

CBCS SCHEME

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BEC503

Fifth Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026 Digital Communication

Time: 3 hrs.

Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks, L: Bloom's level, C: Course outcomes.*

Module - 1			M	L	C															
Q.1	a.	What is Hilbert Transform? Summarize the properties of Hilbert Transform.	10	L2	CO1															
	b.	Describe with the mathematical expression of canonical representation of Bandpass signals.	10	L2	CO1															
OR																				
Q.2	a.	Tabulate the steps used in Gram-Schmidt orthogonalization procedure.	10	L2	CO1															
	b.	Outline the detector diagram of correlation receiver.	10	L2	CO1															
Module - 2																				
Q.3	a.	Derive an expression for probability of error calculation binary PSK using coherent detection.	10	L2	CO2															
	b.	Explain with neat diagrams generation and coherent detection of QPSK signals.	10	L2	CO2															
OR																				
Q.4	a.	Develop signal space diagram of M-array QAM for M = 16.	10	L3	CO2															
	b.	Outline the generation and detection of non-coherent DPSK signals with a necessary diagram.	10	L2	CO2															
Module - 3																				
Q.5	a.	Outline the properties of entropy. Derive an expression for average information content of symbols in long independent sequences.	10	L2	CO3															
	b.	A source has an alphabet of 7 symbols with probabilities as given below : <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <tr> <td>Symbol</td> <td>S1</td> <td>S2</td> <td>S3</td> <td>S4</td> <td>S5</td> <td>S6</td> <td>S7</td> </tr> <tr> <td>Probability</td> <td>1/4</td> <td>1/4</td> <td>1/8</td> <td>1/8</td> <td>1/8</td> <td>1/16</td> <td>1/16</td> </tr> </table> Construct Huffman binary code and find its efficiency.	Symbol	S1	S2	S3	S4	S5	S6	S7	Probability	1/4	1/4	1/8	1/8	1/8	1/16	1/16	10	L3
Symbol	S1	S2	S3	S4	S5	S6	S7													
Probability	1/4	1/4	1/8	1/8	1/8	1/16	1/16													
OR																				
Q.6	a.	Briefly explain the following properties of mutual information <ul style="list-style-type: none"> i) Symmetry property ii) Non negativity property 	10	L2	CO3															
	b.	Derive an expression for Information Capacity Law.	10	L2	CO3															

Module - 4					
Q.7	a.	Consider a (6, 3) linear code whose generator matrix is $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ Calculate : i) All code vectors ii) All the hamming weight and distances iii) Minimum weight parity check matrix iv) Draw the encoder circuit.	10	L3	CO4
	b.	For a systematic (6, 3) linear block code, the parity matrix 'P' is given by $[P] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ Calculate all possible vectors.	10	L3	CO4
OR					
Q.8	a.	The parity check bits of a (7, 4) Hamming code are generated by $C_5 = d_1 + d_3 + d_4$, $C_6 = d_1 + d_2 + d_3$, $C_7 = d_2 + d_3 + d_4$, where d_1, d_2, d_3 and d_4 are the message bits. i) Compute the generator matrix [G] and parity check matrix [H] for this code ii) Show that $GH^T = 0$.	10	L3	CO4
	b.	Briefly, explain the following terms Hamming weight, Hamming distance and minimum distance of LBC with suitable examples.	10	L2	CO4
Module - 5					
Q.9	a.	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (101)$ and $g^{(3)} = (111)$. i) Draw the encoder block diagram ii) Calculate the generator matrix	10	L3	CO5
	b.	Briefly, describe the following terms : i) Code tree ii) Trellis graph iii) State Graph	10	L2	CO5
OR					
Q.10	a.	Describe the steps of viterbi algorithm in detail.	10	L3	CO5
	b.	Discuss with diagram of the following transconvolutional encoder, generator polynomial of path 1 and path 2.	10	L3	CO5

Ques 8 a)

$$C_5 = d_1 + d_3 + d_4 \quad (7,4) - \text{Block}$$

$$C_6 = d_1 + d_2 + d_3$$

$$C_7 = d_2 + d_3 + d_4$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

from the above expression.

c) a) Generator Matrix.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

b) $H = [P^T \mid I_{n-k}]$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

ii) We know that $C H^T = 0$

Code vector

$$\text{eg } C H^T = D \cdot G H^T$$

$$\text{Now, } G = [I_k \mid P]_{k \times n}$$

$$\text{eg } H = [P^T \mid I_{n-k}]_{(n-k) \times n}$$

$$\therefore H^T = \begin{bmatrix} P \\ I_{n-k} \end{bmatrix}_{n \times (n-k)}$$

$$\therefore GH^T = [I_k | P] \begin{bmatrix} P \\ I_{n-k} \end{bmatrix}_{k \times (n-k)} \quad \text{--- (1)}$$

Consider an element P_{ij} in submatrix P in the above equation. When the particular row in G is multiplied with the particular column in H^T , due to the identity matrices in both of them,

$$(1 \times P_{ij}) + (P_{ij} \times 1) = 0$$

Hence, the result is a null matrix.

$$\therefore GH^T = [0]_{k \times (n-k)}$$

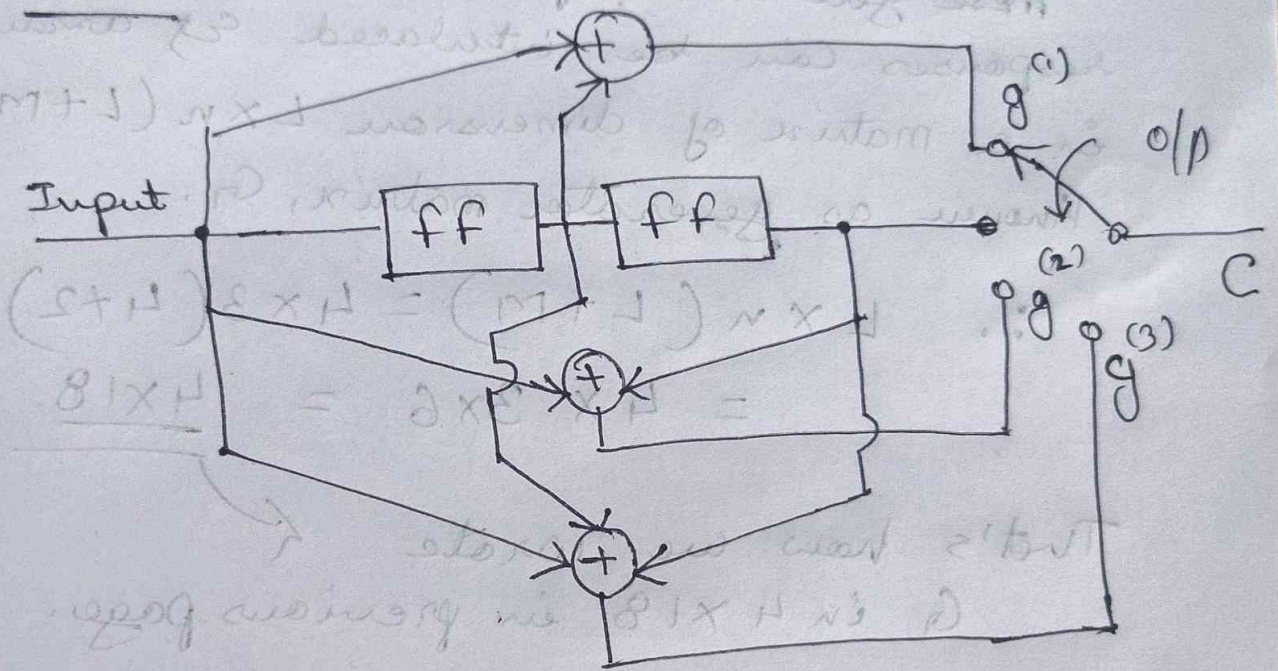
$$\boxed{GH^T = 0}$$

Question - 9 (a)

(3, 1, 2)

$$g^{(1)} = (110), g^{(2)} = (101) \text{ \& } g^{(3)} = (111)$$

a) Encoder Block Diagram



b)

Generator Matrix

$$G = \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix} \quad \underline{\underline{4 \times 6}}$$

We have $M=2$ (no. of fifs)

$n=3$ no. of generator sequences.

Let us assume message sequence has 4 samples.

$$\therefore L=4.$$

These generator sequences or gener impulse responses can be interleaved & arranged in a matrix of dimension $L \times n (L+M)$, known as generator matrix, G .

$$\therefore L \times n (L+M) = 4 \times 3 (4+2) \\ = 4 \times 3 \times 6 = \underline{4 \times 18}.$$

That's how we wrote G in 4×18 in previous page.

The second row of G is obtained by applying a cyclic shift to first row & so on.

$$\begin{bmatrix} 000 & 000 & 000 & 110 & 101 & 111 \\ 000 & 000 & 110 & 101 & 111 & 000 \\ 000 & 110 & 101 & 111 & 000 & 000 \\ 110 & 101 & 111 & 000 & 000 & 000 \end{bmatrix}$$

DC VTU QUESTION PAPER SOLUTION

ANSWER 1(a).

When the phase angles of positive frequency components of a signal $x(t)$ are shifted by -90° and phase angles of negative frequency components are shifted by 90° , the resulting function of time is called Hilbert Transform of the signal, denoted as $\hat{x}(t)$.

However, the amplitude of all frequency components are unaffected by this operation.

Hilbert transformer is an LTI system with the following frequency response.

$$H(f) = \begin{cases} e^{-j\frac{\pi}{2}} & , \text{ for } f > 0 \\ 0 & , \text{ for } f = 0 \\ e^{j\frac{\pi}{2}} & , \text{ for } f < 0 \end{cases} \dots (1)$$

$$\begin{aligned} \text{But } e^{\pm j\frac{\pi}{2}} &= \cos\left(\frac{\pi}{2}\right) \pm j \sin\left(\frac{\pi}{2}\right) \\ &= \pm j \dots (2) \end{aligned}$$

$\therefore H(f)$ can be written as,

$$H(f) = \begin{cases} -j & , \text{ for } f > 0 \\ 0 & , \text{ for } f = 0 \\ j & , \text{ for } f < 0 \end{cases} \dots (3)$$

Alternatively, $H(f)$ can be written as,

$$H(f) = -j \operatorname{sgn}(f) \dots (4)$$

where,

$$\operatorname{sgn}(f) = \begin{cases} 1 & , \text{ for } f > 0 \\ 0 & , \text{ for } f = 0 \\ -1 & , \text{ for } f < 0 \end{cases} \dots (5)$$

Properties of the Hilbert Transform

The Hilbert transform is a linear operator that produces a 90° phase-shifted version of a signal and is widely used in communication systems, signal analysis, and analytic signal generation.

1. Linearity

The Hilbert transform is a linear operation:

$$H\{ax_1(t)+bx_2(t)\}=aH\{x_1(t)\}+bH\{x_2(t)\}$$

where a, b, a, b are constants.

2. Phase Shift Property

It introduces a $\pm 90^\circ$ phase shift:

Positive frequencies \rightarrow phase shift of -90°

Negative frequencies \rightarrow phase shift of $+90^\circ$

Amplitude remains unchanged.

3. Frequency Domain Representation

If $X(f)$ is the Fourier transform of $x(t)$, then:

$$F\{\hat{x}(t)\} = -j \operatorname{sgn}(f) X(f)$$

where $\operatorname{sgn}(f)$ is the sign function.

4. Orthogonality

A signal and its Hilbert transform are orthogonal:

$$\int_{-\infty}^{\infty} x(t) \hat{x}(t) dt = 0$$

5. Double Hilbert Transform

Applying the Hilbert transform twice gives:

$$H\{H\{x(t)\}\} = -x(t)$$

6. Magnitude Preservation

The Hilbert transform does not change the magnitude spectrum, only the phase.

7. Analytic Signal Formation

It is used to form the analytic signal:

$$z(t) = x(t) + j\hat{x}(t)$$

which contains only positive frequency components.

8. Time-Invariant Property

A time shift in the input produces the same time shift in the output:

$$H\{x(t - t_0)\} = \hat{x}(t - t_0)$$

Answer-1(b)

⑤ Derive an expression of bandpass signals extracting in-phase and quadrature components. Obtain a block diagram. Draw the corresponding block diagram.

consider a real valued bandpass signal $x(t)$

$\hat{x}(t) \rightarrow$ Hilbert transform of $x(t)$

$X(f) \rightarrow$ Fourier transform of $x(t)$

$x_+(t) \rightarrow$ Pre-envelope of $x(t)$ with +ve frequencies

$\tilde{x}(t) \rightarrow$ complex envelope of $x(t)$

We know that $x_+(t) = x(t) + j\hat{x}(t) \rightarrow (1)$

and $\tilde{x}(t) = x_+(t) e^{-j2\pi f_c t} \rightarrow (2)$

$x_+(t) = \tilde{x}(t) e^{j2\pi f_c t} \rightarrow (3)$

Using (1) (2) can be written as

$$\begin{aligned} \tilde{x}(t) &= [x(t) + j\hat{x}(t)] e^{-j2\pi f_c t} \\ &= [x(t) + j\hat{x}(t)] [\cos(2\pi f_c t) - j\sin(2\pi f_c t)] \\ &= [x(t)\cos(2\pi f_c t) - j\sin(2\pi f_c t)x(t) + j\hat{x}(t)\cos(2\pi f_c t) + \hat{x}(t)\sin(2\pi f_c t)] \\ &= x(t)\cos(2\pi f_c t) + \hat{x}(t)\sin(2\pi f_c t) \\ &\quad + j[\hat{x}(t)\cos(2\pi f_c t) - x(t)\sin(2\pi f_c t)] \end{aligned} \rightarrow (4)$$

where $x_I(t) = x(t)\cos(2\pi f_c t) + \hat{x}(t)\sin(2\pi f_c t) \rightarrow (5)$

$x_Q(t) = \hat{x}(t)\cos(2\pi f_c t) - x(t)\sin(2\pi f_c t) \rightarrow (6)$

Using (4) (5) can be written as

$$\begin{aligned} x_+(t) &= [x_I(t) + jx_Q(t)] [\cos(2\pi f_c t) + j\sin(2\pi f_c t)] \\ &= x_I(t)\cos(2\pi f_c t) + jx_I(t)\sin(2\pi f_c t) + jx_Q(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) \\ &= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) + j[x_I(t)\sin(2\pi f_c t) + x_Q(t)\cos(2\pi f_c t)] \end{aligned} \rightarrow (7)$$

But (7) suggests that

$$x(t) = \text{Real part of } x_+(t) \rightarrow (8)$$

Using (8) we can write

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t) \rightarrow (9)$$

Proof of Gram-Schmidt orthogonalization procedure.

Stage 1:

First we have to establish whether or not the given set of signals $\{x_1(t), x_2(t), \dots, x_M(t)\}$ is linearly independent.

If not, there exists a set of coefficients $\{a_1, a_2, \dots, a_M\}$, not all equal to zero, such that,

$$a_1 x_1(t) + a_2 x_2(t) + \dots + a_M x_M(t) = 0 \quad \dots (1)$$

$0 \leq t \leq T$

Suppose that $a_M \neq 0$. Then, ③

$$x_M(t) = - \left[\frac{a_1}{a_M} x_1(t) + \frac{a_2}{a_M} x_2(t) + \dots + \frac{a_{M-1}}{a_M} x_{M-1}(t) \right] \quad \dots (2)$$

Consider next the set of signals

$$\{x_1(t), x_2(t), \dots, x_{M-1}(t)\}$$

If this set is linearly dependent, then there exists a set of numbers

$\{b_1, b_2, \dots, b_{M-1}\}$ not all equal to zero,

such that

$$b_1 x_1(t) + b_2 x_2(t) + \dots + b_{M-1} x_{M-1}(t) = 0 \quad \dots (2)$$

$0 \leq t \leq T$

Suppose that $b_{m-1} \neq 0$, then

$$x_{m-1}(t) = - \left[\frac{b_1}{b_{m-1}} x_1(t) + \frac{b_2}{b_{m-1}} x_2(t) + \dots + \frac{b_{m-2}}{b_{m-1}} x_{m-2}(t) \right] \dots (3)$$

Continuing in this fashion, we will end up with a linearly independent set of signals $\{ x_1(t), x_2(t), \dots, x_N(t) \}$

Stage 2 :

Next, we wish to show that it is possible to construct a set of orthonormal basis functions $\{ \phi_1(t), \phi_2(t), \dots, \phi_N(t) \}$ from the linearly independent signals $x_1(t), x_2(t), \dots, x_N(t)$.

As a starting point define the first basis function as.

$$\phi_1(t) = \frac{x_1(t)}{\sqrt{E_1}} \dots (4)$$

where E_1 is the energy of the signal $x_1(t)$ $\phi_1(t)$ has unit energy and

$$x_1(t) = \sqrt{E_1} \phi_1(t) \dots (5)$$

Hence, the coefficient $x_{11} = \sqrt{E_1}$.

Next, using the signal $x_2(t)$, we define the coefficient x_{21} as,

$$x_{21} = \int_0^T x_2(t) \phi_1(t) dt \dots (6)$$

We may define an intermediate function,

$$\phi_2'(t) = x_2(t) - x_{21} \phi_1(t) \dots (7)$$

which is orthogonal to $\phi_1(t)$ over the interval $0 \leq t \leq T$.

Now, we may define the second basis function as

$$\phi_2(t) = \frac{\phi_2'(t)}{\sqrt{E_2}} \dots (8)$$

where E_2 is the energy of $\phi_2'(t)$.

$\phi_2(t)$ is orthonormal to $\phi_1(t)$ from 0 to T.

Continuing in this fashion, we may define

$$\phi_i'(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij} \phi_j(t) \dots (9)$$

where the coefficients x_{ij} , $j=1, 2, \dots, i-1$ are defined by

$$x_{ij} = \int_0^T x_i(t) \phi_j(t) dt \dots (10)$$

Basis function $\phi_i(t)$ is given by

$$\phi_i(t) = \frac{\phi_i'(t)}{\sqrt{E_i}} \dots (11)$$

where E_i is the energy of $\phi_i'(t)$.

Each one of the set of signals $\{x_1(t), x_2(t), \dots, x_N(t)\}$ may be expressed as a linear combination of basis functions $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$.

It follows that each one of $x_1(t), x_2(t), \dots, x_M(t)$ may be expressed as a linear combination of this set of basis functions.

Answer-2(b)

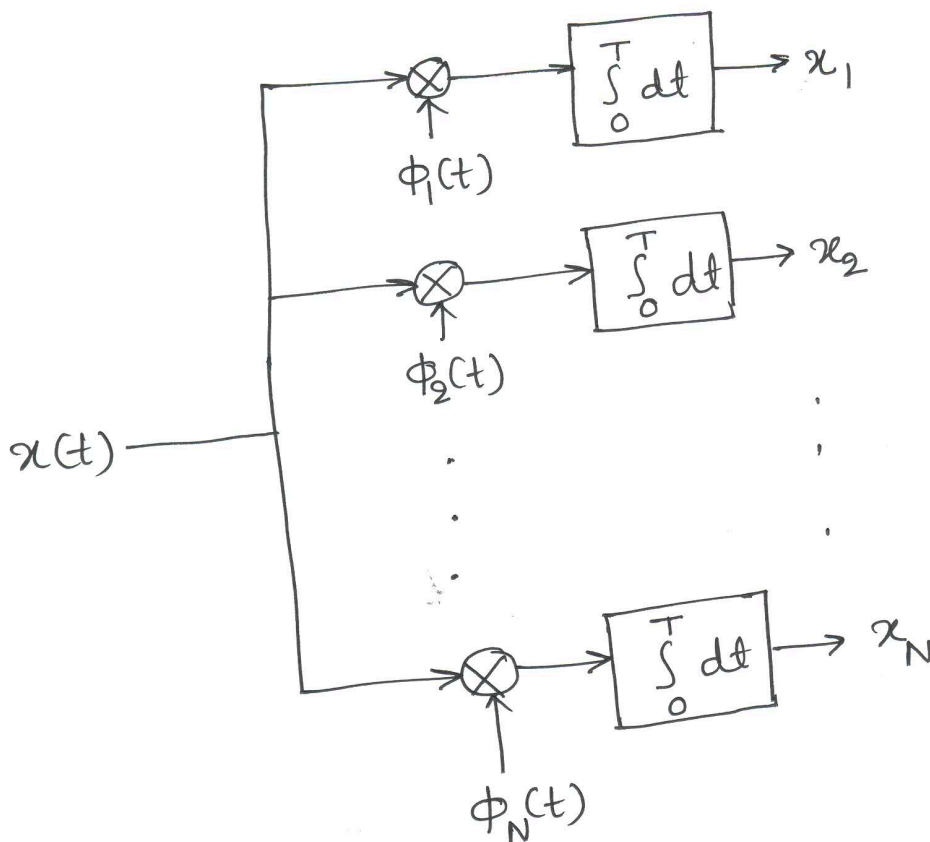
The optimum receiver for an AWGN channel and for the case when the transmitted signals $s_1(t), s_2(t), \dots, s_M(t)$ are equally likely is called correlation receiver.

l.m.c.

It consists of 2 subsystems.

1. Detector which consists of N correlators supplied with orthonormal basis functions $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$ that are generated locally.

This bank of correlators operates on the received signal $x(t), 0 \leq t \leq T$ to produce the observation vector x .



$$\begin{aligned}
 P_e(0) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{z^2}{2N_0}} \sqrt{\frac{N_0}{2}} dz \\
 &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2N_0}} dz \\
 &= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \dots (12)
 \end{aligned}$$

2. Maximum likelihood decoder which operates on the observation vector X to produce an estimate \hat{m} of the transmitted symbol m_i , $i=1,2,\dots,M$ in such a way that average probability of symbol error is minimized.

Answer-3(a)

In binary phase shift keying (BPSK), bit '1' and bit '0' are represented by the following symbols.

Bit 1:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_b \quad (11)$$

$$f_c = \frac{n}{T_b}$$

n - non zero integer
 T_b - bit duration

Bit 0 :

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi), \quad 0 \leq t \leq T_b$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_b$$

To find basis function.

$$\text{Energy of } s_1(t) = \int_0^{T_b} |s_1(t)|^2 dt$$

$$= \int_0^{T_b} \frac{2E_b}{T_b} \cos^2(2\pi f_c t) dt$$

$$= \frac{2E_b}{T_b} \int_0^{T_b} \frac{1 + \cos(4\pi f_c t)}{2} dt$$

$$= \frac{E_b}{T_b} \int_0^{T_b} 1 dt$$

$$= E_b \quad (12)$$

$$\therefore \text{Basis function, } \phi_1(t) = \frac{s_1(t)}{\sqrt{E_b}}$$

$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

$$0 \leq t \leq T_b$$

$$\therefore s_1(t) = \sqrt{E_b} \phi_1(t), \quad 0 \leq t \leq T_b$$

$$s_2(t) = -\sqrt{E_b} \phi_1(t), \quad 0 \leq t \leq T_b$$

Signal-space diagram



Suppose that bit '0' was transmitted.
 i.e., $s_2(t)$ was transmitted.

Then, from the block diagram of the receiver, we may write,

output of the correlator,

$$\begin{aligned} x_1 &= \int_0^{T_b} x(t) \phi_1(t) dt \\ &= \int_0^{T_b} [s_2(t) + w(t)] \phi_1(t) dt \\ &= \int_0^{T_b} s_2(t) \phi_1(t) dt + \int_0^{T_b} w(t) \phi_1(t) dt \\ &= -\sqrt{E_b} + w_1 \quad \dots (3) \end{aligned}$$

↑
coordinate of $s_2(t)$.

Mean of x_1 when '0' was transmitted.

$$\begin{aligned} \mu &= E[x_1] \\ &= -\sqrt{E_b} \quad \dots (3) \end{aligned}$$

Variance of x_1 when '0' was transmitted,

$$\begin{aligned} \sigma^2 &= \text{VAR}[w_1] \quad (15) \\ &= \frac{N_0}{2} \quad \dots (4) \end{aligned}$$

(∵ Variance does not change by the addition of a constant to a random variable.)

∴ Probability density function (PDF) of output of correlator when bit '0' was transmitted,

$$\begin{aligned} f_{x_1}(x_1/0) &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1 - \mu)^2}{2\sigma^2}} \\ &= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1 + \sqrt{E_b})^2}{N_0}} \quad \dots (5) \end{aligned}$$

Wrong decision is made when $s_2(t)$ was transmitted and $x_1 > 0$.

∴ Probability of error when bit '0' was transmitted,

$$P_e(0) = P(X_1 > 0 | 0) \\ = \int_0^{\infty} f_{X_1}(x_1/0) dx_1$$

$$= \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1 + \sqrt{E_b})^2}{N_0}} dx_1 \dots (6) \quad (16)$$

We know that,

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\frac{z^2}{2}} dz \dots (7)$$

Let us represent $P_e(0)$ in terms of Q function.

$$\text{Put } \frac{(x_1 + \sqrt{E_b})^2}{N_0} = \frac{z^2}{2} \dots (8)$$

$$\text{i.e., } \frac{x_1 + \sqrt{E_b}}{\sqrt{N_0}} = \frac{z}{\sqrt{2}}$$

$$\therefore \frac{dx_1}{\sqrt{N_0}} = \frac{dz}{\sqrt{2}}$$

$$\therefore dx_1 = \sqrt{\frac{N_0}{2}} dz \dots (9)$$

$$\text{when } x_1 = 0, \quad z = \sqrt{\frac{2E_b}{N_0}} \dots (10)$$

$$\text{when } x_1 = \infty, \quad z = \infty \dots (11)$$

Using (8), (9), (10), (11), we may write (6) as

Similarly, we may prove that, probability of error when bit '1' was transmitted,

$$P_e(1) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \dots (13)$$

∴ Average probability of error

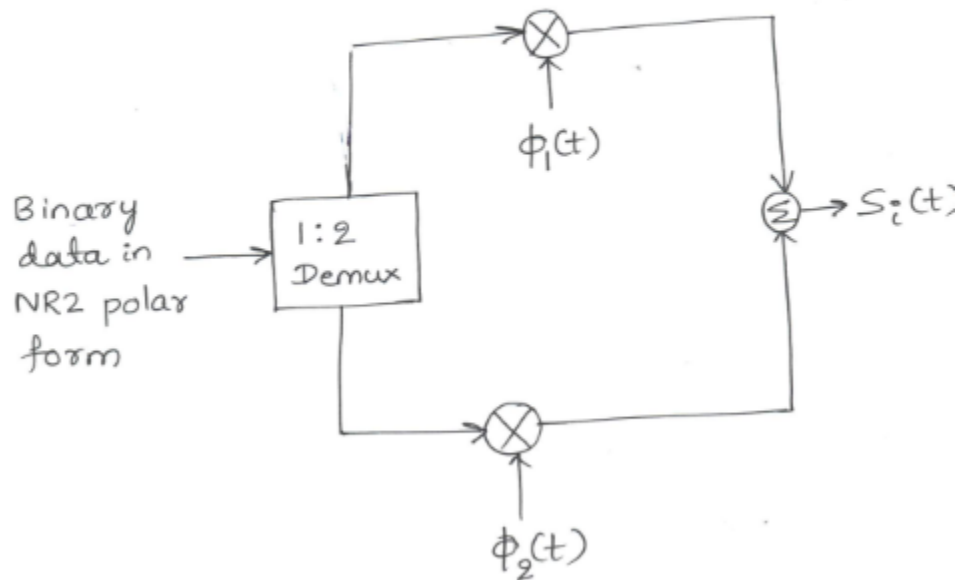
$$= \frac{1}{2} P_e(0) + \frac{1}{2} P_e(1)$$

(Assuming equiprobable 0s & 1s)

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \dots (14)$$

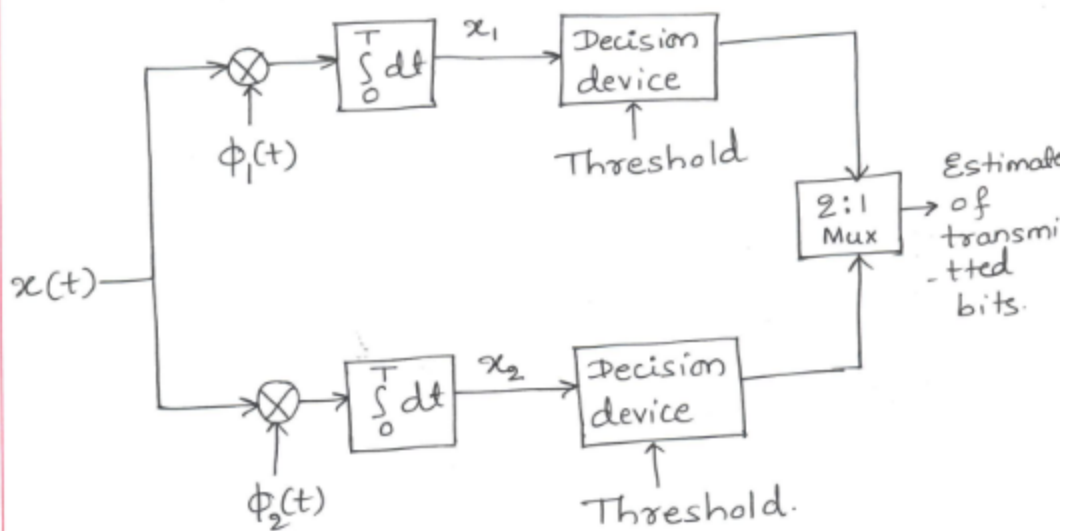
Answer-3(b)

Block diagram of transmitter



Block diagram of receiver

(33)



Let $x(t)$, $0 \leq t \leq T$ be the received symbol.

$$x(t) = s_i(t) + w(t), \quad 0 \leq t \leq T$$
$$i = 1, 2, 3, 4$$

where $w(t)$ represents additive, white Gaussian noise with zero mean and PSD $\frac{N_0}{2}$ i.e., variance $\frac{N_0}{2}$.

In QPSK, the phase of the carrier takes

one of the four equally spaced values ⁽³⁰⁾
 such as $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

For this set of values, we may define the transmitted signal as

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], \quad 0 \leq t \leq T$$

$i = 1, 2, 3, 4$

$$f_c = \frac{n}{T}$$

n - non-zero integer

Here, T is the symbol duration and E is the energy of each symbol.

Each possible value of the phase corresponds to a pair of bits (dibit).

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

$0 \leq t \leq T$

$$E = 2E_b \text{ and } T = 2T_b$$

$$i = 1, 2, 3, 4$$

Basis functions are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \leq t \leq T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \leq t \leq T$$

(31)

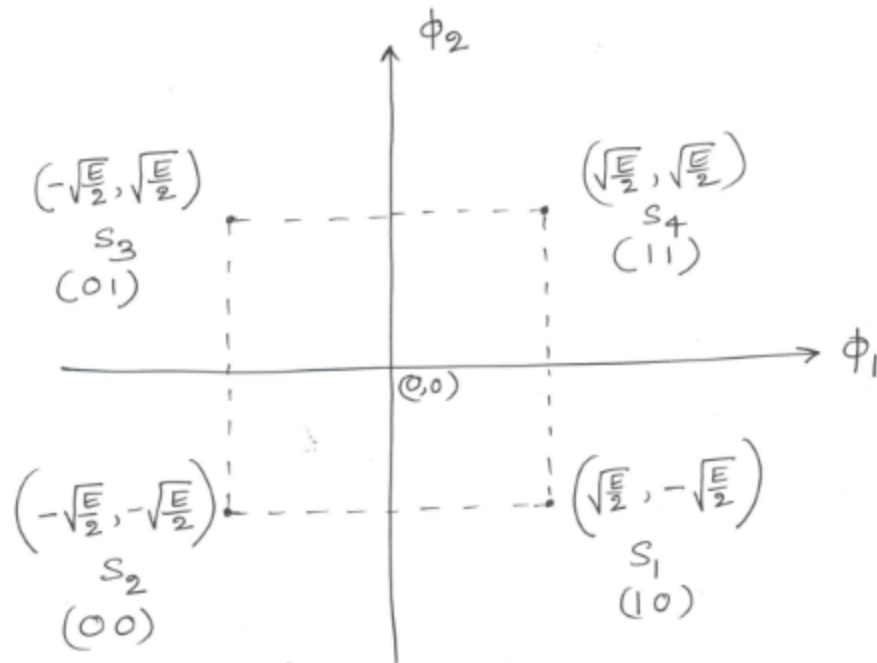
$$\therefore S_i(t) = \sqrt{E} \cos[(2i-1)\frac{\pi}{4}] \phi_1(t) - \sqrt{E} \sin[(2i-1)\frac{\pi}{4}] \phi_2(t), \quad 0 \leq t \leq T$$

\therefore The coordinates of message points are

$$\begin{bmatrix} \sqrt{E} \cos[(2i-1)\frac{\pi}{4}] \\ -\sqrt{E} \sin[(2i-1)\frac{\pi}{4}] \end{bmatrix} \quad i = 1, 2, 3, 4.$$

<u>i</u>	<u>phase</u>	<u>coordinates</u>	<u>dibits</u>
1	$\frac{\pi}{4}$	$\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	10
2	$\frac{3\pi}{4}$	$-\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	00
3	$\frac{5\pi}{4}$	$-\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	01
4	$\frac{7\pi}{4}$	$\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	11

Based on these coordinates, signal space diagram of QPSK system may be drawn as follows.



Answer-4(a)

In quadrature amplitude modulation, (QAM), carrier experiences amplitude as well as phase modulation. (44)

M-ary QAM is two dimensional in that it involves two basis functions.

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \leq t \leq T$$

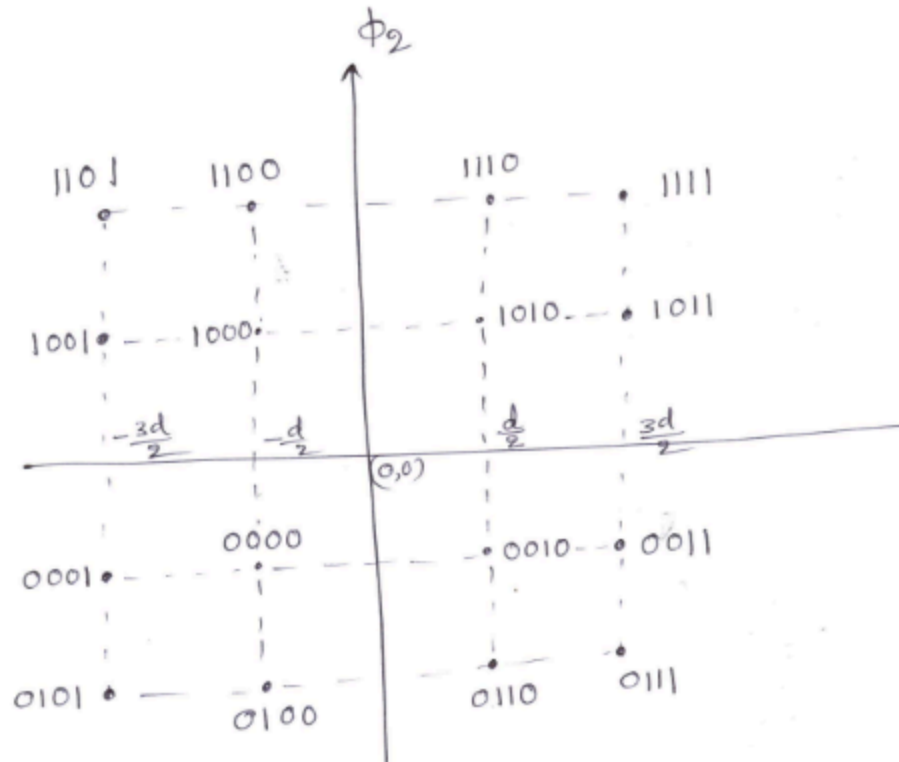
$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \leq t \leq T$$

The QAM signal is given by

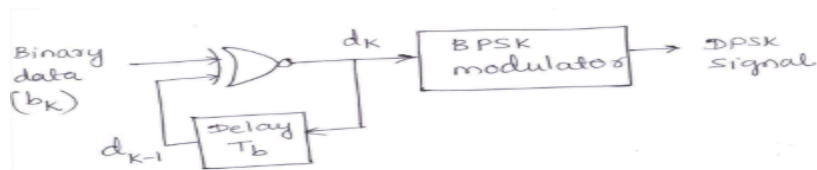
$$s_k(t) = \sqrt{\frac{2E_0}{T}} a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}} b_k \sin(2\pi f_c t), \quad 0 \leq t \leq T$$

$k = 1, 2, \dots, M$

Signal space diagram for $M=16$

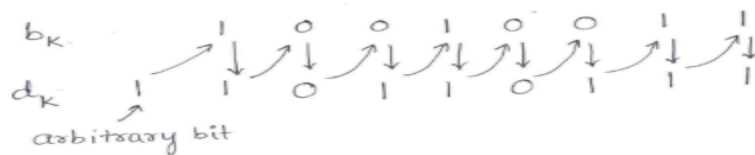


Answer-4(b)
Transmitter:-



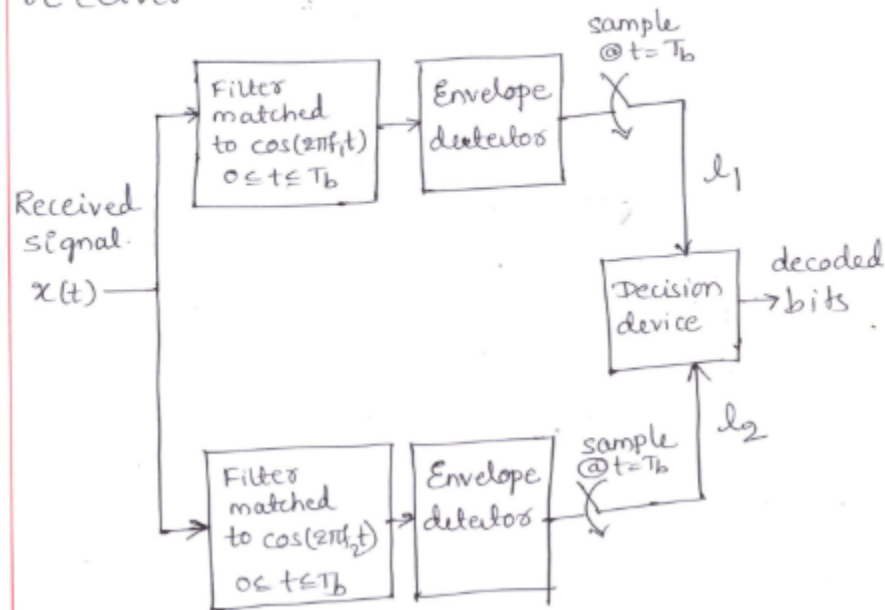
Let b_k be the input binary data and d_k be the differentially encoded data.

$d_k = b_k \odot d_{k-1}$ where \odot represents XOR operation.



Receiver:-

Block diagram of non-coherent FSK receiver.



The non-coherent receiver consists of a pair of matched filters followed by envelope detectors.

One of the filters is matched to $\cos(2\pi f_1 t)$, $0 \leq t \leq T_b$ and the other

one is matched to $\cos(2\pi f_2 t)$, $0 \leq t \leq T_b$ (51)

When bit '1' is transmitted, i.e. $\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$, $0 \leq t \leq T_b$ is transmitted, I_1 will be greater than I_2 in the absence of noise.

When bit '0' is transmitted, i.e. $\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t)$, $0 \leq t \leq T_b$ is transmitted, I_2 will be greater than I_1 in the absence of noise.

Hence, by comparing I_1 and I_2 , a decision will be made.

Decision rule:

Decide in favor of '1' if $I_1 > I_2$

Decide in favor of '0' if $I_1 < I_2$.

Note: Probability of error of BFSK with non-coherent detection is given by

$$P_e = \frac{1}{2} e^{-\left(\frac{E_b}{2N_0}\right)}$$

Answer-5(a)

Definition of Entropy:-

The **entropy** of a discrete source X with symbols x_1, x_2, \dots, x_n and probabilities $p(x_i)$ is the **average information per symbol**:

$$H(X) = - \sum_{i=1}^n p(x_i) \log_b p(x_i)$$

Units: bits/symbol (if base 2).

Properties of Entropy:-

(i) Non-negativity

$$H(X) \geq 0$$

because $\log(1/p(x)) \geq 0$.

Entropy is zero only when one symbol has probability 1.

(ii) Maximum Entropy for Uniform Distribution

If all symbols are equally likely:

$$p(x_i) = \frac{1}{n}$$

$$H_{\max} = \log_b n$$

This represents maximum uncertainty.

(iii) Minimum Entropy

$$H_{\min} = 0$$

Occurs when one symbol occurs with probability 1.

(iv) Entropy depends only on probability distribution

Not on the actual symbols.

(v) Additivity for Independent Sources

For two independent sources X and Y :

$$H(X, Y) = H(X) + H(Y)$$

(vi) Continuity Property

Small change in probability \rightarrow small change in entropy.

(vii) Entropy increases with uncertainty

More randomness \rightarrow higher entropy.

(viii) Bounds

$$0 \leq H(X) \leq \log_b n$$

Average Information Content of Long Independent Sequences (Derivation)

Consider a source emitting symbols independently.
Let symbols:

$$x_1, x_2, \dots, x_n$$

with probabilities:

$$p_1, p_2, \dots, p_n$$

Consider a long sequence of length N

Suppose symbol x_i appears:

$$Np_i$$

times (for large N , by law of large numbers).

Total probability of the sequence

Because symbols are independent:

$$P(\text{sequence}) = p_1^{Np_1} p_2^{Np_2} \cdots p_n^{Np_n}$$

Information content of the sequence

$$\begin{aligned} I(\text{sequence}) &= -\log_b P(\text{sequence}) \\ &= -\log_b (p_1^{Np_1} p_2^{Np_2} \cdots p_n^{Np_n}) \end{aligned}$$

Using log property

$$\begin{aligned} &= -N (p_1 \log_b p_1 + p_2 \log_b p_2 + \cdots + p_n \log_b p_n) \\ &= NH(X) \end{aligned}$$

Average information per symbol

$$\begin{aligned} \text{Average information per symbol} &= \frac{I(\text{sequence})}{N} \\ &= H(X) \\ &= -\sum_{i=1}^n p_i \log_b p_i \end{aligned}$$

For long independent sequences, the **average information content per symbol equals entropy**:

$$H(X) = -\sum_{i=1}^n p(x_i) \log_b p(x_i)$$

and total information in a sequence of length N :

$$I = NH(X)$$

1. Entropy represents average uncertainty
2. Also represents minimum number of bits needed per symbol
3. Fundamental limit for data compression (Shannon's source coding theorem)

Answer-6(a)

Properties of Mutual Information

- Measures the amount of information one random variable contains about another
- Represents reduction in uncertainty between X and Y

$$I(X;Y) = H(X) - H(X|Y) \text{ or } I(X;Y) = H(X,Y)$$

1) Symmetry

- Mutual information is symmetric:

$$I(X;Y) = I(Y;X)$$

Proof:

- Starting from the definition:

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

- Based on the relationship:

$$H(X;Y) = H(Y;X)$$

Soe that:

$$I(Y;X) = H(Y) + H(X) - H(Y,X)$$

$$\Rightarrow I(X;Y) = I(Y;X)$$

Interpretation:

- Information that X provides about Y is equal to information that Y provides about X
- Mutual information is a shared quantity

Alternative proof (using probability):

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

Logarithm of ratio ≥ 0 on average:
 $\Rightarrow I(X;Y) \geq 0$

2) Non-Negativity

- Mutual information is always non-negative:

$$I(X;Y) \geq 0$$

- Represents reduction in uncertainty
- Uncertainty reduction cannot be negative
- If variables are independent \rightarrow no information shared

Proof:

- From definition:

$$I(X;Y) = H(X) - H(X|Y)$$

$$\rightarrow I(X;Y) \geq 0$$

Equality: $I(X;Y) = 0$ when

$$p(x,y) = p(x)p(y)$$

i.e., X and Y are independent:

Property	Expression	Meaning
Symmetry	$I(X;Y) = I(Y;X)$	Equal both ways.
Non-negativity	$I(X;Y) \geq 0$	Information shared is never negative.
Zero condition	$I(X;Y) = 0$	X and Y are independent.

Answer-6(b)

Information Capacity Law

(Shannon-Hartley Theorem)

• Channel Model:

- Bandwidth = B Hz
- Signal Power = S
- Noise Power = N (AWGN)
- Nyquist Theorem: Max. samples/sec = $2B$

• Info. per Sample:

$$I = \frac{1}{2} \log_2 \left(1 + \frac{S}{N} \right) \text{ (bits/sample)}$$

• Total Info. per Second:

$$C = 2B \times \frac{1}{2} \log_2 \left(1 + \frac{S}{N} \right) \\ = B \log_2 \left(1 + \frac{S}{N} \right)$$

$$C = B \log_2 \left(1 + \frac{S}{N} \right) \\ \text{(Capacity Formula)}$$

• Where:

- C = Channel Capacity (bits/sec)
- B = Bandwidth (Hz)
- S = Signal Power
- N = Noise Power
- $\frac{S}{N}$ = Signal-to-Noise Ratio (SNR)

Max theoretical data rate!

- \uparrow Bandwidth
- \uparrow SNR
- \uparrow Power

• Alternative Form:

- Since $N = N_0 B$

$$C = B \log_2 \left(1 + \frac{S}{N_0 B} \right)$$

Question-8(b)

Hamming Weight, Hamming Distance & Minimum Distance of LBC

• Hamming Weight

- Number of 1's in a binary vector

Example: $w(1\bar{0}1\bar{1}\bar{0}1\bar{0}) = 4$

- Property: Denoted as $w(x)$

• Hamming Distance

- Number of positions which differ between two binary vectors

Example: $x = 1011010$ and $y = 1100110$

$0111100 \rightarrow = 0111100$

$d(x,y) = 3$

- Property: Denoted as $d(x,y)$

• Minimum Distance of LBC

- Smallest Hamming distance of any pair of codewords in the code

Example: $x = 1011010$ and $y = 1100110$

0111100

$d(x,y) = 3$

- Property: Denoted as $d(x,y)$

• Minimum Distance of LBC

$n = 4, k = 2$ Linear Block Code:

$00 \rightarrow c_1 = 0000$

$01 \rightarrow c_2 = 1011$

$10 \rightarrow c_3 = 0101$

$11 \rightarrow c_4 = 1110$

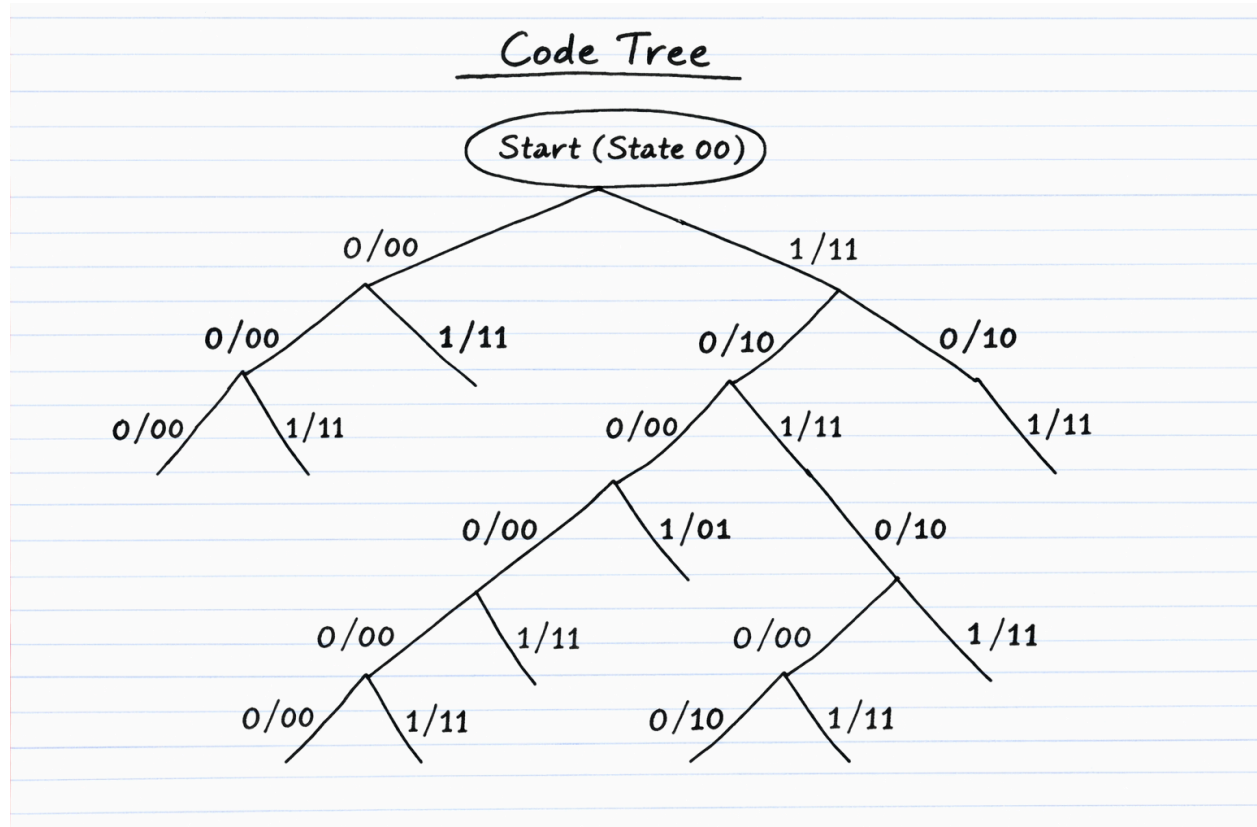
Property	Expression	Meaning
Minimum distance:	$d_{min} = \text{smallest } d(c_i, c_j)$	$d(c_1, c_2)$ $d(c_3, c_4) = 2$

$d_{min} = 2$ Minimum distance = 2

Question-9(b)

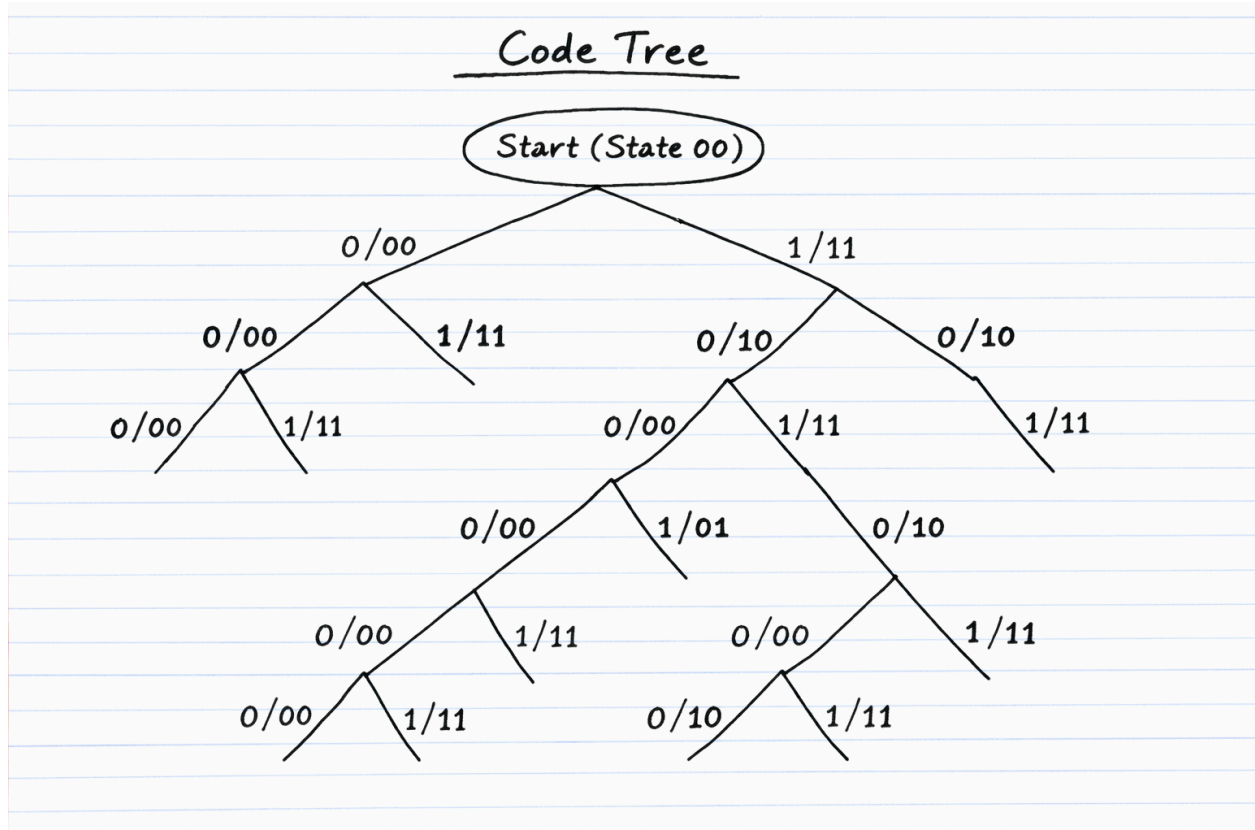
1. Code Tree:

- A code tree is a graphical representation showing all possible output sequences of a convolutional encoder.
- It starts from an initial state and branches for every possible input bit (0 or 1).
- Each branch shows encoder output corresponding to that input.
- Number of branches increases exponentially with time.
- Used mainly for understanding encoder operation, but not efficient for Decoding.



2. State Graph:

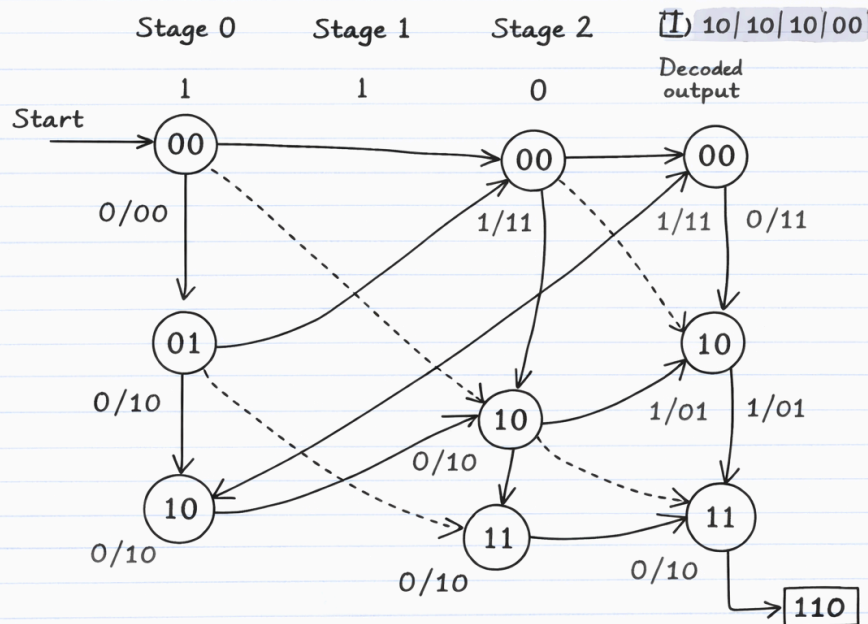
- A state graph shows all encoder states and transitions between them.
- Each node represents one state of the encoder.
- Each directed branch represents transition due to input bit.
- Output corresponding to transition is labeled on branch.
- Does not show time progression explicitly.



3. Trellis Graph:

- A trellis graph is a time-expanded version of the state graph.
- It shows state transitions at each time instant.
- States are repeated at every time step.
- Used in Viterbi algorithm for decoding convolutional codes.
- Makes it easier to find the most likely transmitted sequence

Trellis Code for 101



Question-10(a)

Steps of the Viterbi Algorithm

The Viterbi algorithm is a maximum likelihood decoding algorithm used to decode convolutional codes. It finds the most probable transmitted sequence by selecting the path with the minimum distance metric in the trellis diagram.

Key Terms:

- State: Condition of encoder shift registers
- Trellis: Time expanded state diagram
- Branch Metric (BM): Distance between received and expected output
- Path Metric (PM): Sum of branch metrics
- Survivor Path: Path with minimum metric
- ACS Operation: Add-Compare-Select process

Steps of Viterbi Algorithm:

1. Draw the trellis diagram representing all possible state transitions.
2. Initialize path metrics: Initial state metric = 0, other states = ∞ .
3. Calculate branch metric using Hamming distance (hard decision) or

Euclidean distance (soft decision).

4. Perform Add–Compare–Select operation to find minimum path metric.

5. Store survivor path for each state.

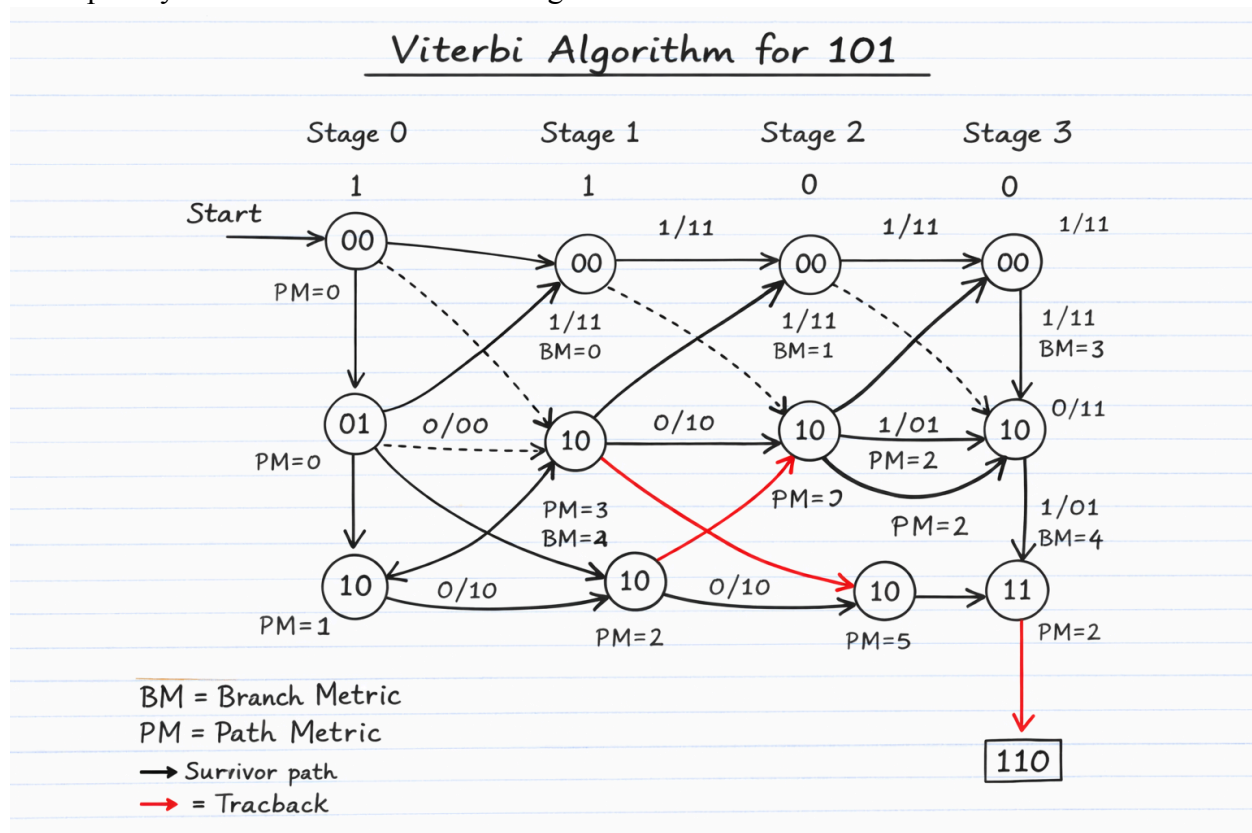
6. Repeat the process for entire received sequence.

7. Perform traceback from final state with minimum metric.

8. Obtain decoded sequence from survivor path.

Important Points:

- Provides optimal decoding.
- Minimizes probability of error.
- Used in wireless, satellite and mobile communication.
- Complexity increases with constraint length.



Answer-7(a)

Given Generator Matrix G:

G =

[1 0 0 1 0 1
 0 1 0 1 1 0
 0 0 1 0 1 1]

All possible message vectors and codewords:

000 → 000000

001 → 001011

010 → 010110

011 → 011101

100 → 100101

101 → 101110

110 → 110011

111 → 111000

Hamming weights:

000000 → w=0

001011 → w=3

010110 → w=3

011101 → w=4

100101 → w=3

101110 → w=4

110011 → w=4

111000 → w=3

Minimum distance $d_{min} = 3$

Parity Check Matrix H:

H =

[1 1 0 1 0 0

0 1 1 0 1 0

1 0 1 0 0 1]

Encoder circuit equations:

$c_1 = m_1$

$c_2 = m_2$

$c_3 = m_3$

$c_4 = m_1 \oplus m_2$

$c_5 = m_2 \oplus m_3$

$c_6 = m_1 \oplus m_3$

