CBCS SCHEME

USN | | | | | | | | | | |

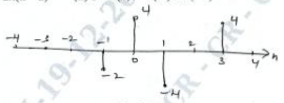
BEC502

Fifth Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026

Digital Signal Processing

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	A discrete time signal x(n) is shown below Fig.Q1(a). Sketch (i) $2x(n-2)$ (ii) $3-x(n)$ (iii) $2x(-n)-4$  Fig.Q1(a)	08	L3	CO1
	b.	Determine whether each of the following signals is periodic or not. If periodic find the fundamental period. (i) $x(n)=\sin(3n)$ (ii) $x(n)=\cos(0.3\pi n+\pi/4)$ (iii) $x(n)=\sin\left(\frac{7\pi n}{37}\right)$	06	L3	CO1
	c.	Write a program to generate the following discrete time signals: (i) Unit sample sequence (ii) Exponential sequence (iii) Random sequence	06	L3	CO1
		OR			
Q.2	a.	The following are the impulse response of discrete time LTI systems. Determine whether each system is memoryless, causal and stable. (i) $h(n)=e^{-n}\cos(n)*u(n)$ (ii) $h(n)=(0.99)^n*u(n+3)$ (iii) $h(n)=(1/2)^n*u(n)$	09	L3	CO2
	b.	Determine whether the following systems represented by impulse response are causal and stable: (i) $h(n)=5\,\delta(n)$ (ii) $h(n)=(1/4)^{\lvert n\rvert}$ (iii) $h(n)=(1/2)^{-n}u(-n)$	06	L3	CO1
	c.	Write a program to perform the following operation on signals: (i) Signal addition (ii) Signal multiplication (iii) Scaling (iv) Shifting (v) Folding	05	L3	CO1
		Module – 2			
Q.3	a.	Explain the frequency domain sampling of discrete time signals and obtain the DFT and IDFT expressions.	08	L2	CO3
	b.	Find the 4 point DFT of the sequence $x(n)=[1, 0, 0, 1]$ using matrix method and verify the answer by taking the 4-point IDFT of the result.	06	L3	CO3
	c.	Find the 4 point DFT of $x(n)=\cos\frac{\pi n}{4}+\sin\frac{\pi n}{4}$ using linearity property.	06	L3	CO3

BEC502

		OR	M	L	C
Q.4	a.	Show that the multiplication of two DFT's lead to circular convolution of the corresponding time sequences.	06	L2	CO3
	b.	Consider the finite N sequence $x(n)=\delta(n)+2\delta(n-5)$ Find (i) The 10 point DFT X(k) (ii) The sequence that has a DFT $Y(k)=e^{\frac{-j4\pi k}{10}}X(k)$ (iii) Find the 10 point sequence y(n) that has DFT $Y(k)=X(k)W(k)$ where X(k) is the 10 point DFT of x(n) and W(k) is the 10 point DFT of $w(n)=u(n)-u(n-7)$.	06	L3	CO3
	c.	Find the circular convolution of sequences $x_1(n)=[1, 2, 3, 1]$ and $x_2(n)=[4, 3, 2, 1]$ using time domain approach and verify the result using frequency domain approach.	08	L3	CO3
		Module – 3			
Q.5	a.	State and prove the following properties : (i) Circular time shift of a sequence (ii) Parseval's Theorem	06	L2	CO1
	b.	Find the output y(n) of a filter whose impulse response is $h(n)=[1, 1, 1]$ and the input signal $x(n)=[3, -1, 0, 1, 3, 2, 0, 1, 2, 1]$ using overlap save method. Assume the length of each block N is 5.	07	L3	CO3
	c.	Given $x(n)=[1, 2, 3, 4, 4, 3, 2, 1]$. Find X(k) using Radix – 2 DIT-FFT Algorithm.	07	L3	CO3
		OR			
Q.6	a.	Derive the radix – 2 DIT-FFT algorithm and draw the signal flow graph for N = 8.	08	L2	CO3
	b.	Consider a FIR filter with impulse response $h(n)=[3, 2, 1, 1]$ .If the input is $x(n)=[1, 2, 3, 3, 2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1]$, find the output y(n). Use overlap add method assuming the length of the block is 7.	07	L3	CO3
	c.	A length 8 sequence $x(n)=[-4, 5, 2, -3, 0, -2, 3, 4]$ with 8-point DFT given ny X(k). Determine the sequence y(n) whose 8-point DFT is given by $Y(k)=W_4^{3k}X(k)$.	05	L3	CO3
		Module – 4			
Q.7	a.	A low pass filter is to be designed for the desired frequency response $H_d(e^{jw})=H_d(w)=\begin{cases}e^{-j2w} & \lvert w\rvert<\pi/4\\ 0 & \pi/4<\lvert w\rvert<\pi\end{cases}$ Determine the filter coefficients $h_d(n)$ and h(n) if rectangular window is used. Also find the frequency H(w) of the resulting FIR filter.	10	L3	CO4
	b.	Determine the Direct form realization of the system function $H(z)=1+2z^{-1}-3z^{-2}+4z^{-3}+5z^{-4}$	04	L3	CO4
	c.	Write a program to design digital low pass FIR filter using a window.	06	L3	CO4

BEC502

		OR	M	L	C
Q.8	a.	Design a FIR filter with desired frequency response $H_d(e^{jw})=\begin{cases}e^{-j4w} & -\pi/4\le w\le\pi/4\\ 0 & \pi/4\le\lvert w\rvert\le\pi\end{cases}$ Find filter specifications and transfer function using Bartlett window.	10	L3	CO4
	b.	Realize the system function in cascade form $H(z)=1+\frac{5}{2}z^{-1}+2z^{-2}+2z^{-3}$	04	L3	CO4
	c.	Write a program to design digital high pass FIR filter using a window.	06	L3	CO4
		Module – 5			
Q.9	a.	Design an analog Butterworth lowpass filter that has – 2dB or better (ie., lesser than – 2 dB) at frequency of 20 rad/sec and atleast – 10 dB of attenuation at 30 rad/sec.	10	L3	CO5
	b.	Obtain the direct form – I and direct form – II structure for the filter given by system function $H(z)=\frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$	04	L3	CO5
	c.	Write a program to design digital IIR Butterworth low pass filter.	06	L3	CO5
		OR			
Q.10	a.	Design a digital Butterworth lowpass filter with frequency specifications given by (i) Passband ≤ 3.01 dB (ii) Passband edge frequency : 500 Hz (iii) Stopband attenuation ≥ 15 dB (iv) Stopband edge frequency : 750 Hz (v) Sampling rate f_s = 2 KHz Use Bilinear transformation metod.	10	L3	CO5
	b.	A filter is given by the difference equation $y(n)-\frac{1}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)+\frac{1}{2}x(n-2)$ Draw direct form – I and direct form – II realizations.	04	L3	CO5
	c.	Write a program to design digital IIR Butterworth high pass filter.	06	L3	CO5

* * * * *

Digital Signal Processing

VTU SOLUTIONS DEC 2025-JAN 2026

1.

		Module – 1	M	L	C
Q.1	a.	A discrete time signal x(n) is shown below Fig.Q1(a). Sketch (i) 2x(n – 2) (ii) 3 – x(n) (iii) 2x(– n) – 4	08	L3	CO1

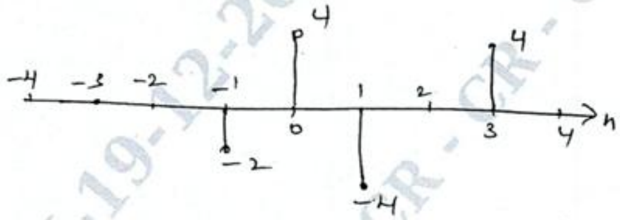


Fig.Q1(a)

We are given:

$$x[n] = \{0, 0, 0, -2, 4, -4, 0, 4, 0\}$$

From:

$$n = -4 \text{ to } 4$$

Let us write it clearly in tabular form.

📊 Original Signal Representation

n	-4	-3	-2	-1	0	1	2
x[n]	0	0	[illegible]	-2	4	-4	0

✅ (i) Find 2x(n−2)

🔹 Step 1: Understand x(n−2)

This means:

➡️ **Shift right by 2 units**

Because:

$$x(n-k) \Rightarrow \text{Right Shift by } k$$

So every sample moves **2 steps right**.

✅ Final Answer (i)

$$2x(n-2) = \{0, 0, 0, -4, 8, -8, 0, 8, 0\}$$

for

$$n = -2 \text{ to } 6$$

✅ (ii) Find 3 − x(n)

This is an amplitude operation only.

No shifting.

🔹 Step 1: Subtract each value from 3

$$y[n] = 3 - x[n]$$

n	-4	-3	-2	-1	0	1	2	3	4
x[n]	0	0	0	-2	4	-4	0	4	0
3 - x[n]	3	3	3	5	-1	7	3	-1	3

✅ Final Answer (ii)

$$3 - x(n) = \{3, 3, 3, 5, -1, 7, 3, -1, 3\}$$

✅ (iii) Find 2x(-n) − 4

This involves:

1️⃣ Time reversal
2️⃣ Amplitude scaling
3️⃣ Constant shift

✅ Final Answer (iii)

$$2x(-n) - 4 = \{-4, 4, -4, -12, 4, -8, -4, -4, -4\}$$

for

$$n = -4 \text{ to } 4$$

	b.	Determine whether each of the following signals is periodic or not. If periodic find the fundamental period. (i) $x(n) = \sin(3n)$ (ii) $x(n) = \cos(0.3\pi n + \pi/4)$ (iii) $x(n) = \sin\left(\frac{7\pi n}{37}\right)$	06	L3	CO1

Sol:

✅ (i) $x[n] = \sin(3n)$

Here:

$$\omega_0 = 3$$

We check:

$$\frac{\omega_0}{2\pi} = \frac{3}{2\pi}$$

Since π is irrational,

$$\frac{3}{2\pi}$$

is irrational.

👉 Therefore it **cannot be written as a rational number**.

❌ Hence:

Signal is **NOT periodic**.

✅ (ii) $x[n] = \cos(0.3\pi n + \pi/4)$

Here:

$$\omega_0 = 0.3\pi$$

Ignore phase $\pi/4$ (it does NOT affect periodicity).

Step 1: Apply periodicity condition

$$\omega_0 N = 2\pi k$$

$$0.3\pi N = 2\pi k$$

Cancel π:

$$0.3N = 2k$$

$$\frac{3}{10}N = 2k$$

Multiply both sides by 10:

Step 2: Find smallest integer solution

We want smallest N.

Since:

$$N = \frac{20k}{3}$$

To make N integer:

Choose $k = 3$

$$N = \frac{20 \times 3}{3}$$

$$N = 20$$

✅ Therefore:

Signal is **periodic**

Fundamental period:

$$\boxed{N = 20}$$

✅ (iii) $x[n] = \sin\left(\frac{7\pi n}{37}\right)$

Here:

$$\omega_0 = \frac{7\pi}{37}$$

Step 1: Apply periodicity condition

$$\omega_0 N = 2\pi k$$

$$\frac{7\pi}{37} N = 2\pi k$$

Cancel π:

$$\frac{7}{37} N = 2k$$

Multiply both sides by 37:

$$7N = 74k$$

$$N = \frac{74k}{7}$$

Step 2: Smallest integer solution

To make N integer:

Choose $k = 7$

$$N = \frac{74 \times 7}{7}$$

$$N = 74$$

✅ Therefore:

Signal is **periodic**

Fundamental period:

$$\boxed{N = 74}$$

	c.	Write a program to generate the following discrete time signals: (i) Unit sample sequence (ii) Exponential sequence (iii) Random sequence	06	L3	CO1

Sol:

clc

clear all

close all

n=0:9 %Discrete time index

%Unit Sample Sequence (Unit Impulse)

x1=[1,zeros(1,9)]

subplot(2,1,1)

stem(n,x1)

grid on

xlabel('Discrete Time')

ylabel('Amplitude')

title('Unit Impulse')

%Exponential Sequence

a=2

x3=a.^n

figure

subplot(2,1,1)

stem(n,x3)

grid on

xlabel('Discrete Time')

ylabel('Amplitude')

title('Exponential Sequence')

%Random Sequence

x5=5*rand(1,10)

figure

stem(n,x5)

grid on

xlabel('Discrete Time')

ylabel('Amplitude')

title('Random Sequence')

2.

Q.2	**a.**	The following are the impulse response of discrete time LT1 systems. Determine whether each system is memoryless, causal and stable. (i) $h(n) = e^{-n}\cos(n) * u(n)$ (ii) $h(n) = (0.99)^n * u(n+3)$ (iii) $h(n) = (1/2)^n * u(n)$	**09**	**L3**	**CO2**

Sol:

a) $h[n] = (e)^{-n}\cos n\, u[n]$

$$h(n) = \begin{cases} (e)^{-n}\cos n, & n \geq 0 \\ 0, & otherwise \end{cases}$$

The given impulse response $h[n] = 0$ for $n < 0$

Hence the system is **causal**.

$h(n)$ is not form $c\delta(n)$, Hence the system is **not memoryless**.

$$\sum_{k=0}^{\infty} |h(k) < \infty|$$

In the given $h(n)$, due to decaying exponential term the given DT system is **stable.**

b) $h[n] = 0.999\, u[n+3]$

$$h(n) = \begin{cases} 0.999, & n \geq -3 \\ 0, & otherwise \end{cases}$$

The given impulse response $h[n] \neq 0$ for $n < 0$

Hence the system is **non causal.**

$h(n)$ is not form $c\delta(n)$, Hence the system is **not memoryless**.

$$\sum_{k=0}^{\infty} |h(k)| < \infty$$

$$= 0.999 + 0.999 + 0.9999 + \cdots \ldots \ldots \ldots = \infty$$

In the given $h(n)$, due to decaying exponential term the given DT system is **unstable.**

c) $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & otherwise \end{cases}$$

The given impulse response $h[n] = 0$ for $n < 0$

Hence the system is **causal.**

$h(n)$ is not form $c\delta(n)$, Hence the system is **not memoryless**.

$$\sum_{k=0}^{\infty} |h(k)| < \infty$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + . \ . \ . \ . < \infty$$

In the given $h(n)$, due to decaying exponential term the given DT system is **stable.**

b.

b.	Determine whether the following systems represented by impulse response are causal and stable: (i) $h(n) = 5\,\delta(n)$ (ii) $h(n) = (1/4)^{\lvert n \rvert}$ (iii) $h(n) = (1/2)^{-n}u(-n)$

Sol:

a) $h(n) = 5\delta(n)$
   - **Causal and Stable**

b) $h(n) = \left(\frac{1}{4}\right)^{|n|}$

$$h(n) = \begin{cases} \left(\frac{1}{4}\right)^n for\ n \geq 0 \\ \left(\frac{1}{4}\right)^{-n} for\ n < 0 \end{cases}$$

   - **Non-causal and unstable**

c) $h(n) = \left(\frac{1}{2}\right)^{-n} u(-n)$

   - **Non-causal and stable**

$$S = \sum_{k=-\infty}^{0} |h(k)| = \sum_{k=-\infty}^{0} 2^n$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + . \ . \ . \ . < \infty$$

c.

	c.	Write a program to perform the following operation on signals: (i) Signal addition (ii) Signal multiplication (iii) Scaling (iv) Shifting (v) Folding	05	L3	CO1

%Matlab program to simulate addition, multiplication of signals

```
clc
clear all
close all

x1=input('Enter the samples of first signal')
x2=input('Enter the samples of second signal')

if length(x1)>length(x2)
   x2=[x2,zeros(1,length(x1)-length(x2))]
end

if length(x2)>length(x1)
   x1=[x1,zeros(1,length(x2)-length(x1))]
end

%Addition of signals
x3=x1+x2

%Multiplication of signals
x4=x1.*x2

n=0:length(x4)-1

subplot(2,1,1)
stem(n,x1)
grid on
```

```
xlabel('Discrete Time')
ylabel('Amplitude')
title('First Signal')

subplot(2,1,2)
stem(n,x2)
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Second Signal')

figure
subplot(2,1,1)
stem(n,x3)
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Sum Signal')

subplot(2,1,2)
stem(n,x4)
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Product Signal')
```

%Matlab program to simulate amplitude scaling & time scaling

```
clc
clear all
close all
```

```
%Original Signal
x=[1,2,3,4,5,6,7]
n=0:length(x)-1
subplot(2,1,1)
stem(n,x)
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Original Signal')

%Amplitude Scaling
k=2 %Scaling Factor
y1=k*x
subplot(2,1,2)
stem(n,y1)
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Amplitude Scaled Signal')

%Time Scaling (Compression)
k=3 %Compression Factor
y2=zeros(1,length(x))
for n =0:length(x)-1
    if (k*n+1)<=length(x)
        y2(n+1)=x(k*n+1)
    end
end
```

```
%Plotting the original signal
figure
n=0:length(x)-1
subplot(3,1,1)
stem(n,x)
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Original Signal')

%Plotting the compressed signal
n=0:length(y2)-1
subplot(3,1,2)
stem(n,y2)
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Compressed Signal')

%Time Scaling (Expansion)
k=3 %Expansion Factor
y3=zeros(1,k*length(x))
for n =0:length(y3)-1
    if mod(n,k)==0
        y3(n+1)=x(n/k+1)
    end
end
n=0:length(y3)-1
subplot(3,1,3)
stem(n,y3)
```

```
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Expanded Signal')
```

%Matlab program to simulate shifting of signals

```
clc
clear all
close all

%Original Signal
x=[1,2,3,4,5,6]
n=0:length(x)-1
subplot(3,1,1)
stem(n,x)
xlim([-4,8])%To limit x axis from -4 to 8
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Original Signal')

%Right Shift
n1=n+2
subplot(3,1,2)
stem(n1,x)
xlim([-4,8])%To limit x axis from -4 to 8
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Right Shifted Signal')
```

```
%Left Shift
n2=n-2
subplot(3,1,3)
stem(n2,x)
xlim([-4,8])%To limit x axis from -4 to 8
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Left Shifted Signal')
```

%Matlab program to simulate time reversal or reflection or folding of signals

```
clc
clear all
close all

%Original Signal
x=[1,2,3,4]
n=0:length(x)-1
subplot(3,1,1)
stem(n,x)
xlim([-4,4]) %To limit x axis from -4 to 4
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Original Signal')

%Reflection
n1=-n
```

```
subplot(2,1,2)
stem(n1,x)
xlim([-4,4]) %To limit x axis from -4 to 4
grid on
xlabel('Discrete Time')
ylabel('Amplitude')
title('Reflected Signal')
```

Module – 2						
Q.3	a.	Explain the frequency domain sampling of discrete time signals and obtain the DFT and IDFT expressions.	08	L2	CO3

Sol:

We consider sampling of the Fourier Transform (DTFT) of an aperiodic discrete time sequence $x(n)$. We establish the relationship between the sampled Fourier Transform and DTFT.

The Fourier Transform of an aperiodic discrete time signal $x(n)$ is

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{j\omega n} \quad ----------> \quad (1)$$

- $X(\omega)$ is periodic with a period of 2π.
- Recall aperiodic finite energy signals have continuous spectra.

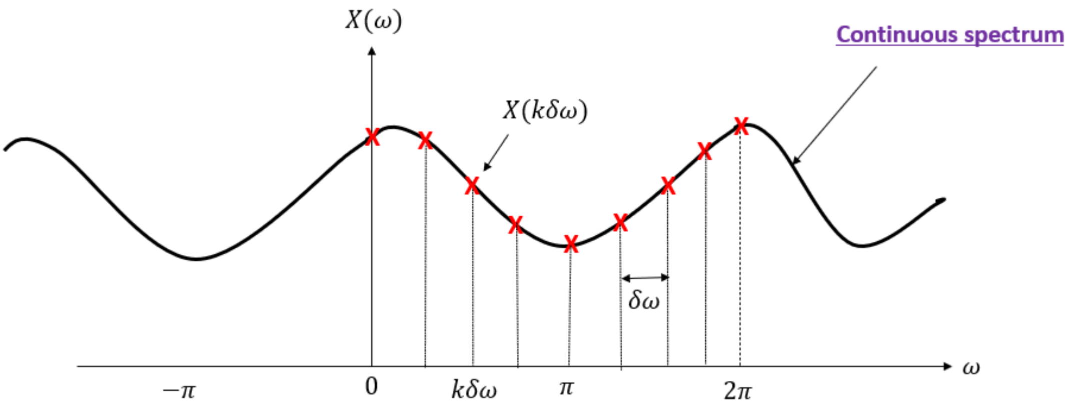


Fig 1: Frequency-domain sampling of the Fourier Transform

N samples ⟷ 2π

1 sample ⟷ $\delta\omega = \frac{2\pi}{N}$ *(Spacing between successive samples)*

- $X(\omega)$ is periodically sampled at a frequency spacing of $\delta\omega$ radians between successive samples.

- The samples are necessary only in the fundamental frequency range.
- N equidistant samples in the interval $0 \leq \omega \leq 2\pi$ with spacing of $\boldsymbol{\delta\omega} = \frac{\mathbf{2\pi}}{\boldsymbol{N}}$
- N is the number of samples in the frequency domain.

If we evaluate (1) at $\omega = \frac{2\pi k}{N}$

$$X\left(\frac{2\pi k}{N}\right) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi kn}{N}}, \quad k = 0,1,2...(N-1) \qquad ------> \ (2)$$

This summation can be divided into infinite number of summations where each sum contains N terms.

$$X\left(\frac{2\pi k}{N}\right) = \ ...\sum_{n=-N}^{-1} x(n)\, e^{-j\frac{2\pi kn}{N}} + \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}} + \sum_{n=N}^{2N-1} x(n)\, e^{-j\frac{2\pi kn}{N}} + ...$$

$$X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j\frac{2\pi kn}{N}}$$

Put $m = n - lN$

when $n = lN$, $\quad m = 0$

when $n = lN + N - 1$, $\quad m = N - 1$

$$X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{m=0}^{N-1} x(m + lN)\, e^{-j\frac{2\pi k}{N}(m+lN)}$$

since $e^{-j2\pi kl} = 1$

Interchange the order of summation, we obtain the result

$$X\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{N-1} \underbrace{\left[\sum_{l=-\infty}^{\infty} x(n - lN)\right]}_{x_p(n)} e^{-j\frac{2\pi kn}{N}} \qquad ------> \ (3)$$

for $k = 0,1,2 ....., (N-1)$

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n - lN) \qquad ------> \ (4)$$

The signal $x_p(n)$ is periodic repetition of $x(n)$ every N samples. It can be expanded using Fourier Series.

$$x_p(n) = \sum_{k=0}^{N-1} c_k\, e^{j\frac{2\pi kn}{N}}, \qquad ------> \ (5)$$

$n = 0,1,2 ....., (N-1)$

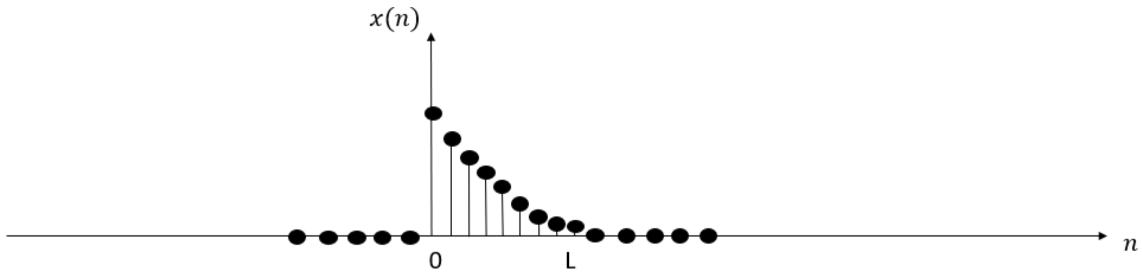


Fig a: Aperiodic sequence $x(n)$ of length L

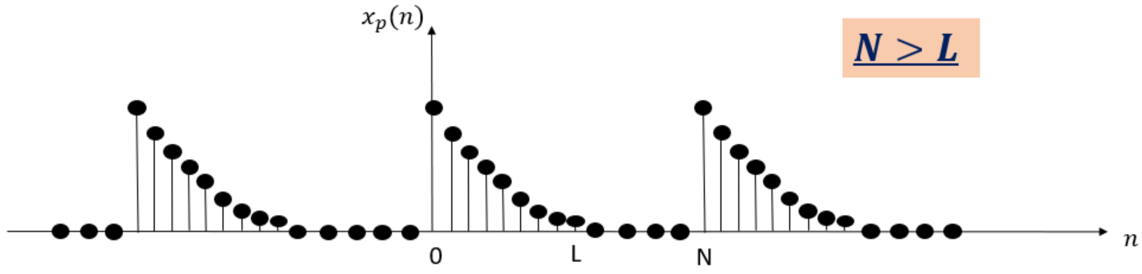


Fig b: Periodic extension for $N \geq L$ (no aliasing)

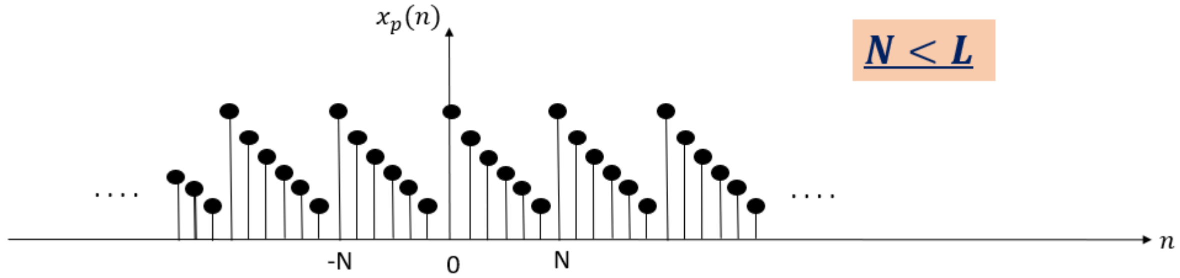


Fig b: Periodic extension for $N < L$ (aliasing)

Fourier coefficients,

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi kn}{N}}, \qquad -------> \ (6)$$

$$k = 0,1,2 \, . \; . \; . \; . \; , (N-1)$$

Comparing equations (3) and (6), we can conclude that

$$c_k = \frac{1}{N} X\left(\frac{2\pi k}{N}\right) \qquad ------> \ (7)$$

$$k = 0,1,2 \, . \; . \; . \; . \; , (N-1)$$

Hence,

$$\boldsymbol{x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi kn}{N}}} \qquad ------> \ (8)$$

$$\boldsymbol{n = 0, 1, 2 \, . \; . \; . \; . \; , (N-1)}$$

i. This relationship provides the reconstruction of periodic signal $x_p(n)$ from samples of the spectrum $X(\omega)$.

ii. We need to consider relationship between $x(n)$ and $x_p(n)$.

$$x(n) = x_p(n) \ , 0 \le n \le N-1$$

iii. $x(n)$ can be recovered from $x_p(n)$ if there is no time aliasing.

iv. The spectrum of an aperiodic discrete time signal with finite duration L can be recovered from its samples at frequencies $\omega_k = \frac{2\pi k}{N}$ if $N \ge L$.

$$\boldsymbol{x(n)} = \begin{cases} \boldsymbol{x_p(n)}, & \boldsymbol{0 \le n \le N-1} \\ \boldsymbol{0}, & \boldsymbol{elsewhere} \end{cases} \qquad ----->\ (9)$$

v. It is possible to express the spectrum $X(\omega)$ directly in terms of its samples $X\left(\frac{2\pi}{N}k\right)$, $k = 0,1....(N-1)$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right) e^{j\frac{2\pi kn}{N}}, \qquad 0 \le n \le (N-1) \qquad ----->\ (10)$$

$3b.$

	b.	Find the 4 point DFT of the sequence x(n) = [1, 0, 0, 1] using matrix method and verify the answer by taking the 4-point IDFT of the result.	06	L3	CO3

Sol:

$$4 \text{ –point DFT}, \quad \boldsymbol{X}_k = \boldsymbol{W}_4 \boldsymbol{x}_4$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 2 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$

$$\boldsymbol{X(k) = \{2, 1+j, 0, 1-j\}}$$

c.

	c.	Find the 4 point DFT of $x(n) = \cos\frac{\pi n}{4} + \sin\frac{\pi n}{4}$ using linearity property.	06	L3	CO3

Sol:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}, \qquad k = 0,1, \dots .. N-1 \qquad ---->\ (1)$$

$$x(n) = \cos\left(\frac{\pi n}{4}\right) + \sin\left(\frac{\pi n}{4}\right)$$

Applying linearity property, we get

$$X(k) = X_1(k) + X_2(k)$$

We get $X_1(k)$ and $X_2(k)$ and adding the results to get final DFT $X(k)$.

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0.7071 \\ 0 \\ -0.7071 \end{bmatrix}$$

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - j1.142 \\ 1 \\ 1 + j1.142 \end{bmatrix}$$

Similarly

$$\begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 0.7071 \\ 1 \\ 0.7071 \end{bmatrix}$$

$$\begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} 2.4142 \\ -1 \\ -0.4142 \\ -1 \end{bmatrix}$$

$$X(k) = X_1(k) + X_2(k)$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} + \begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 3.4142 \\ -j1.4142 \\ 0.58 \\ j1.4142 \end{bmatrix}$$

4.

Q.4	**a.**	Show that the multiplication of two DFT's lead to circular convolution of the corresponding time sequences.	**06**	**L2**	**CO3**

Sol:

If we have finite-duration sequences of length N, $x_1(n)$ and $x_2(n)$ and their DFTs are

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)e^{-j\frac{2\pi kn}{N}}, \qquad 0 \le k \le N-1 \qquad --------> (1)$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n)e^{-j\frac{2\pi kn}{N}}, \qquad 0 \le k \le N-1 \qquad --------> (2)$$

Let $X_3(k) = X_1(k).X_2(k)$, $0 \le k \le N-1$ $\qquad --------> (3)$

IDFT$\{X_3(k)\}$

$$x_3(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k)e^{j\frac{2\pi kn}{N}}$$

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_1(k)X_2(k)e^{j\frac{2\pi km}{N}}$$

Substitute for $X_1(k)$ and $X_2(k)$ using (1) and (2)

$$= \frac{1}{N}\sum_{k=0}^{N-1}\underbrace{\left[\sum_{n=0}^{N-1} x_1(n)e^{-j\frac{2\pi kn}{N}}\right]}_{X_1(k)}\underbrace{\left[\sum_{l=0}^{N-1} x_2(l)e^{-j\frac{2\pi kl}{N}}\right]}_{X_2(k)}e^{j\frac{2\pi km}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x_1(n)\sum_{l=0}^{N-1} x_2(l)\left[\sum_{k=0}^{N-1} e^{j\frac{2\pi k}{N}(m-n-l)}\right] \qquad --------> (4)$$

with $a = e^{j\frac{2\pi}{N}(m-n-l)}$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & a = 1 \\ \dfrac{1-a^N}{1-a}, & a \ne 1 \end{cases}$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & l = m-n+pN = ((m-n))_N, \quad p\ an\ integer \\ 0, & otherwise \end{cases}$$

$$--------> (5)$$

Substitute (5) in (4)

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)x_2((m-n))_N\ , \quad m = 0,1,\dots.(N-1) \qquad --------> (6)$$

This expression has a form of convolution sum.

The convolution sum in (6) involves the index $((m-n))_N$ is called ***circular convolution***.

Thus we can conclude that multiplication of the DFTs of two sequences is equivalent to circular convolution of two sequences in the time domain.

b.

b.	Consider the finite N sequence $x(n) = \delta(n) + 2\delta(n-5)$ Find (i) The 10 point DFT X(k) (ii) The sequence that has a DFT $Y(k) = e^{\frac{-j4\pi k}{10}} X(k)$ (iii) Find the 10 point sequence y(n) that has DFT Y(k) = X(k)W(k) where X(k) is the 10 point DFT of x(n) and W(k) is the 10 point DFT of w(n) = u(n) – u(n – 7).	**06**	**L3**	**CO3**

Sol:

$$x(n) = \{1,0,0,0,0,2,0,0,0,0\}$$

a)

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}, \qquad k = 0,1,\ldots\ldots N-1 \qquad ----> \quad (1)$$

$$X(k) = 1 + 2W_{10}^{5k}, \qquad k = 0,1,\ldots\ldots 9$$

$$X(k) = 1 + (-1)^k, \quad k = 0,1,\ldots\ldots 9$$

$$X(k) = \{2,-2,2,-2,2,-2,2,-2,2,-2\}$$

b)

$$Y(k) = e^{-j\frac{4\pi k}{10}} X(k)$$

$$Y(k) = e^{-j\frac{2\pi}{10}2k} X(k) = W_{10}^{2k} X(k)$$

Using circular frequency shift property,

$$y(n) = x((n-2))_{10}$$

$$y(n) = \{0,0,1,0,0,0,0,2,0,0\}$$

c)

$$Y(k) = X(k)W(k)$$

$$w(n) = u(n) - u(n-7)$$

$$w(n) = \{1,1,1,1,1,1,1,0,0,0\}$$

$$X(k) = 1 + 2W_{10}^{5k}, \qquad W(k) = 1 + W_{10}^{k} + W_{10}^{2k} + W_{10}^{3k} + W_{10}^{4k} + W_{10}^{5k} + W_{10}^{6k}$$

Multiply the DFTs and apply inverse DFT to find the sequence $y(n)$

$$y(n) = \{3,3,1,1,1,3,3,2,2,2\}$$

c.

	c.	Find the circular convolution of sequences $x_1(n) = [1, 2, 3, 1]$ and $x_2(n) = [4, 3, 2, 1]$ using time domain approach and verify the result using frequency domain approach.	08	L3	CO3

Sol:

$$x_1(n) = \{1,2,3,1\}, \qquad x_2(n) = \{4,3,2,1\}$$

The circular convolution is given by

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n) x_2\big((m-n)\big)_N \ , \ \ m = 0,1, \dots .(N-1)$$

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix} = \begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix}$$

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix} = \begin{bmatrix} 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$x_3(n) = \{15,16,21,18\}$$

		Module – 3			
Q.5	a.	State and prove the following properties : (i) Circular time shift of a sequence (ii) Parseval's Theorem	06	L2	CO1

Sol:

If

$$x(n) \xleftrightarrow[N]{DFT} X(k)$$

then

$$x((n-l))_N \xleftrightarrow[N]{DFT} X(k) e^{-j\frac{2\pi}{N}kl}$$

Proof:

$$X(k) = DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$DFT\{x((n-l))_N\} = \sum_{n=0}^{N-1} x((n-l))_N \, e^{-j\frac{2\pi kn}{N}}$$

$$= \sum_{n=0}^{l-1} x((n-l))_N \, e^{-j\frac{2\pi kn}{N}} + \sum_{n=l}^{N-1} x((n-l))_N \, e^{-j\frac{2\pi kn}{N}} \; -------> (1)$$

we can write $x((n-l))_N = x(N-l+n)$

$$\sum_{n=0}^{l-1} x((n-l))_N \, e^{-j\frac{2\pi kn}{N}} = \sum_{n=0}^{l-1} x(N-l+n) \, e^{-j\frac{2\pi kn}{N}}$$

Put $m = N-l+n$

$$= \sum_{n=N-l}^{N-1} x(m) \, e^{-j\frac{2\pi k(m+l-N)}{N}}$$

$$= \sum_{m=N-l}^{N-1} x(m) \, e^{-j\frac{2\pi k(m+l)}{N}} \; ---------> (2)$$

Furthermore,

$$\sum_{n=l}^{N-1} x((n-l))_N \, e^{-j\frac{2\pi kn}{N}} = \sum_{n=l}^{N-1} x(n-l) \, e^{-j\frac{2\pi kn}{N}}$$

Put $m = n-l$

$$= \sum_{n=0}^{N-1-l} x(m) \, e^{-j\frac{2\pi k(m+l)}{N}} \quad -------> (3)$$

Substituting (2) and (3) in (1), we get

$$DFT\{x((n-l))_N\} = \sum_{m=N-l}^{N-1} x(m) \, e^{-j\frac{2\pi k(m+l)}{N}} + \sum_{n=0}^{N-1-l} x(m) \, e^{-j\frac{2\pi k(m+l)}{N}}$$

$$= \sum_{m=0}^{N-1} x(m) \, e^{-j\frac{2\pi k(m+l)}{N}}$$

$$DFT\{x((n-l))_N\} = e^{-j\frac{2\pi kl}{N}} \sum_{m=0}^{N-1} x(m) \, e^{-j\frac{2\pi km}{N}}$$

$$= \boldsymbol{e^{-j\frac{2\pi kl}{N}} X(k)}$$

<u>Parseval's Theorem:</u>

If

$$x(n) \xleftrightarrow[N]{DFT} X(k)$$

and

$$y(n) \xleftrightarrow[N]{DFT} Y(k)$$

then

$$\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$$

Proof:
From Cross-correlation property,

$$\tilde{r}_{xy}(l) = \sum_{n=0}^{N-1} x(n)y^*\big((n-l)\big)_N$$

When $l = 0$,

$$\tilde{r}_{xy}(0) = \sum_{n=0}^{N-1} x(n)y^*(n) \qquad ----------->(1)$$

$$\tilde{r}_{xy}(l) \xleftrightarrow[N]{DFT} \tilde{R}_{xy}(k)$$

$$\tilde{r}_{xy}(l) = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{R}_{xy}(k)\, e^{j\frac{2\pi kl}{N}}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)\, e^{j\frac{2\pi kl}{N}}$$

Evaluating IDFT at $l = 0$

$$\tilde{r}_{xy}(0) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k) \qquad ---------->(2)$$

Comparing (1) and (2), we get

$$\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$$

Hence Proved
when $y(n) = x(n)$,

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2$$

It can be seen that DFT can be used to compute the energy of a discrete time sequence $x(n)$.

b.

	b.	Find the output y(n) of a filter whose impulse response is h(n) = [1, 1, 1] and the input signal x(n) = [3, –1, 0, 1, 3, 2, 0, 1, 2, 1] using overlap save method. Assume the length of each block N is 5.	**07**	**L3**	**CO3**

Sol:

$$x(n) = \{3,-1,0,1,3,2,0,1,2,1\}$$
$$h(n) = \{1,1,1\}$$

Length of impulse response is $M = 3$
Given $N = 5$

$$N = L + M - 1,$$
$$5 = L + 3 - 1,$$
$$\boldsymbol{L = 3}$$

Forming the block of sequences each of length N by taking L samples from $x(n)$ each time.

$x_1(n)$ contains $(M-1)$ zeros in the beginning and L samples are chosen from $x(n)$

$$x_1(n) = \{\underbrace{0,\ 0,}_{\substack{M-1\\ Zeros}}\ \underbrace{3,-1,0\}}_{\substack{L\ samples\\ from\ x(n)}}$$

$$x_2(n) = \{\underbrace{-1,\ 0,}_{\substack{Last\\ M-1\\ samples\\ from\\ x_1(n)}}\ \underbrace{1,3,2\}}_{\substack{next\ L\ samples\\ from\ x(n)}}$$

$$x_3(n) = \{\underbrace{3,2,}_{\substack{Last\\ M-1\\ samples\\ from\\ x_1(n)}}\ \underbrace{0,1,2\}}_{\substack{next\ L\ samples\\ from\ x(n)}}$$

$$x_4(n) = \{\underbrace{1,\ 2,}_{\substack{Last\\ M-1\\ samples\\ from\\ x_1(n)}}\ \underbrace{1,0,0\}}_{\substack{next\ L\ samples\\ from\ x(n)}}$$

<u>**Note**</u>: If we make one more block it will have all zeros. Hence $x_4(n)$ not needed.

$h(n)$ has $\mathrm{M} = 3$ samples. Pad $(\mathrm{L}-1) = 2$ zeros to increase its length to $N = 5$.

$$h(n) = \{1,1,1\ 0,\ 0\}$$

Obtain the output blocks by performing 5 −point circular convolution.

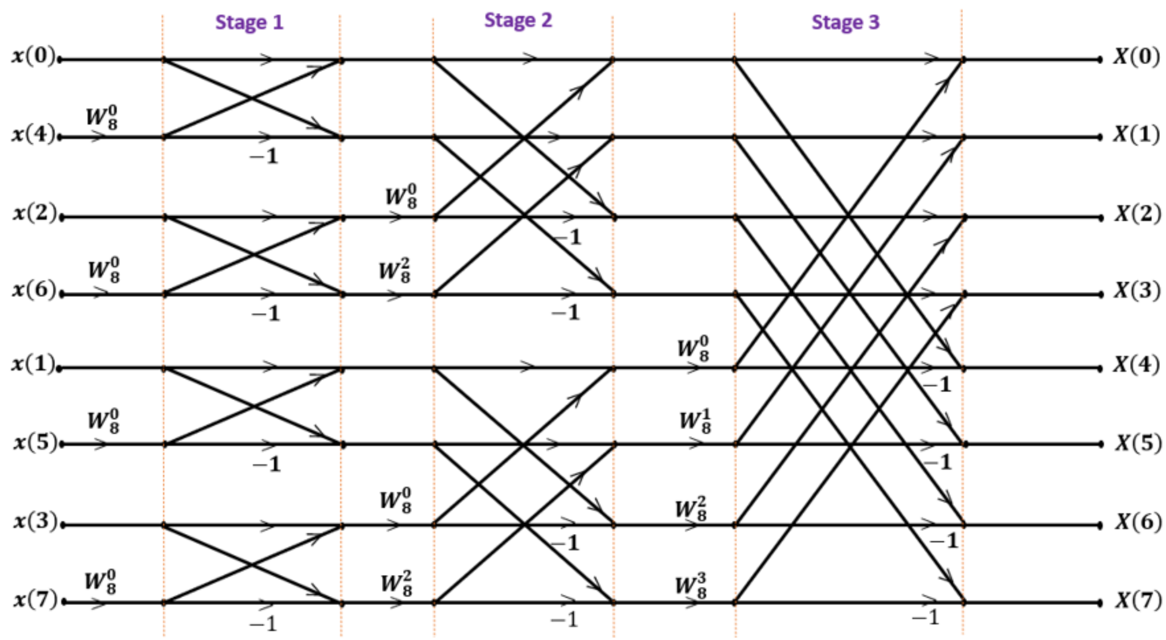
$$y_1(n) = x_1(n) \circledast h(n)$$
$$y_1(n) = \{-1,0,3,2,2\}$$
$$y_2(n) = \{4,1,0,4,6\}$$
$$y_3(n) = \{-1,0,3,2,2\}$$
$$y_4(n) = \{1,3,4,3,1\}$$

The final output is

$$\boldsymbol{y(n) = \{3, 2, 2, 0, 4, 6, 3, 2, 2, 4, 3, 1\}}$$

	c.	Given x(n) = [1, 2, 3, 4, 4, 3, 2, 1]. Find X(k) using Radix – 2 DIT-FFT Algorithm.	07	L3	CO3

Sol:



Using this flowchart, the DFT is obtained as

$$X(k) = \{20,\ -5.828 - j2.414,\ 0,\ -0.172 - j0.414,\ 0, -0.172 + j0.414,\ 0,\ -5.828 + j2.414\}$$

Q.6	a.	Derive the radix – 2 DIT-FFT algorithm and draw the signal flow graph for N = 8.	08	L2	CO3

Sol:

1. When N is highly composite that is when $N = r_1 r_2 \ldots\ldots r_v$ where $\{r_j\}$ are prime.
2. When $r_1 r_2 \ldots\ldots r_v = r$ so that $N = r^v$. The DFTs are of size r, so that the computation of DFT has regular pattern.
3. The number r is called the radix of the FFT algorithm.
4. Let us consider the computation of $N = 2^v$ point DFT by divide and conquer approach.
5. Split the N −point data sequence into $\frac{N}{2}$ point data sequences $f_1(n)$ and $f_2(n)$ corresponding to even-numbered and odd-numbered samples of $x(n)$.

$$f_1(n) = x(2n)$$
$$f_2(n) = x(2n+1), \qquad ------> (1)$$
$$n = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{2} - 1$$

$f_1(n)$ and $f_2(n)$ are obtained by decimating $x(n)$ by a factor of 2 and hence the resulting FFT algorithm is called a **<u>"Decimation-in-time-algorithm"</u>**.

N −point DFT can be expressed in terms of DFTs of the decimated sequence as follows:

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, \ \ k = 0,1,\ldots \ldots (N-1)$$

$$= \sum_{n\ even} x(n) W_N^{kn} + \sum_{n\ odd} x(n) W_N^{kn} \qquad -------->(2)$$

$$= \sum_{m=0}^{\frac{N}{2}-1} x(2m) W_N^{2km} + \sum_{m=0}^{\frac{N}{2}-1} x(2m+1) W_N^{k(2m+1)}$$

But $W_N^2 = W_{\frac{N}{2}}$

With this substitution (2) can be expressed as

$$X(k) = \sum_{m=0}^{\frac{N}{2}-1} f_1(m) W_{N/2}^{km} + W_N^k \sum_{m=0}^{\frac{N}{2}-1} f_2(m) W_{N/2}^{km}$$

$$= F_1(k) + W_N^k F_2(k), \ \ k = 0,\ 1,\ \ldots\ldots\ldots\ldots N-1 \qquad ---->(3)$$

$F_1(k)$ and $F_2(k)$ are $\frac{N}{2}$ –point DFTs of the sequences $f_1(m)$ and $f_2(m)$ respectively

$F_1(k)$ and $F_2(k)$ are periodic with period $\frac{N}{2}$.

$$F_1\left(k + \frac{N}{2}\right) = F_1(k),$$

$$F_2\left(k + \frac{N}{2}\right) = F_2(k),$$

$$X(k) = F_1(k) + W_N^k F_2(k), \ \ k = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{2}-1 \qquad ---------->(4)$$

$$X\left(k + \frac{N}{2}\right) = F_1(k) - W_N^k F_2(k), \ \ k = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{2}-1 \qquad ------->(5)$$

Direct computation of $F_1(k)$ requires $\left(\frac{N}{2}\right)^2$ complex multiplications.

The same applies to the computation of $F_2(k)$.

$\frac{N}{2}$ additional complex multiplications required for the computation of $W_N^k F_2(k)$.

Computation of $X(k)$ requires $2\left(\frac{N}{2}\right)^2 + \frac{N}{2} = \frac{N^2}{2} + \frac{N}{2}$ Complex multiplications

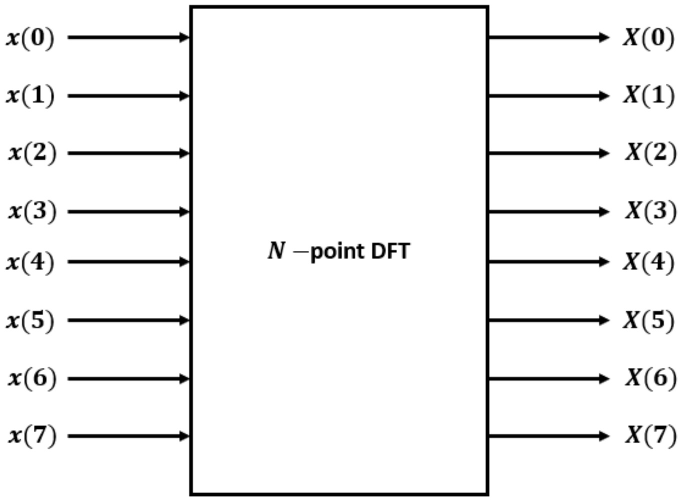


Fig 1: Computation of N −point DFT (for $N = 8$)

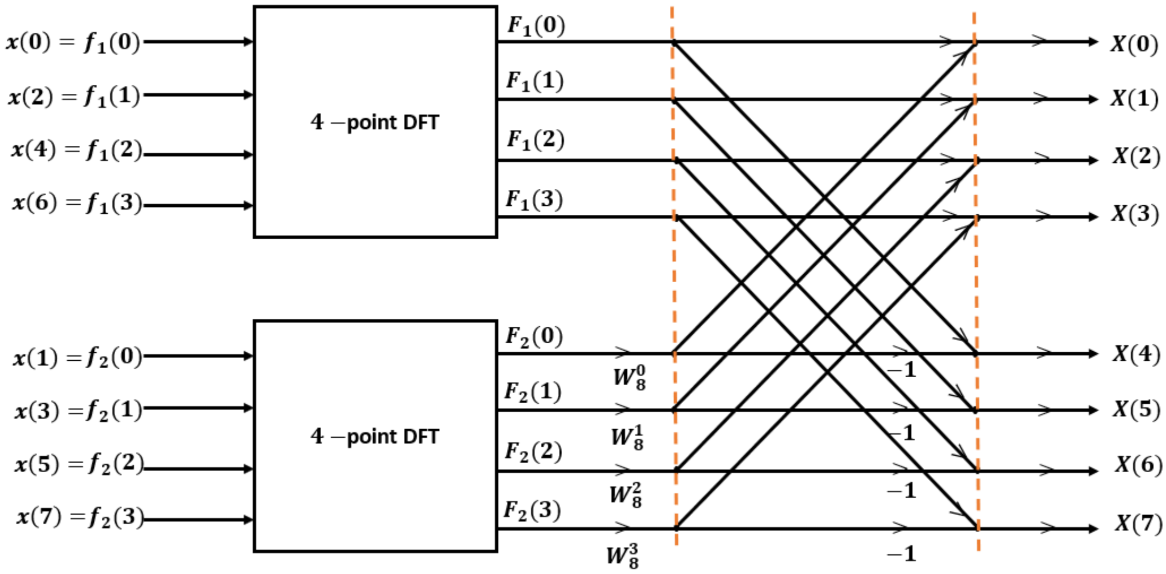


Fig 2: Decimation in time FFT algorithm for N = 8 (Stage 1)

Having performed the decimation-in-time once, we can repeat the process for each of the sequences $f_1(n)$ and $f_2(n)$. Thus $f_1(n)$ results in two $\frac{N}{4}$ point sequences.

$$v_{11}(n) = f_1(2n), \ \ n = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{4} - 1$$

$$v_{12}(n) = f_1(2n+1), \ n = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{4} - 1 \quad -----> (6)$$

$f_2(n)$ would yield

$$v_{21}(n) = f_2(2n), \ \ n = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{4} - 1$$

$$v_{22}(n) = f_2(2n+1), \ n = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{4} - 1 \quad -----> (7)$$

By computing $\frac{N}{4}$ −point DFTs, we would obtain $\frac{N}{2}$ −point DFTs $F_1(k)$ and $F_2(k)$ from the relations

$$F_1(k) = V_{11}(k) + W_{N/2}^{k} V_{12}(k), \quad k = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{4} - 1$$

$$F_1\left(k + \frac{N}{4}\right) = V_{11}(k) - W_{N/2}^{k} V_{12}(k), \quad k = 0,\ 1,\ \ldots\ldots\ldots\ldots \frac{N}{4} - 1 \qquad ------> (8)$$

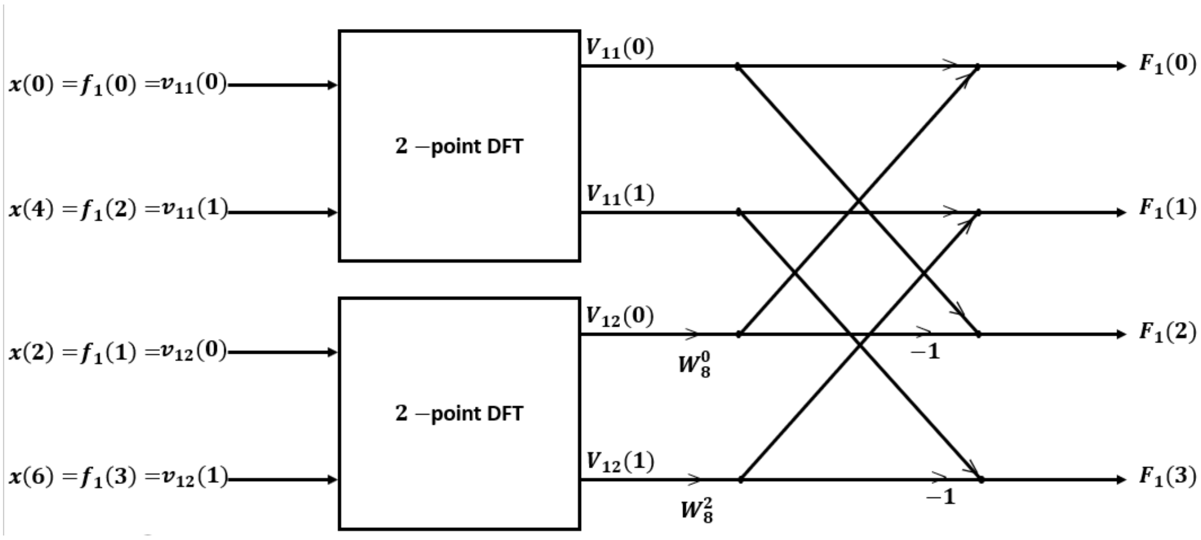


Fig 3: $\frac{N}{2}$ −DIT-FFT algorithm (N = 8)

The computation of $\{V_{ij}(k)\}$ requires $4\left(\frac{N}{4}\right)^2$ multiplications.

The computation of $F_1(k)$ and $F_2(k)$ can be accomplished by $\frac{N^2}{4} + \frac{N}{2}$ complex multiplications.

An additional $\frac{N}{2}$ complex multiplications are required to compute $X(k)$ from $F_1(k)$ and $F_2(k)$.

The total number of complex multiplications is reduced approximately by a factor of 2 again to $\frac{N^2}{4} + N$.

The decimation of the data sequence can be repeated again and again until the resulting sequences are reduced to one point sequences.

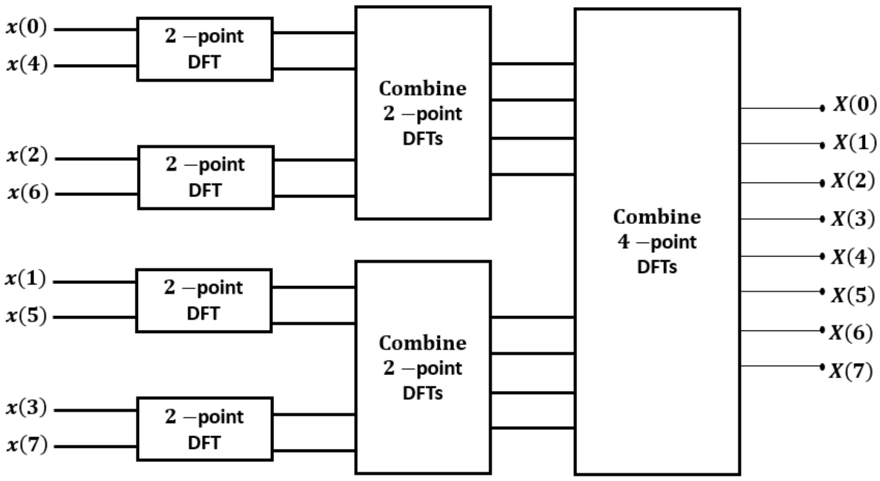


Fig 4: Three Stages in the computation of an $N = 8$ −point DFT

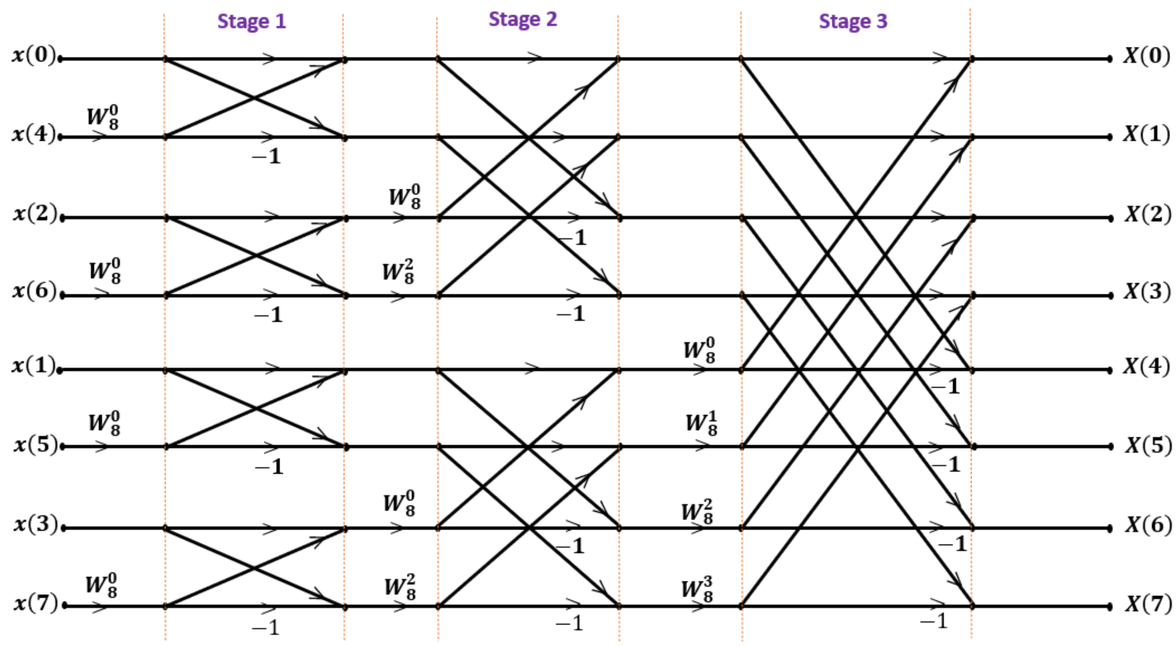


Fig 5: Decimation-in-time FFT algorithm for $N = 8$

- Each butterfly involves one complex multiplication and two complex additions.
- For $N = 2^v$, there are $N/2$ butterflies per stage of the computation process.
- The number of stages are $\log_2 N$.
- Total number of complex multiplications are $\frac{N}{2}\log_2 N$.
- Total number of complex additions are $N\log_2 N$.

$2N$ storage locations are used throughout the computation of N −point DFT, we say the ***computations are done in place***.

b.

	b.	Consider a FIR filter with impulse response h(n) = [3, 2, 1, 1] .If the input is x(n) = [1, 2, 3, 3, 2, 1 , −1, −2, −3, 5, 6, −1, 2, 0, 2, 1] , find the output y(n). Use overlap add method assuming the length of the block is 7.	07	L3	CO3

Sol:

$$x(n) = \{1,2,3,3,2,1,-1,-2,-3,5,6,-1,2,0,2,1\}$$

$$h(n) = \{3,2,1,1\}$$

Length of impulse response is $M = 4$
Given $N = 7$

$$N = L + M - 1,$$

$$7 = L + 4 - 1,$$

$$\boldsymbol{L = 4}$$

Obtain the block of sequences using Overlap-Add method as

$$x_1(n) = \{\underbrace{1,2,3,3}_{\substack{\text{L samples}\\ \text{from x(n)}}}, \underbrace{0,\ 0,0}_{\substack{\text{M}-1\\ \text{Zeros}}}\}$$

$$x_2(n) = \{\underbrace{2,1,-1,-2}_{\substack{\text{next}\\ \text{L samples}\\ \text{from x(n)}}}, \underbrace{0,\ 0,0}_{\substack{\text{M}-1\\ \text{Zeros}}}\}$$

$$x_3(n) = \{\underbrace{-3,5,6,-1}_{\substack{\text{next}\\ \text{L samples}\\ \text{from x(n)}}}, \underbrace{0,\ 0,0}_{\substack{\text{M}-1\\ \text{Zeros}}}\}$$

$$x_4(n) = \{\underbrace{2,0,2,1}_{\substack{\text{next}\\ \text{L samples}\\ \text{from x(n)}}}, \underbrace{0,\ 0,0}_{\substack{\text{M}-1\\ \text{Zeros}}}\}$$

$h(n)$ has $\text{M} = 4$ samples. Pad $(\text{L} - 1) = 3$ zeros to increase its length to $N = 7$.

$$h(n) = \{3,\ 2,\ 1,\ 1,\ 0,\ 0,\ 0,\ 0\}$$

Obtain the output blocks by performing 8 −point circular convolution.

$$y_1(n) = x_1(n) \circledast h(n)$$

$$y_1(n) = \{\ 3,8,14,18,11,6,3\}$$

$$y_2(n) = \{\ 6,7,1,-5,-4,-3,-2\}$$

$$y_3(n) = \{\ -9,9,25,11,9,5,-1\}$$

$$y_4(n) = \{\ 6,4,8,9,4,3,1\}$$

$$y(n) = \{3,8,14,18,17,13,4,-5,-13,6,23,11,15,9,7,9,4,3,1\}$$

	c.	A length 8 sequence $x(n) = [-4, 5, 2, -3, 0, -2, 3, 4]$ with 8-point DFT given ny X(k). Determine the sequence y(n) whose 8-point DFT is given by $Y(k) = W_4^{3k} X(k)$.	05	L3	CO3

Sol:

$$x(n) = \{-4,5,2,-3,0,-2,3,4\}$$

$$Y(k) = W_4^{3k} X(k) = e^{-j\frac{2\pi}{4}3k} X(k)$$

$$Y(k) = e^{-j\frac{2\pi}{8}6k} X(k)$$

$$Y(k) = W_8^{6k} X(k)$$

Using circular freq shift property

$$y(n) = x((n-6))_8$$

$$y(n) = \{2, -3, 0, -2, 3, 4, -4, 5\}$$

Q.7	a.	A low pass filter is to be designed for the desired frequency response $H_d(e^{jw}) = H_d(w) = \begin{cases} e^{-j2w} & \lvert w \rvert < \pi/4 \\ 0 & \pi/4 < \lvert w \rvert < \pi \end{cases}$ Determine the filter coefficients $h_d(n)$ and $h(n)$ if rectangular window is used. Also find the frequency $H(w)$ of the resulting FIR filter.	10	L3	CO4

Sol:

$$H_d(\omega) = \begin{cases} e^{-j2\omega}, & 0 \le |\omega| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left\{ \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, e^{j\omega n} d\omega \right\}$$

$$= \frac{1}{2\pi j n} e^{j\omega n} \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{2\pi j n} \left[e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n} \right]$$

$$h(n) = \frac{1}{2\pi j n} \left(2j \sin\left(\frac{\pi}{4} n\right) \right)$$

$$h(n) = \frac{\sin\left(\frac{\pi}{4} n\right)}{\pi n}, -\infty < n < \infty, n \neq 0$$

For $n = 0$,

$$h(0) = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} H(\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, d\omega = \frac{1}{4}$$

$$h(n) = \begin{cases} \frac{\sin\left(\frac{\pi}{4} n\right)}{\pi n}, n \neq 0 \\ \frac{1}{4}, \quad n = 0 \end{cases}$$

Given , $N = 5$

$$\alpha = \frac{N-1}{2} = 2$$

$$h_d(n-\alpha) = \begin{cases} \dfrac{\sin\left(\frac{\pi}{4}(n-2)\right)}{\pi(n-2)}, & n \neq 2 \\ \dfrac{1}{4}, & n = 2 \end{cases}$$

Impulse response of practical filter is

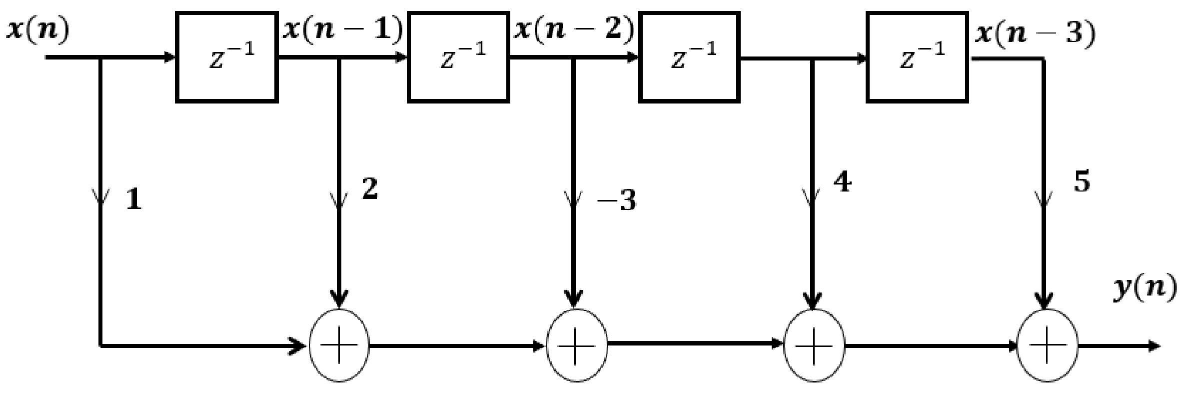
$$h(n) = h_d(n-\alpha)w(n), \quad 0 \le n \le 4$$

	b.	Determine the Direct form realization of the system function $H(z) = 1 + 2z^{-1} - 3z^{-2} + 4z^{-3} + 5z^{-4}$	04	L3	CO4

Sol:

The difference equation of the system is given by

$$y(n) = x(n) + 2x(n-1) - 3x(n-2) + 4x(n-3) + 5x(n-4)$$



	c.	Write a program to design digital low pass FIR filter using a window.	06	L3	CO4

Sol:

```
%Matlab Program To Simulate Digital LPF
clc
clear all
close all
load handel.mat
N=61; %Length of Impulse Response of LPF
n=0:N-1;
alpha=(N-1)/2; %Impulse Response will be shifted to right by Alpha steps
               %so that the system becomes CAUSAL
```

```
wc=pi/4; %Cut-off frequency of LPF

%Simulate Window Functions
rectangular_window=ones(1,N);
hamming_window=0.54-0.46*cos(2*pi*n/(N-1));

%To Plot Window Sequences
subplot(2,1,1)
stem(n,rectangular_window)
grid on
xlabel('n')
ylabel('amplitude')
title('Rectangular Window')

subplot(2,1,2)
stem(n,hamming_window)
grid on
xlabel('n')
ylabel('amplitude')
title('Hamming Window')

% Impulse Response of LPF
h=sin(wc*(n-alpha))./(pi*(n-alpha));
h(alpha+1)=wc/pi;

%Multiply Impulse Response with Window Functions
h_rect=h.*rectangular_window;
h_hamm=h.*hamming_window;

%To see the frequency response of the filter designed
%Using rectangular window
fvtool(h_rect)

%fv tool stands for filter visualization tool
%fvtool is the Matlab Command to show Impulse Response,
%Magnitude Response,Phase Response and Pole-Zero plot

%To see the frequency response of the filter designed
%Using Hamming window
fvtool(h_hamm)

%To obtain pole-zero plot
zplane(h_rect)


x_noise=y+0.05*randn(length(y),1);
y_filt = filter(h_hamm,1,x_noise);
soundsc(x_noise,Fs)
```

		OR			
Q.8	a.	Design a FIR filter with desired frequency response $H_d(e^{jw}) = \begin{cases} e^{-j4w} & -\pi/4 \le w \le \pi/4 \\ 0 & \pi/4 \le \lvert w \rvert \le \pi \end{cases}$ Find filter specifications and transfer function using Bartlett window.	10	L3	CO4

Sol:

$$H_d(\omega) = \begin{cases} e^{-j4\omega}, & -\frac{\pi}{4} \le |\omega| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left\{ \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, e^{j\omega n} d\omega \right\}$$

$$= \frac{1}{2\pi j n} e^{j\omega n} \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{2\pi j n} \left[e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n} \right]$$

$$h(n) = \frac{1}{2\pi j n} \left(2j \sin\left(\frac{\pi}{4} n\right) \right)$$

$$h(n) = \frac{\sin\left(\frac{\pi}{4} n\right)}{\pi n}, -\infty < n < \infty, n \ne 0$$

For $n = 0$,

$$h(0) = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} H(\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, d\omega = \frac{1}{4}$$

$$h(n) = \begin{cases} \frac{\sin\left(\frac{\pi}{4} n\right)}{\pi n}, n \ne 0 \\ \frac{1}{4}, \quad n = 0 \end{cases}$$

Given , $N = 9$

$$\alpha = \frac{N-1}{2} = 4$$

$$h_d(n-\alpha) = \begin{cases} \dfrac{\sin\left(\frac{\pi}{4}(n-2)\right)}{\pi(n-2)}, n \neq 2 \\ \dfrac{1}{4}, \quad n = 2 \end{cases}$$

Impulse response of practical filter is

$$h(n) = h_d(n-\alpha)w(n), \quad 0 \leq n \leq 8$$

where bartlett window function is given by

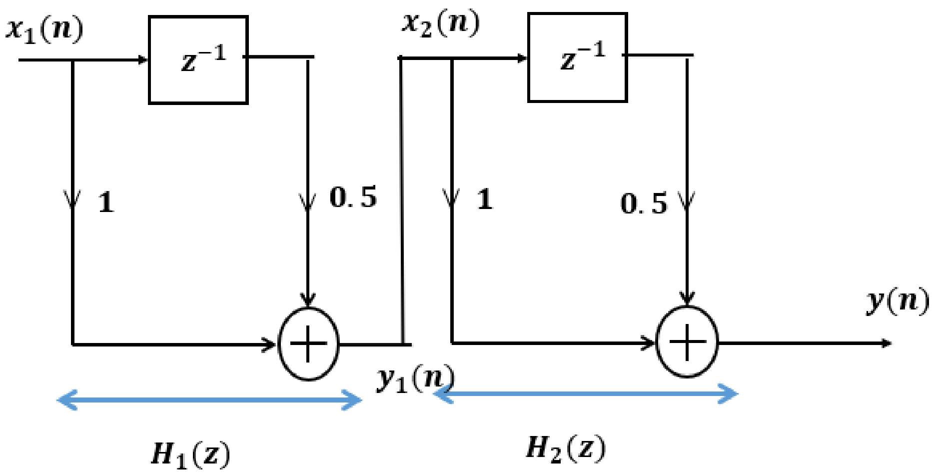
$$w(n) = 1 - \frac{2\left|n - \frac{N-1}{2}\right|}{N-1}, \quad 0 \leq n \leq 8$$

	b.	Realize the system function in cascade form $H(z) = 1 + \frac{5}{2}z^{-1} + 2z^{-2} + 2z^{-3}$	04	L3	CO4

Sol:

$$H(z) = (z+2)(z^2 + 0.5z + 1)$$

$$H(z) = 2(1 + 0.5z^{-1})(1 + 0.5z^{-1} + z^{-2})$$



	c.	Write a program to design digital high pass FIR filter using a window.	06	L3	CO4

Sol:

```
%Matlab Program To Simulate Digital HPF
clc
clear all
close all

N=61; %Length of Impulse Response of HPF
```

```
n=0:N-1;
alpha=(N-1)/2; %Impulse Response will be shifted to right by Alpha steps
                   %so that the system becomes CAUSAL
wc=pi/4; %Cut-off frequency of HPF

%Simulate Window Functions
rectangular_window=ones(1,N);
hamming_window=0.54-0.46*cos(2*pi*n/(N-1));

%To Plot Window Sequences
subplot(2,1,1)
stem(n,rectangular_window)
grid on
xlabel('n')
ylabel('amplitude')
title('Rectangular Window')

subplot(2,1,2)
stem(n,hamming_window)
grid on
xlabel('n')
ylabel('amplitude')
title('Hamming Window')

% Impulse Response of HPF
h=-sin(wc*(n-alpha))./(pi*(n-alpha));
h(alpha+1)=1-wc/pi;

%Multiply Impulse Response with Window Functions
h_rect=h.*rectangular_window;
h_hamm=h.*hamming_window;

%To see the frequency response of the filter designed
%Using rectangular window
fvtool(h_rect)

%fv tool stands for filter visualization tool
%fvtool is the Matlab Command to show Impulse Response,
%Magnitude Response,Phase Response and Pole-Zero plot

%To see the frequency response of the filter designed
%Using Hamming window
fvtool(h_hamm)

%To obtain pole-zero plot
zplane(h_rect)
```

Q.9	a.	Design an analog Butterworth lowpass filter that has – 2dB or better (ie., lesser than – 2 dB) at frequency of 20 rad/sec and atleast – 10 dB of attenuation at 30 rad/sec.	10	L3	CO5

Sol:

Order $N = 4$,

Butterworth Polynomial

N	$B_N(s)$
1	$s + 1$
2	$s^2 + 1.414s + 1$
3	$s^3 + 2s^2 + 2s + 1$
4	$(s^2 + 0.765s + 1)(s^2 + 1.618s + 1\ )$

$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.618s + 1\ )}$$

	b.	Obtain the direct form – I and direct form – II structure for the filter given by system function $H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$	04	L3	CO5

Sol:

Step 1: Difference Equation

$$(1 - 0.5z^{-1} + 0.06z^{-2})Y(z) = (1 + 0.4z^{-1})X(z)$$

Taking inverse Z-transform:

$$y(n) - 0.5y(n-1) + 0.06y(n-2) = x(n) + 0.4x(n-1)$$

Rearranging:

$$\boxed{y(n) = 0.5y(n-1) - 0.06y(n-2) + x(n) + 0.4x(n-1)}$$

Implement the direct form 1 and 2 structures

	c.	Write a program to design digital IIR Butterworth low pass filter.	06	L3	CO5

```
clc;
clear;
close all;

wp = 0.4*pi;   % normalized passband
ws = 0.6*pi;   % normalized stopband
Rp = 2;        % passband ripple (dB)
Rs = 20;       % stopband attenuation (dB)

[N, Wn] = buttord(wp/pi, ws/pi, Rp, Rs);
[b,a] = butter(N, Wn);

freqz(b,a,1024);
title('IIR Butterworth Low Pass Filter');
```

OR

Q.10	a.	Design a digital Butterworth lowpass filter with frequency specifications given by (i) Passband ≤ 3.01 dB (ii) Passband edge frequency : 500 Hz (iii) Stopband attenuation ≥ 15 dB (iv) Stopband edge frequency : 750 Hz (v) Sampling rate f_s = 2 KHz Use Bilinear transformation metod.	10	L3	CO5

Sol:

◆ Step 1: Convert to Digital Angular Frequencies

$$\Omega_p = 2\pi \frac{f_p}{f_{samp}}$$

$$\Omega_p = 2\pi \frac{500}{2000} = \frac{\pi}{2}$$

$$\Omega_s = 2\pi \frac{750}{2000} = \frac{3\pi}{4}$$

🔹 Step 2: Prewarping (Bilinear Transform)

Using:

$$\omega = \frac{2}{T}\tan\left(\frac{\Omega}{2}\right)$$

Since:

$$T = \frac{1}{2000}$$

$$\frac{2}{T} = 4000$$

Prewarped Passband Frequency

$$\omega_p = 4000\tan\left(\frac{\pi}{4}\right)$$

$$\tan(\pi/4) = 1$$

$$\boxed{\omega_p = 4000}$$

Prewarped Stopband Frequency

$$\omega_s = 4000\tan\left(\frac{3\pi}{8}\right)$$

$$\tan(67.5^\circ) = 2.414$$

$$\boxed{\omega_s = 9656}$$

🔹 Step 3: Convert Attenuations to Linear Form

$$10^{A_p/10} - 1$$

$$10^{3.01/10} - 1$$

$$= 2 - 1 = 1$$

$$10^{A_s/10} - 1$$

$$10^{15/10} - 1$$

$$= 31.62 - 1 = 30.62$$

Step 4: Order Calculation

Butterworth order formula:

$$N = \frac{\log\left(\frac{30.62}{1}\right)}{2\log\left(\frac{9656}{4000}\right)}$$

$$\frac{\log(30.62)}{2\log(2.414)}$$

$$\frac{1.486}{2(0.382)}$$

$$\frac{1.486}{0.764}$$

$$N = 1.94$$

Final Order:

$$\boxed{N = 2}$$

(Round up to next integer)

Step 7: Apply Bilinear Transform

Substitute:

$$s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}$$

After simplification, final digital filter form:

$$\boxed{H(z) = \frac{0.0675 + 0.135z^{-1} + 0.0675z^{-2}}{1 - 1.143z^{-1} + 0.412z^{-2}}}$$

	b.	A filter is given by the difference equation $y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-2)$ Draw direct form – I and direct form – II realizations.	04	L3	CO5

Sol:

Given:

$$y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-2)$$

Step 1: Write in Standard Form

$$\boxed{y(n) = \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{2}x(n-2)}$$

◆ Direct Form I

- Two delay elements for input
- Two delay elements for output
- Feedforward multipliers: $1, 0, 1/2$
- Feedback multipliers: $1/4, -1/8$

◆ Direct Form II

- Only two delay elements
- Combined input-output delay line
- Same coefficients

	c.	Write a program to design digital IIR Butterworth high pass filter.	06	L3	CO5

Sol:

```
clc;
clear;
close all;

fp = 750;     % Passband frequency
fs = 500;     % Stopband frequency
Fs = 2000;    % Sampling frequency

Wp = fp/(Fs/2);
Ws = fs/(Fs/2);

Ap = 3;       % Passband ripple
As = 15;      % Stopband attenuation

[N,Wn] = buttord(Wp,Ws,Ap,As);
[b,a] = butter(N,Wn,'high');

freqz(b,a,1024);
title('Digital IIR Butterworth High Pass Filter');
```