CBCS SCHEME

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First Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notation, prove that
$$\frac{1}{P^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2.$$ (06 Marks)

b. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 4$ at the point where it cuts and line passing through the origin making an angle 45° with the x-axis. (06 Marks)

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. (08 Marks)

OR

2 a. Find the angle between pair of curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$. (06 Marks)

b. Show that for the curve $r(1 - \cos\theta) = 2a$, ρ^2 varies as r^3. (06 Marks)

c. Find the pedal equation of the curve $r^n = a^n \sin n\theta + b^n \cos n\theta$. (08 Marks)

Module-2

3 a. Using Maclaurin's series, prove that
$$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - - - - -$$ (06 Marks)

b. Evaluate : $\lim_{x \to 0}\left[\frac{a^x + b^x + c^x}{3}\right]^{1/x}$. (07 Marks)

c. Find the extreme values of the function : $x^3 + y^3 - 3x - 12y + 20$. (07 Marks)

OR

4 a. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ P.T $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. (06 Marks)

b. If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, S.T $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. (07 Marks)

c. A rectangular box open at the top is to have a volume of 32 cubic feet. Find its dimensions, if the total surface area is minimum. (07 Marks)

Module-3

5 a. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$ by changing the order of integration. (06 Marks)

b. Evaluate : $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)dz\, dy\, dx$. (07 Marks)

c. Prove that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)

OR

6 a. Evaluate : $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing in to polar form. (06 Marks)

b. Find the volume of the tetrahedron bounded by the planes x = 0, y = 0, z =0 and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (07 Marks)

c. Prove that $\int_0^{\pi/2}\sqrt{\sin\theta}\, d\theta \cdot \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} = \Pi$. (07 Marks)

Module-4

7 a. Solve $(2xy + y - \tan y)\, dx + (x^2 - x\tan^2 y + \sec^2 y)\, dy = 0$. (06 Marks)

b. Find the orthogonal trajectories of the family $r = a(1 + \sin\theta)$. (07 Marks)

c. Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. (07 Marks)

OR

8 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. (06 Marks)

b. A body in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes. (07 Marks)

c. Solve $e^{4x}(P - 1) + e^{2y}P^2 = 0$ by using the substitution $u = e^{2x}$ and $v = e^{2y}$. (07 Marks)

Module-5

9 a. Find the rank of the matrix :

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$ by elementary row operation. **(06 Marks)**

b. Test for consistency and solve

$5x_1 + x_2 + 3x_3 = 20$
$2x_1 + 5x_2 + 2x_3 = 18$
$3x_1 + 2x_2 + x_3 = 14$. **(07 Marks)**

c. Find the dominant eigan value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

By power method taking the initial eigen vector $[1 \ 1 \ 1]'$. **(07 Marks)**

OR

10 a. Solve the following system of equations by Gauss-Jordan method :

$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$. **(06 Marks)**

b. Solve the following system of equations by Gauss-Seidal method :

$10x + y + z = 12$
$x + 10y + z = 12$
$x + y + 10z = 12$. **(07 Marks)**

c. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(07 Marks)**

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