CBCS SCHEME

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Second Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\cdot dx\cdot dz$ **(07 Marks)**

b. By changing the order of integration, Evaluate $\int_{0}^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xy\cdot dy\cdot dx$ **(07 Marks)**

c. Shot that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

OR

2 a. Change the integral $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}}\sqrt{x^2+y^2}\cdot dy\cdot dx$ into polar and hence evaluate. **(06 Marks)**

b. Find by double integration the area enclosed by the curve r = a(1 + cosθ) between θ = 0 and θ = Π. **(07 Marks)**

c. Show that $\int_{0}^{\Pi/2}\dfrac{d\theta}{\sqrt{\sin\theta}}\times\int_{0}^{\Pi/2}\sqrt{\sin\theta}\cdot d\theta=\Pi$. **(07 Marks)**

Module-2

3 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, −2, −1) along $2\hat{i}-\hat{j}-2\hat{k}$. **(06 Marks)**

b. If $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$. Find div $\vec{F}$ and curl $\vec{F}$. **(07 Marks)**

c. Show that $\vec{F}$ = (y + z) i + (z + x) j + (x + y)k is irrotational. Also a scalar function ϕ such that $\vec{F}$ ∇ϕ. **(07 Marks)**

OR

4 a. If $\vec{F}= xy\hat{i}+yz\hat{j}+zx\hat{k}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the curve represented by $x=t,\ y=t^2,\ z=t^3,\ -1\le t\le 1$. **(06 Marks)**

b. Verify Green's theorem in a plane for $\int(3x^2-8y^2)dx+(4y-6xy)dy$ where C is the boundary of the region enclosed by $y=\sqrt{x}$ and $y=x^2$. **(07 Marks)**

c. Verify Stokes theorem $\vec{F}$ = (2x − y) i − yz² j − y²zk, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$, C is its boundary. **(07 Marks)**

Module-3

5 a. Form the PDE by eliminating the arbitrary function from Z = y f(x) + x g(y). **(06 Marks)**

b. Solve $\dfrac{\partial^2 z}{\partial x^2}=xy$ subject to the conditions that $\dfrac{\partial z}{\partial x}=\log(1+y)$ when x = 1 and z = 0 when x = 0. **(07 Marks)**

c. Derive one dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2}=C^2\dfrac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

OR

6 a. Form a partial differential equation by eliminating arbitrary constant from $2z=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$. **(06 Marks)**

b. Solve : (y − z) p + (z − x)q = x − y. **(07 Marks)**

c. Solve : $\dfrac{\partial^2 z}{\partial y^2}=z$, given the when y = 0, $z=e^x$ and $\dfrac{\partial z}{\partial y}=e^{-x}$. **(07 Marks)**

Module-4

7 a. Find a real root of the equation $x^3-2x-5=0$ correct to three decimal places by using regula – falsi method. **(06 Marks)**

(10 Marks)

b. The area of a circle (A) corresponding to diameter (D) is given below :

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula. **(07 Marks)**

c. Evaluate using Simpson's 1/3rd rule $\int_0^6 \dfrac{e^x}{1+x}\cdot dx$ by taking six equal parts. **(07 Marks)**

OR

8. a. Find the real root of the equation $xe^x-2=0$ correct to three decimal places by applying Newton Raphson method. **(06 Marks)**

b. Use Newton's divided difference formula to find f(4) given the data :

X	0	2	3	6
f(x)	−4	2	14	158

(07 Marks)

c. Use Simpson's 3/8th rule to evaluate $\int_1^4 e^{1/x}dx$ **(07 Marks)**

Module-5

9 a. Use Taylor's series method to find y at x = 0.1, 0.2, 0.3 considering terms up to the third degree given that $dy/dx = x^2 + y^2$ and y(0) = 1. **(06 Marks)**

b. Given dy/dx = 3x + y/2, y(0) = 1. Using compute y(0.2) by taking h = 0.2. Using Runge-Kutta method of fourth order. **(07 Marks)**

c. Apply Milne's method to compute y(1.4) corrector to four decimal places given $dy/dx = x^2 + y/2$ and following data :
y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. **(07 Marks)**

OR

10 a. Employ Taylor's series method to find y at x = 0.1 given $\frac{dy}{dx} - 2y = 3e^x$ whose solution passes through the origin. **(06 Marks)**

b. Given $\frac{dy}{dt} = 1 + y/x$, y = 2, at x = 1 find the approximate value y at x =1.4 by taking step size h = 0.2 applying modified Euler's method. **(07 Marks)**

c. Use forth order Runge – kutta method to find y at x = 0.1, given that $\frac{dy}{dx} = 3e^x + 2y$, y(0) = 0 and h = 0.1. **(07 Marks)**

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