2} \left(R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right) + j\omega \sqrt{LC} \end{aligned} \quad (11-2-8)$$

Thus the attenuation and phase constants are

$$\alpha = \frac{1}{2} \left(R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right) \quad \text{Np/m} \quad (11-2-9)$$

and

$$\beta = \omega \sqrt{LC} \quad \text{rad/m} \quad (11-2-10)$$

Substitution of the distributed parameters of a parallel strip line into Eq.(11-2-9) yields the attenuation constants for the conductor and dielectric losses:

$$\alpha_c = \frac{1}{2} R \sqrt{\frac{C}{L}} = \frac{1}{d} \sqrt{\frac{\pi f \epsilon_d}{\sigma_c}} \quad \text{Np/m} \quad (11-2-11)$$

and

$$\alpha_d = \frac{1}{2} G \sqrt{\frac{L}{C}} = \frac{188 \sigma_d}{\sqrt{\epsilon_{rd}}} \quad \text{Np/m} \quad (11-2-12)$$

5.b.

$$w = \frac{377}{\sqrt{\epsilon_{rd}}} \frac{d}{Z_0} = \frac{377}{\sqrt{6}} \frac{4 \times 10^{-3}}{50}$$
$$= 12.31 \times 10^{-3} \text{ m}$$

b. The strip-line capacitance is

$$C = \frac{\epsilon_d w}{d} = \frac{8.854 \times 10^{-12} \times 6 \times 12.31 \times 10^{-3}}{4 \times 10^{-3}}$$
$$= 163.50 \text{ pF/m}$$

c. The strip-line inductance is

$$L = \frac{\mu_c d}{w} = \frac{4\pi \times 10^{-7} \times 4 \times 10^{-3}}{12.31 \times 10^{-3}}$$
$$= 0.41 \text{ } \mu\text{H/m}$$

d. The phase velocity is

$$v_p = \frac{c}{\sqrt{\epsilon_{rd}}} = \frac{3 \times 10^8}{\sqrt{6}}$$
$$= 1.22 \times 10^8 \text{ m/s}$$

5c.

Directivity and Power Density of an Antenna

Power density is defined as the power radiated by an antenna per unit area in a given direction. It is equal to the magnitude of the Poynting vector. In the far-field region, power density is given by:

$$S = \text{Prad} / (4\pi r^2) \text{ W/m}^2$$

where Prad is the total radiated power and r is the distance from the antenna.

Directivity of an antenna is defined as the ratio of maximum radiation intensity to the average radiation intensity over all directions. It is expressed as:

$$D = U_{\text{max}} / U_{\text{avg}}$$

Directivity indicates the ability of an antenna to concentrate radiated power in a particular direction.

6.a.

Antenna Radiation pattern:

To completely specify the radiation pattern with respect to field intensity and polarization requires three patterns:

1. The θ component of the electric field as a function of the angles θ and ϕ or $E_\theta(\theta, \phi)$ (V m^{-1}) as in Figs. 2-3 and 2-4.
2. The ϕ component of the electric field as a function of the angles θ and ϕ or $E_\phi(\theta, \phi)$ (V m^{-1}).
3. The phases of these fields as a function of the angles θ and ϕ or $\delta_\theta(\theta, \phi)$ and $\delta_\phi(\theta, \phi)$ (rad or deg).

Definition:

The antenna radiation pattern is a graphical representation of the variation of radiated power (or field strength) of an antenna as a function of direction in space, at a constant distance from the antenna.

Types of Radiation Patterns

Field pattern – variation of electric or magnetic field intensity

Power pattern – variation of radiated power per unit solid angle

Common pattern planes

E-plane: Plane containing electric field vector and direction of maximum radiation

H-plane: Plane containing magnetic field vector and direction of maximum radiation

Main lobe: Direction of maximum radiation

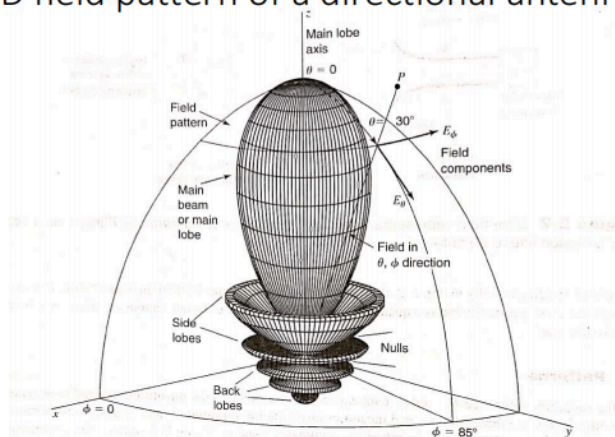
Side lobes: Minor radiation lobes

Nulls: Directions of zero radiation

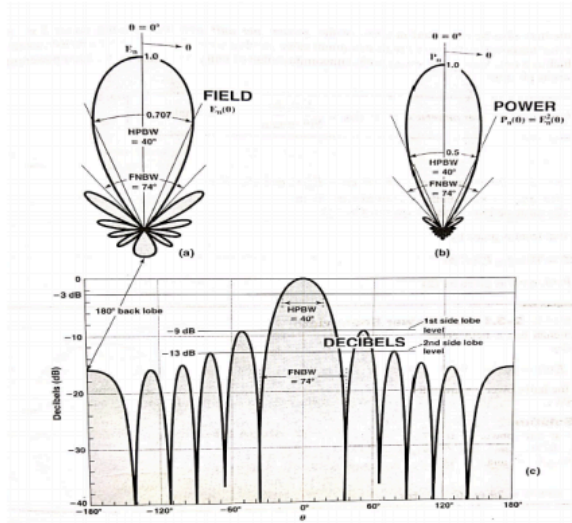
Beamwidth: Angular width between half-power points

Directivity: Ability of antenna to focus energy in one direction

3D field pattern of a directional antenna



2D pattern



$$\text{Normalized field pattern} = E_{\theta}(\theta, \phi)_n = \frac{E_{\theta}(\theta, \phi)}{E_{\theta}(\theta, \phi)_{\max}} \quad (\text{dimensionless})$$

$$\text{Normalized power pattern} = P_n(\theta, \phi)_n = \frac{S(\theta, \phi)}{S(\theta, \phi)_{\max}} \quad (\text{dimensionless})$$

where

$$S(\theta, \phi) = \text{Poynting vector} = [E_{\theta}^2(\theta, \phi) + E_{\phi}^2(\theta, \phi)]/Z_0, \text{ W m}^{-2}$$

$$S(\theta, \phi)_{\max} = \text{maximum value of } S(\theta, \phi), \text{ W m}^{-2}$$

$$Z_0 \text{ intrinsic impedance of space} = 376.7 \, \Omega$$

The decibel level is given by

$$\text{dB} = 10 \log_{10} P_n(\theta, \phi)$$

Maximum Effective Aperture of Short Dipole:

Effective Aperture (Ae)

Effective aperture is the area that an antenna appears to have when receiving power from an incident electromagnetic wave.

$$A_e = \frac{\lambda^2}{4\pi} G$$

Gain of a Short Electric Dipole

For a **short dipole**:

- Directivity $D = 1.5$
- Radiation efficiency ≈ 1

$$G = D = 1.5$$

$$A_{e(\max)} = \frac{\lambda^2}{4\pi} \times 1.5$$

$$A_{e(\max)} = \frac{1.5}{4\pi} \lambda^2$$

$$A_{e(\max)} = 0.119 \lambda^2$$

6. B. Frii's transmission formula:

Radio communication link

- Power per unit area available at the receiving antenna is

$$S_r = \frac{P_t}{4\pi r^2} \quad (W)$$

If the antenna has gain G_t , the power

Per unit area at the receiving antenna will be increased in proportion given by

$$S_r = \frac{P_t G_t}{4\pi r^2} \quad (W)$$

- The power collected by the lossless, matched receiving antenna of effective aperture, A_{er} is,

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2} \quad (W)$$

The gain of the transmitting antenna can be expressed as

$$G_t = \frac{4\pi A_{et}}{\lambda^2}$$

$$\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2} \quad (\text{dimensionless}) \quad \text{Friis transmission formula}$$

where

- P_r = received power, W
- P_t = transmitted power, W
- A_{et} = effective aperture of transmitting antenna, m^2
- A_{er} = effective aperture of receiving antenna, m^2
- r = distance between antennas, m
- λ = wavelength, m

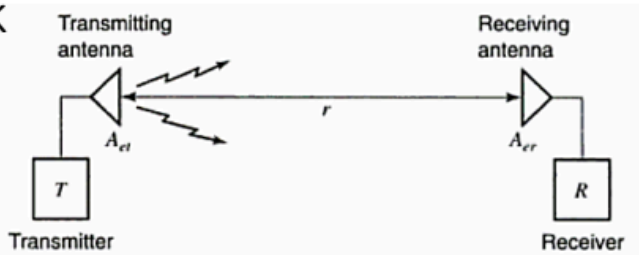


Figure 2-14 Communication circuit with waves from transmitting antenna arriving at the receiving antenna by a direct path of length r .

6. C. Construction and field pattern of microstrip line:

A microstrip line is a planar transmission line commonly used in microwave integrated circuits (MICs).

Structure

It consists of:

1. **Conducting strip (signal line)**
 - A thin metallic strip (copper/gold)
 - Width = W, thickness = t

2. **Dielectric substrate**

Relative permittivity ϵ_r

Thickness = h

3. **Ground plane**

A continuous conducting plane on the bottom of the substrate

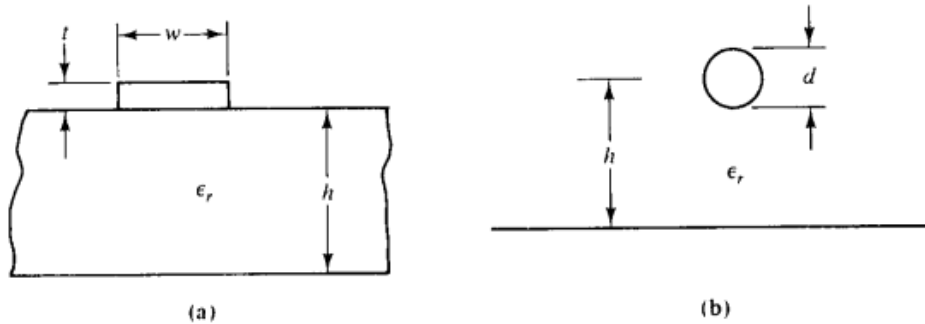


Figure 11-1-1 Cross sections of (a) a microstrip line and (b) a wire-over-ground line.

Characteristic impedance is given by

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \frac{4h}{d} \quad \text{for } h \gg d$$

where ϵ_r = dielectric constant of the ambient medium

h = the height from the center of the wire to the ground plane

d = diameter of the wire

Field Lines:

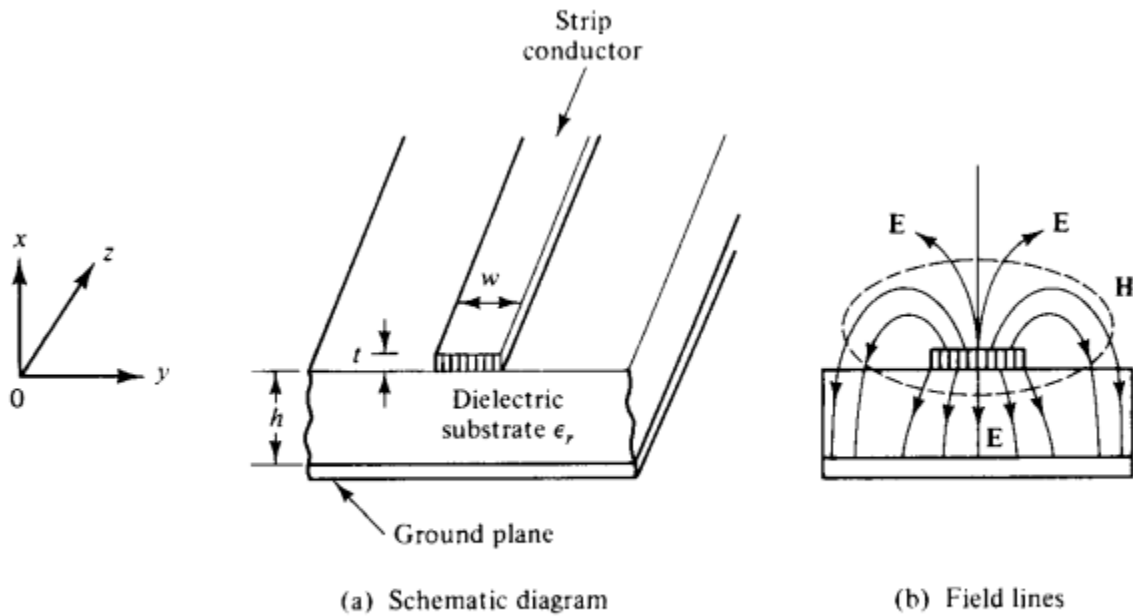
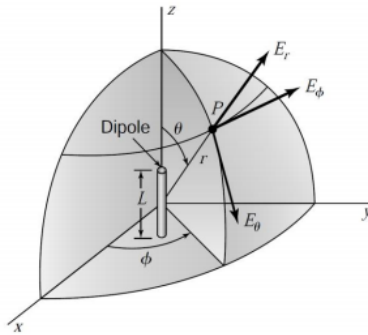


Figure 11-0-1 Schematic diagrams of strip lines.

7. a. Derive the expression for radiation resistance of short electric dipole antennas.



Radiation Resistance of Short Electric Dipole:

The Poynting vector of the far field is integrated over a large sphere to obtain the total power radiated. This power is then equated to $I^2 R$ where I is the rms current on the dipole and R is a resistance, called the radiation resistance of the dipole.

The *average* Poynting vector is given by

$$S = \frac{1}{2} \text{Re}(E \times H^*)$$

The far-field components are E_θ and H_ϕ so that the radial component of the Poynting vector is

$$S_r = \frac{1}{2} \text{Re} E_\theta H_\phi^*$$

where E_θ and H_ϕ^* are complex.

The far-field components are related by the intrinsic impedance of the medium. Hence,

$$E_\theta = H_\phi Z = H_\phi \sqrt{\frac{\mu}{\epsilon}}$$

$$S_r = \frac{1}{2} \text{Re} Z H_\phi H_\phi^* = \frac{1}{2} |H_\phi|^2 \text{Re} Z = \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu}{\epsilon}}$$

The total power P radiated is then

$$P = \iint S_r ds = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \int_0^{2\pi} \int_0^\pi |H_\phi|^2 r^2 \sin \theta d\theta d\phi$$

<p>Magnetic fields of short dipole</p> $ \mathbf{H} = H_\phi = \frac{I_0 L \sin \theta e^{j\omega[t-(r/c)]}}{4\pi} \left(\frac{j\omega}{cr} + \frac{1}{r^2} \right)$ <p style="text-align: center;">$H_r = H_\theta = 0$</p>	<p>General case</p>
--	----------------------------

$$|H_\phi| = \frac{\omega I_0 L \sin \theta}{4\pi cr}$$

$$P = \iint S_r ds = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \int_0^{2\pi} \int_0^\pi |H_\phi|^2 r^2 \sin \theta d\theta d\phi$$

$$P = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \int_0^{2\pi} \int_0^\pi |H_\phi|^2 r^2 \sin \theta d\theta d\phi$$

$$P = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \int_0^{2\pi} \int_0^\pi \left| \frac{\omega I_0 L \sin \theta}{4\pi cr} \right|^2 r^2 \sin \theta d\theta d\phi$$

$$P = \frac{1}{32} \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_0^2 L^2}{\pi^2} \int_0^{2\pi} \int_0^\pi \sin^3 \theta d\theta d\phi$$

$$P = \frac{1}{32} \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_0^2 L^2}{\pi^2} (2\pi)^{(4/3)}$$

$$P = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi}$$

This is the *average* power or rate at which energy is streaming out of a sphere surrounding the dipole.

Hence, it is equal to the power radiated. Assuming no losses, it is also equal to the power delivered to the dipole.

Therefore, P must be equal to the square of the rms current I flowing on the dipole times a resistance R_r called the *radiation resistance* of the dipole.

Thus,

$$\sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi} = \left(\frac{I_0}{\sqrt{2}} \right)^2 R_r$$

$$R_r = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 L^2}{6\pi}$$

For air or vacuum $\sqrt{\mu/\epsilon} = \sqrt{\mu_0/\epsilon_0} = 377 = 120\pi \Omega$

<p>Dipole with uniform current $R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 = 80\pi^2 L_\lambda^2 = 790 L_\lambda^2 \quad (\Omega)$ Radiation resistance</p>
--

Modifying the equation of power for the general case where the current is not uniform on the dipole, the *radiated power* is

$$P = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_{av}^2 L^2}{12\pi} \quad (\text{W})$$

↓
 $I_{av} \neq I_0$

where I_{av} = amplitude of *average current* on dipole (peak value in time).

The *power delivered* to the dipole is

$$P = \frac{1}{2} I_0^2 R_r$$

where I_0 = amplitude of *terminal current* of center-fed dipole (peak value in time).

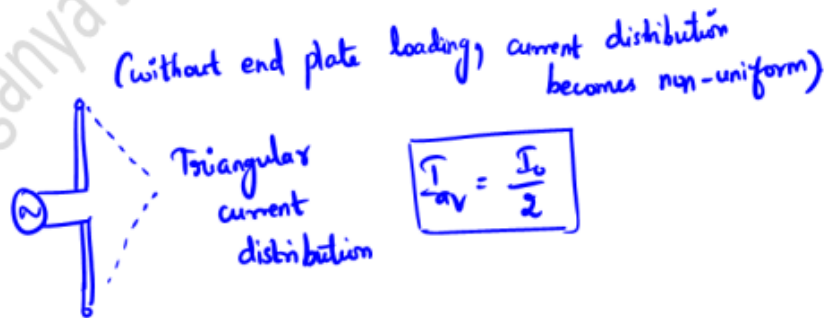
Equating the *power radiated* to the *power delivered*, for free space ($\mu = \mu_0$ and $\epsilon = \epsilon_0$), yields the radiation resistance as,

$$R_r = 790 \left(\frac{I_{av}}{I_0}\right)^2 L_\lambda^2$$

For a short dipole without end loading,

$$I_{av} = \frac{I_0}{2},$$

$$R_r = 197 L_\lambda^2$$



As an example,

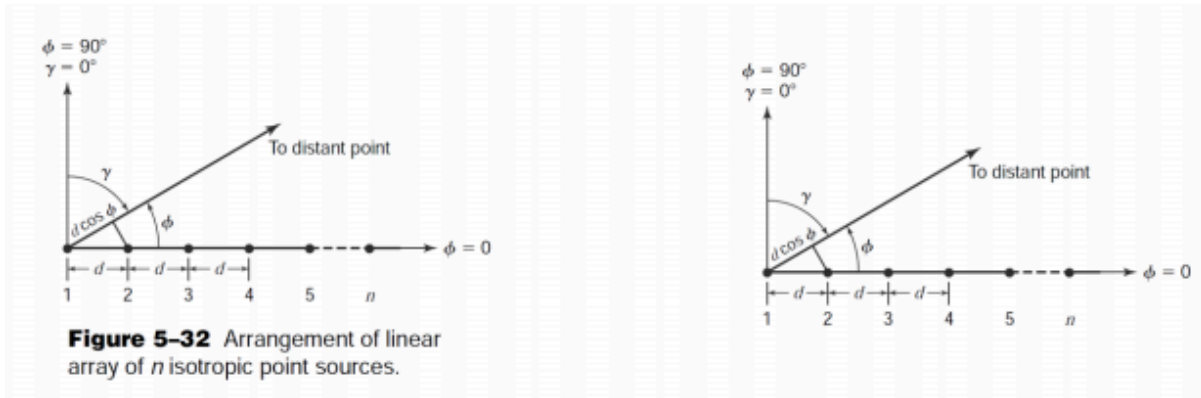
Suppose that, $L_\lambda = 0.1$, then $R_r = 7.9 \Omega$.

If $L_\lambda = 0.01$, then $R_r = 0.08 \Omega$.

Thus, the radiation resistance of a short dipole is small.

7. b. Obtain the expression for total electric field for array of n-point sources; consider uniform linear array.

Let us now proceed to the case of n isotropic point sources of equal amplitude and spacing arranged as a linear array, as indicated in Fig. 5–32, where n is any positive integer.



The amplitudes of the fields from the sources are all equal
Source 1 is the phase reference.

Thus, at a distant point in the direction ϕ
the field from source 2 is advanced in phase with respect to source 1 by ψ ,
the field from source 3 is advanced in phase with respect to source 1 by 2ψ , etc.

where ψ is the total phase difference of the fields from adjacent sources as given by

$$\psi = \frac{2\pi d}{\lambda} \cos \phi + \delta = d_r \cos \phi + \delta \quad (2)$$

where δ is the phase difference of adjacent sources,
i.e., source 2 with respect to 1, 3 with respect to 2, etc.

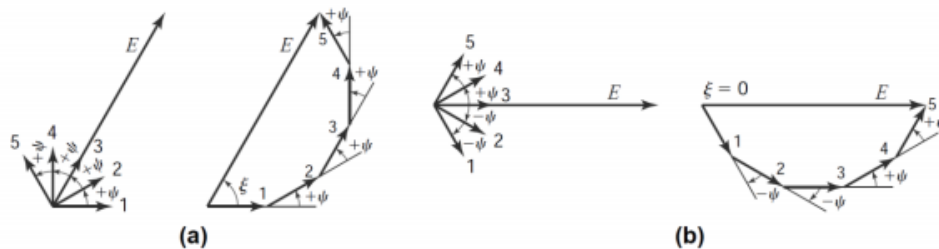


Figure 5-33 (a) Vector addition of fields at a large distance from the linear array of five isotropic point sources of equal amplitude with source 1 as the phase center (reference for phase). (b) Same, but with midpoint of array (source 3) as phase center.

The total field E at a large distance in the direction ϕ is given by

$$E = E_0(1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{j(n-1)\psi}) \rightarrow \textcircled{1}$$

Equation (1) is a geometric series.

Each term represents a phasor, and the amplitude of the total field E and its phase angle ξ can be obtained by phasor (vector) addition

Multiply (1) by $e^{j\psi}$, giving

$$Ee^{j\psi} = E_0(e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi}) \rightarrow \textcircled{2}$$

Now subtract (2) from (1) and divide by $1 - e^{j\psi}$, yielding

$$E(1 - e^{j\psi}) = E_0(1 - e^{jn\psi})$$

$$E = \frac{E_0(1 - e^{jn\psi})}{(1 - e^{j\psi})} \rightarrow \textcircled{3}$$

$$E = E_0 \frac{e^{jn\psi/2} (e^{jn\psi/2} - e^{-jn\psi/2})}{e^{j\psi/2} (e^{j\psi/2} - e^{-j\psi/2})} = E_0 e^{j(n-1)\psi/2} \frac{(\sin n\psi/2) \cdot 2j}{\sin(\psi/2) \cdot 2j}$$

$$E = E_0 e^{j(n-1)\psi/2} \frac{(\sin n\psi/2)}{\sin(\psi/2)} \rightarrow \textcircled{4}$$

where ξ is referred to the field from source 1.

$$\xi = \left(\frac{n-1}{2}\right)\psi \rightarrow \textcircled{7}$$

$$E_1 = E_0 \left\{ \frac{\sin(n\psi/2)}{\sin(\psi/2)} \right\} e^{j\xi} \rightarrow \textcircled{5}$$

$$E_1 = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)} \angle \xi \rightarrow \textcircled{6}$$

When $\psi = 0$, (6) or (8) is indeterminate so that for this case E must be obtained as the limit of (8) as ψ approaches zero. Thus, for $\psi = 0$ we have the relation that

$$\lim_{\psi \rightarrow 0} E_t = E_0 \cdot n \cdot \frac{\frac{d}{d\psi} (\sin(n\psi/2))}{\frac{d}{d\psi} (\sin(\psi/2))}$$

$$= E_0 \lim_{\psi \rightarrow 0} \frac{\cos(n\psi/2) \cdot (n/2)}{\cos(\psi/2) \cdot (1/2)}$$

$$\Rightarrow E_{tmax} = E_0 \cdot \frac{n/2}{1/2}$$

$$E_{tmax} = E_0 \cdot n \rightarrow \text{9}$$

\therefore field from array is max^m in any direction ϕ when $\psi = 0$.

Hence, normalized value of total field for $E_{tmax} = n E_0$ is

$$E_0 = \frac{E_t}{E_{tmax}} = \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)} \rightarrow \text{10}$$

We may conclude from the above discussion that the field from the array will be a maximum in any direction ϕ for which $\psi = 0$.

Stated in another way, *the fields from the sources all arrive at a distant point in the same phase when $\psi = 0$.*

In special cases, ψ may not be zero for any value of ϕ , and in this case the field is usually a maximum at the minimum value of ψ .

8. a. Explain principle of pattern multiplication.

Nonisotropic but Similar Point Sources and the Principle of Pattern Multiplication

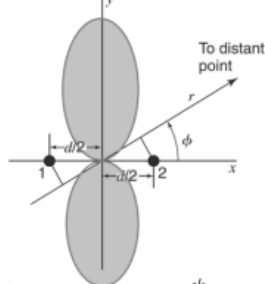
The word **similar** is here used to indicate that *the variation with absolute angle ϕ of both the amplitude and phase of the field is the same.*

The patterns not only must be of the same shape but *also must be oriented in the same direction* to be called "similar."

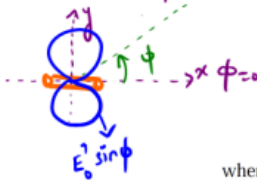
The *maximum amplitudes* of the individual sources *may be unequal.*

If, however, they are also equal, the sources are not only similar but are **identical**.

Case (i) : Broadside array



Pattern of short dipole:



$$E = 2E_0 \cos \frac{\psi}{2}$$

$$E = \sin \phi \cos \frac{\psi}{2}$$

$$\text{where } \psi = d_r \cos \phi + \delta$$

This result is the same as obtained by multiplying the pattern of the individual source ($\sin \phi$) by the pattern of two isotropic point sources ($\cos \psi/2$).

If the similar but unequal point sources of Case 5 have patterns as given by $E_0 = E'_0 \sin \phi$ the total normalized pattern is

$$E = \sin \phi \sqrt{(1 + a \cos \psi)^2 + a^2 \sin^2 \psi}$$

Here again, the result is the same as that obtained by multiplying the pattern of the individual source by the pattern of an array of isotropic point sources.

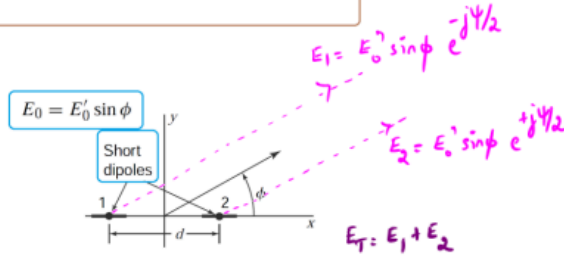


Figure 5-21 Two nonisotropic sources with respect to the coordinate system.

Patterns of this type might be produced by short dipoles oriented parallel to the x axis as suggested by Fig. 5-21.

$$E_T = E'_0 \sin \phi e^{-j\psi/2} + E'_0 \sin \phi e^{+j\psi/2}$$

$$= E'_0 \sin \phi (e^{-j\psi/2} + e^{+j\psi/2})$$

$$E_T = E'_0 \sin \phi \left[2 \cos \frac{\psi}{2} \right]$$

These are examples illustrating the **principle of pattern multiplication**, which may be expressed as follows:

The field pattern of an array of nonisotropic but similar point sources is the product of the pattern of the individual source and the pattern of an array of isotropic point sources having the same locations, relative amplitudes, and phase as the nonisotropic point sources.

This principle may be applied to arrays of any number of sources provided only that they are similar. The individual nonisotropic source or antenna may be of finite size but can be considered as a point source situated at the point in the antenna to which phase is referred. This point is said to be the "phase center."

The above discussion of pattern multiplication has been concerned only with the field pattern or magnitude of the field. If the field of the nonisotropic source and the array of isotropic sources vary in phase with space angle, i.e., have a phase pattern which is not a constant, the statement of the principle of pattern multiplication may be extended to include this more general case as follows:

The total field pattern of an array of nonisotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources each located at the phase center of the individual source and having the same relative amplitude and phase, while the total phase pattern is the sum of the phase patterns of the individual source and the array of isotropic point sources.

The total phase pattern is referred to the phase center of the array. In symbols, the total field E is then

$$E = \underbrace{f(\theta, \phi)}_{\text{Field pattern}} \underbrace{[f_p(\theta, \phi) + F_p(\theta, \phi)]}_{\text{Phase pattern}} \quad (4)$$

where

$f(\theta, \phi)$ = field pattern of individual source

$f_p(\theta, \phi)$ = phase pattern of individual source

$F(\theta, \phi)$ = field pattern of array of isotropic sources

$F_p(\theta, \phi)$ = phase pattern of array of isotropic sources

EXAMPLE 5-10.1 Assume two identical point sources separated by a distance d , each source having the field pattern given by (1) as might be obtained by two short dipoles arranged as in Fig. 5-21. Let $d = \lambda/2$ and the phase angle $\delta = 0$. Then the total field pattern is

$$E = \sin \phi \cos \left(\frac{\pi}{2} \cos \phi \right) \quad (5)$$

This pattern is illustrated by Fig. 5-22c as the product of the individual source pattern ($\sin \phi$) shown at (a) and the array pattern $\{\cos[(\pi/2) \cos \phi]\}$ as shown at (b). The pattern is sharper than it was in Case 1 (Sec. 5-9) for the isotropic sources. In this instance, the maximum field of the individual source is in the direction $\phi = 90^\circ$, which coincides with the direction of the maximum field for the array of two isotropic sources.

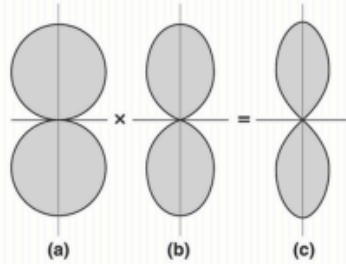


Figure 5-22 Example of pattern multiplication. Two nonisotropic but identical point sources of the same amplitude and phase, spaced $\lambda/2$ apart and arranged as in Fig. 5-21, produce the pattern shown at (c). The individual source has the pattern shown at (a), which, when multiplied by the pattern of an array of two isotropic point sources (of the same amplitude and phase) as shown at (b), yields the total array pattern of (c).

8. b. Write a note on Thin Linear Antenna.

The Thin Linear Antenna

Assumption: The antennas are symmetrically fed at the center by a balanced two-wire transmission line.

The antennas may be of any length, but it is assumed that the current distribution is sinusoidal. Current-distribution measurements indicate that this is a good assumption provided that the antenna is thin, i.e., when the conductor diameter is less than, say, $\lambda/100$.

Thus, the sinusoidal current distribution approximates the natural distribution on thin antennas.

Examples of the approximate natural-current distributions on a number of thin, linear center-fed antennas of different length are illustrated in Fig. 6-7.

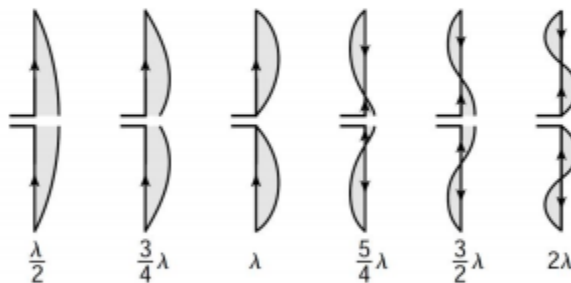


Figure 6-7 Approximate natural-current distribution for thin, linear, center-fed antennas of various lengths.

The currents are in phase over each $\lambda/2$ section and in opposite phase over the next.

Far-field equations for a symmetrical, thin, linear, center-fed antenna of length L :

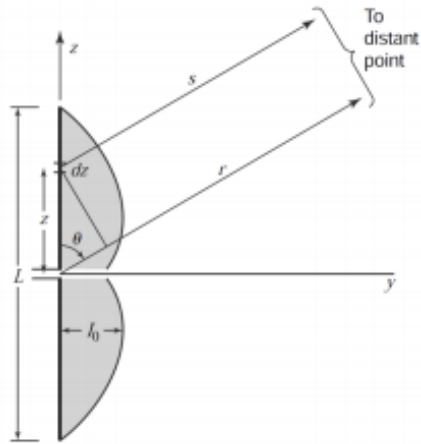


Figure 6-8 Relations for symmetrical, thin, linear, center-fed antenna of length L . The retarded value of the current at any point z on the antenna referred to a point at a distance s is

$$[I] = I_0 \sin \left[\frac{2\pi}{\lambda} \left(\frac{L}{2} \pm z \right) \right] e^{j\omega[t - (r/c)]}$$

The function,

$$\sin \left[\frac{2\pi}{\lambda} \left(\frac{L}{2} \pm z \right) \right]$$

is the form factor for the current on the antenna.

The expression $\left(\frac{L}{2}\right) + z$ is used when $z < 0$ and $\left(\frac{L}{2}\right) - z$ is used when $z > 0$.

The antenna is regarded as made up of a series of infinitesimal dipoles of length dz .

The field of the entire antenna may be obtained by integrating the fields from all of the dipoles.

<i>Far fields of center-fed dipole</i>	$H_\phi = \frac{j[I_0]}{2\pi r} \left[\frac{\cos[(\beta L \cos \theta)/2] - \cos(\beta L/2)}{\sin \theta} \right]$
	$E_\theta = \frac{j60[I_0]}{r} \left[\frac{\cos[(\beta L \cos \theta)/2] - \cos(\beta L/2)}{\sin \theta} \right]$

where $[I_0] = I_0 e^{j\omega[t-(r/c)]}$ and

$$E_\theta = 120\pi H_\phi$$

The shape of the far-field pattern is given by the factor in the brackets.

The factors preceding the brackets give the instantaneous magnitude of the fields as functions of the antenna current and the distance r .

To obtain the rms value of the field, we let $[I_0]$ equal the rms current at the location of the current maximum.

There is no factor involving phase in these expressions, since the center of the antenna is taken as the phase center.

Hence any phase change of the fields as a function of θ will be a jump of 180° when the pattern factor changes sign.

8. c. A thin dipole antenna is $\lambda/10$ long. If its loss resistance is 2.5Ω , find the radiation resistance and efficiency.

For a **short dipole** ($l \ll \lambda$), the radiation resistance is:

$$R_r = 80\pi^2 \left(\frac{l}{\lambda} \right)^2$$

Substitute $l = \lambda/10$:

$$R_r = 80\pi^2 \left(\frac{1}{10} \right)^2$$

$$R_r = 80\pi^2 \times \frac{1}{100}$$

$$R_r = 0.8\pi^2$$

$$R_r \approx \underline{7.9 \Omega}$$

Antenna efficiency is given by:

$$\eta = \frac{R_r}{R_r + R_L}$$

Substitute values:

$$\eta = \frac{7.9}{7.9 + 2.5}$$

$$\eta = \frac{7.9}{10.4}$$

$$\eta \approx 0.76$$

Module – 5

9. a. Explain different types of horn antenna with relevant diagrams.

Horn Antennas

A horn antenna may be regarded as a ***flared-out (or opened-out) waveguide***.

The function of the horn is to produce a ***uniform phase front with a larger aperture*** than that of the waveguide and ***hence greater directivity***.

Jagadis Chandra Bose constructed a pyramidal horn in 1897.

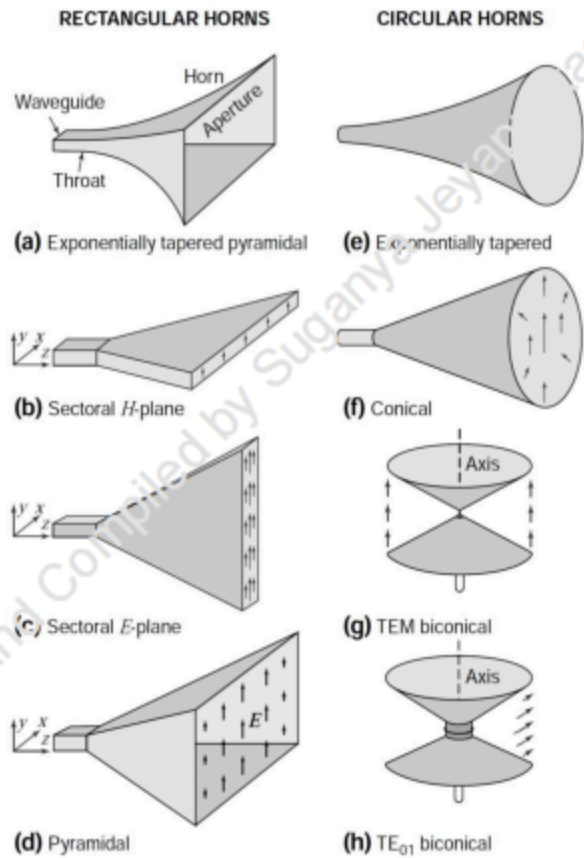


Figure 7-40 Types of rectangular and circular horn antennas. Arrows indicate **E**-field directions.

Those in the left column are **rectangular horns**. All are energized from rectangular waveguides.

Those in the right column are **circular horns**.

To minimize reflections of the guided wave, the transition region or horn between the waveguide at the throat and free space at the aperture could be given a **gradual exponential taper** as in Fig. 7-40a or e.

However, it is the general practice to make horns with **straight flares** as suggested by the other types in Fig. 7-40.

Sectoral horns:

The types in Fig. 7-40*b* and *c* are **sectoral horns**. They are rectangular types with a flare in only one dimension.

Assuming that the rectangular waveguide is energized with a TE_{10} mode wave electric field (\mathbf{E} in the y direction), the horn in Fig. 7-40*b* is flared out in a plane perpendicular to \mathbf{E} . This is the plane of the magnetic field \mathbf{H} . Hence, this type of horn is called a **sectoral horn flared in the H plane** or simply an **H -plane sectoral horn**.

The horn in Fig. 7-40*c* is flared out in the plane of the electric field \mathbf{E} , and, hence, is called an **E -plane sectoral horn**.

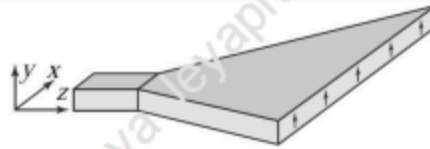
Pyramidal horn:

A rectangular horn with flare in both planes, as in Fig. 7-40*d*, is called a **pyramidal horn**.

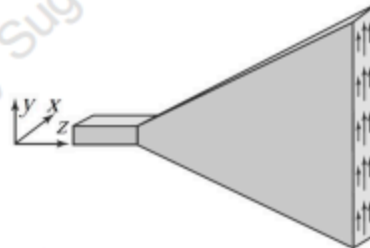
With a TE_{10} wave in the waveguide, the magnitude of the electric field is quite uniform in the y direction across the apertures of the horns of Fig. 7-40*b*, *c* and *d* but tapers to zero in the x direction across the apertures.

This variation is suggested by the arrows at the apertures in Fig. 7-40*b*, *c* and *d*. The arrows indicate the direction of the electric field \mathbf{E} , and their length gives an approximate indication of the magnitude of the field intensity.

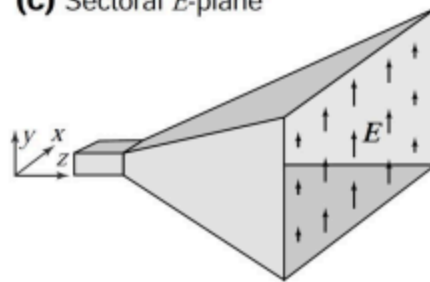
For small flare angles, the field variation across the aperture of the rectangular horns is similar to the sinusoidal distribution of the TE_{10} mode across the waveguide.



(b) Sectoral H -plane



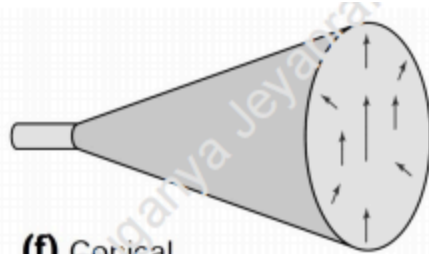
(c) Sectoral E -plane



(d) Pyramidal

Conical type horn:

The horn shown in Fig. 7-40f is a **conical type horn**. When excited with a circular guide carrying a TE₁₁ mode wave, the electric field distribution at the aperture is as shown by the arrows.



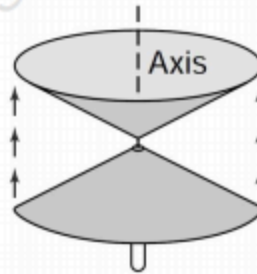
(f) Conical

Bi-conical types horn:

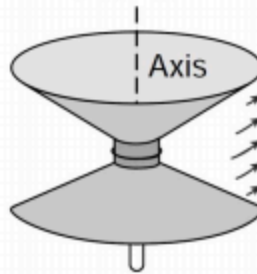
The horns in Fig. 7-40g and h are **bi-conical types**.

The one in Fig. 7-40g is excited in the TEM mode by a vertical radiator while the one in Fig. 7-40h is excited in the TE₀₁ mode by a small horizontal loop antenna.

These bi-conical horns are non-directional in the horizontal plane.



(g) TEM biconical



9. b. The radius of a circular loop antenna is 0.02λ. How many turns of the antenna will give radiation resistance of 35Ω?

Radius of circular loop antenna $a=0.02\lambda$

Required radiation resistance $R_r=35 \Omega$

Let N =number of turns

For a **small loop antenna** with **N turns**:

$$R_r = 31200 \left(\frac{NA}{\lambda^2} \right)^2$$

where

$$A = \pi a^2$$

$$A = \pi(0.02\lambda)^2$$

$$A = \pi \times 0.0004 \lambda^2$$

$$A = 0.001257 \lambda^2$$

$$35 = 31200 \left(\frac{N \times 0.001257 \lambda^2}{\lambda^2} \right)^2$$

$$35 = 31200(0.001257N)^2$$

$$35 = 31200 \times 1.58 \times 10^{-6} \times N^2$$

$$35 = 0.0493 N^2$$

$$N^2 = \frac{35}{0.0493}$$

$$N^2 \approx 710$$

$$N \approx \sqrt{710}$$

$$\boxed{N \approx 27}$$

9. c. Compare the far field components of small loop and short dipole antenna.

The comparison is made in Table 7–1. The presence of the operator j in the dipole expressions and its absence in the loop equations indicate that the fields of the electric dipole and of the loop are in time-phase quadrature, the current I being in the same phase in both the dipole and loop. This quadrature relationship is a fundamental difference between the fields of loops and dipoles..

The formulas in Table 7–1 apply to a loop oriented as in Fig. 7–2 and a dipole oriented parallel to the polar or z axis. The formulas are exact only for vanishingly small loops and dipoles. However, they are good approximations for loops up to $\lambda/10$ in diameter and dipoles up to $\lambda/10$ long.

Table 7–1 Far fields of small electric dipoles and loops

Field	Electric dipole	Loop
Electric	$E_{\theta} = \frac{j60\pi [I] \sin \theta}{r} \frac{L}{\lambda}$	$E_{\phi} = \frac{120\pi^2 [I] \sin \theta}{r} \frac{A}{\lambda^2}$
Magnetic	$H_{\phi} = \frac{j[I] \sin \theta}{2r} \frac{L}{\lambda}$	$H_{\theta} = \frac{\pi [I] \sin \theta}{r} \frac{A}{\lambda^2}$

10. a. Explain Yagi–Uda antenna and list its applications.

The Yagi-Uda Array:

Shintaro Uda, an assistant professor at Tohoku University, had conducted experiments on the use of parasitic reflector and director elements in 1926.

He measured patterns and gains with a single parasitic reflector, a single parasitic director and with a reflector and as many as 30 directors. One of his many experimental arrays is shown in Fig. 8–30.

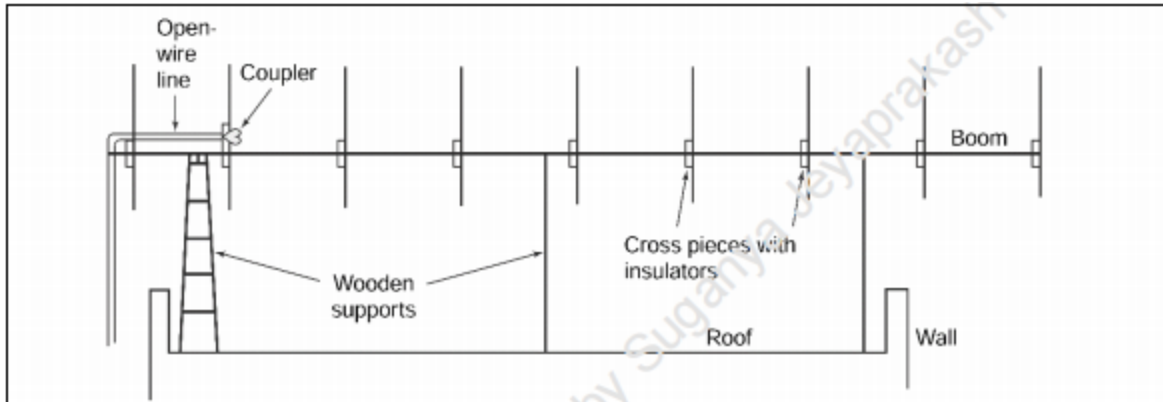


Figure 8-30 Shintaro Uda's experimental antenna with 1 reflector and 7 directors on the roof of his laboratory at Tohoku University for vertically polarized transmission tests during 1927 and 1928 over land and sea paths up to 135 km using a wavelength $\lambda = 4.4$ m. The horizontal wooden boom supporting the array elements is 15 m long.

He found the highest gain with the reflector about $\lambda/2$ in length and spaced about $\lambda/4$ from the driven element, while the best director lengths were about 10 percent less than $\lambda/2$ with optimum spacings about $\lambda/3$.

Even though many patterns were measured in the near field, these lengths and spacings agree remarkably well with optimum values determined since then by further experimental and computer techniques.

After George H. Brown demonstrated the advantages of close spacing, the reflector-to-driven-element spacings were reduced.

Hidetsugu Yagi, professor of electrical engineering at Tohoku University and 10 years Uda's senior, had supported the antenna research done by Uda.

The narrow beams of short waves produced by the guiding action of the multi-director periodic structure, which they called a "wave canal," had encouraged them to suggest using it for short-wave power transmission, an idea now being considered for beaming solar power to the earth from a space station or from earth to a satellite.

The antenna soon came to be called "a Yagi" antenna.

Uda's ingenuity was mainly responsible for the antenna's successful development. In deference to Uda's contributions, we refer to the array as a **Yagi-Uda antenna**, a practice now becoming common.

A typical modern-version 6-element Yagi-Uda antenna is shown in Fig. 8-31.

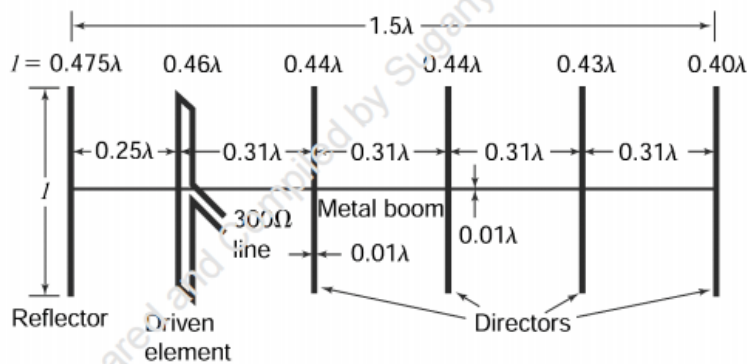


Figure 8-31 Modern-version 6-element Yagi-Uda antenna with dimensions. It has a maximum directivity of about 12 dBi at the center of a bandwidth of 10 percent at half-power.

It consists of a driven element (folded $\lambda/2$ dipole) fed by a $300\ \Omega$ 2-wire transmission line (twin line), a reflector and 4 directors.

Dimensions (lengths and spacings) are indicated on the figure. The antenna provides a gain of about 10 dBi (maximum) with a bandwidth at half-power of 10 percent.

By adjusting lengths and spacings appropriately (tweaking), the dimensions can be optimized, producing an increase in gain of another decibel (Chen-1, 2; Viezbicke-1).

However, the dimensions are critical.

The inherently narrow bandwidth of the Yagi-Uda antenna can be broadened to 1.5 to 1 by lengthening the reflector to improve operation at low frequencies and shortening the directors to improve high-frequency operation.

However, this is accomplished at a sacrifice in gain of as much as 5 dB.

10. b. Explain Parabolic dish antenna or microwave dish antenna with relevant diagram.

The Parabola-General Properties;

Suppose that we have a point source and that we wish to produce a plane-wave front over a large aperture by means of a sheet reflector.

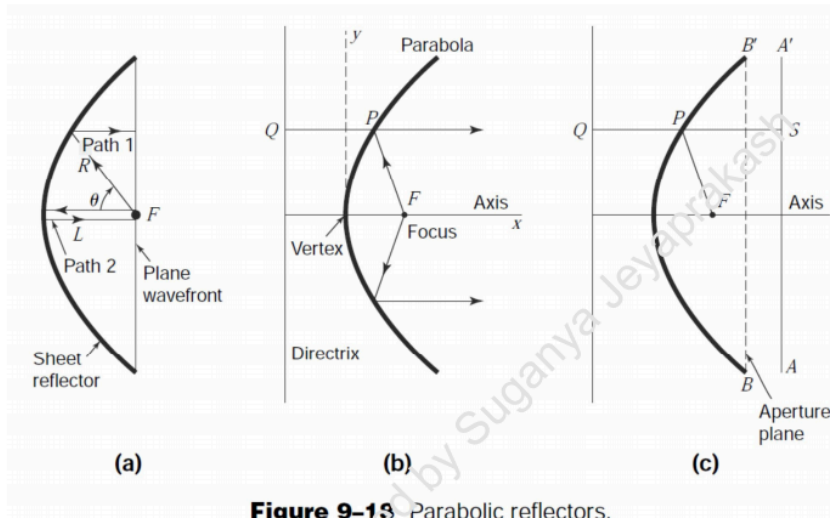


Figure 9-13 Parabolic reflectors.

Referring to Fig. 9-18a, it is then required that the distance from the source to the plane-wave front via path 1 and 2 be equal (This is an application of the *principle of equality of path length (Fermat's principle)* to the special case where all paths are in the same medium)

$$2L = R (1 + \cos \theta) \quad (1)$$

and

$$R = \frac{2L}{(1 + \cos \theta)} \quad (2)$$

This is the equation for the required surface contour.
It is the equation of a parabola with the focus at F.

Referring to Fig. 9-18b, the parabolic curve may be defined as follows.

The distance from any point P on a parabolic curve to a fixed point F , called the **focus**, is equal to the perpendicular distance to a fixed line called the **directrix**.

Thus, in Fig. 9-18b,

$$PS = PQ$$

Referring now to Fig. 9-18c, let AA' be a line normal to the axis at an arbitrary distance QS from the directrix.

Since $PS = QS - PQ$ and $PF = PQ$, it follows that the distance from the focus to S is

$$PF + PS = PF + QS - PQ = QS \quad (3)$$

Thus, a property of a parabolic reflector is that all waves from an isotropic source at the focus that are reflected from the parabola arrive at a line AA' with equal phase.

The "image" of the focus is the directrix, and the reflected field along the line AA' appears as though it originated at the directrix as a plane wave.

The plane BB' (Fig. 9-18c) at which a reflector is cut off is called the **aperture plane**.

A cylindrical parabola converts a cylindrical wave radiated by an in-phase line source at the focus, as in Fig. 9-19a, into a plane wave at the aperture.

A paraboloid-of-revolution converts a spherical wave from an isotropic source at the focus, as in Fig. 9-18b, into a uniform plane wave at the aperture.

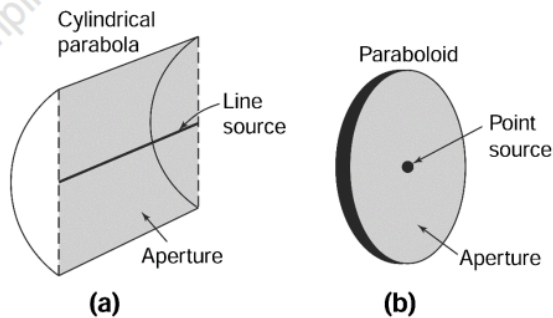


Figure 9-19 Line source and cylindrical parabolic reflector (a) and point source and paraboloidal reflector (b).

Confining our attention to a single ray or wave path, the paraboloid has the property of directing or collimating radiation from the focus into a beam parallel to the axis (see Fig. 9-19).

Any radiation from the primary source or feed antenna at the focus of the parabola which is not directed into the parabola is not collimated but is radiated by direct paths over a large solid angle.

This is not only inefficient but the distributed radiation can degrade the pattern of the radiation from the parabola.

Thus, **it is essential that a parabolic reflector have a directional feed which radiates all or most of the energy into the parabola.**

Paraboloidal Reflector:

The surface generated by the revolution of a parabola around its axis is called a **paraboloid or a parabola of revolution** (Friis-1, Silver-1, Cutler-1, Slater-1).

If an isotropic source is placed at the focus of a paraboloidal reflector, as in Fig. 9-21 a, the portion A of the source radiation that is intercepted by the paraboloid is reflected as a plane wave of circular cross section, only if the reflector surface deviates from a true parabolic surface by no more than a small fraction of a wavelength.

L is the distance between the focus and vertex of the paraboloid.

If L is an even number of $\lambda/4$, the direct radiation in the axial direction from the source will be in opposite phase and will tend to cancel the central region of the reflected wave.

If L is an odd number of $\lambda/4$, the direct radiation in the axial direction from the source will be in the same phase and will tend to reinforce the central region of the reflected wave.

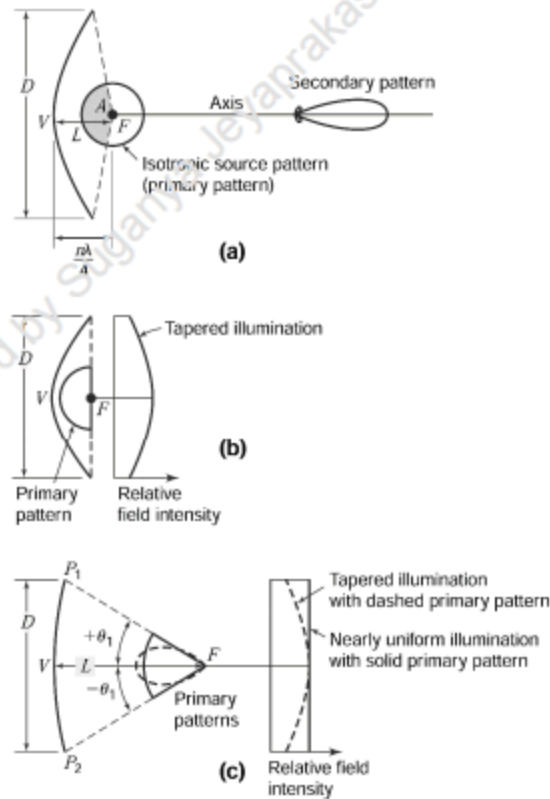


Figure 9-21 Parabolic reflectors of different focal lengths (L) and with sources of different patterns.

Direct radiation from the source can be eliminated by means of a directional source or primary antenna 1 as in Fig. 9-21*b* and *c*.

A primary antenna with the idealized hemispherical pattern shown in Fig. 9-21*b* (solid curve) results in a wave of uniform phase over the reflector aperture. However, the amplitude is tapered as indicated.

To obtain a more uniform aperture field distribution or illumination, it is necessary to make θ_1 small, as suggested in Fig. 9-21*c* by increasing the focal length L while keeping the reflector diameter D constant.

The arrangement of Fig. 9-21*b* illustrates the case of a small focal ratio. The arrangement in Fig. 9-21*c* illustrates the case of a larger focal ratio.

Suitable directional patterns may be obtained with various types of primary antennas.

As examples, a $\lambda/2$ antenna with a small ground plane is shown in Fig. 9-22*a* and a small horn antenna in Fig. 9-22*b*.

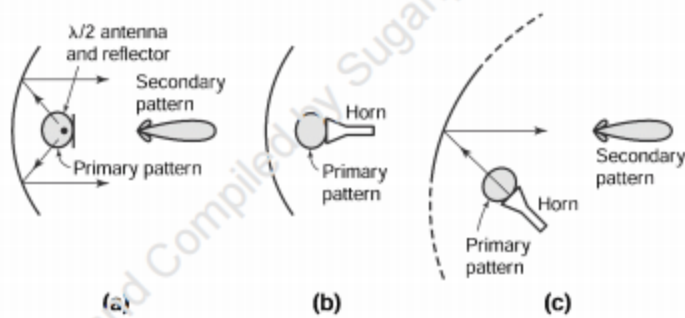


Figure 9-22 Full parabolic reflectors (*a* and *b*) and partial reflector with offset feed (*c*).