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First Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. With usual notations. Prove that $\tan\phi = \dfrac{rd\theta}{dr}$. **(07 Marks)**

b. Find the Pedal equation of the curve $r^m \cos m\theta = a^m$. **(06 Marks)**

c. Show that in the rectangular hyperbola $r^2\cos 2\theta = a^2$, $P = \dfrac{r^3}{a^2}$. **(07 Marks)**

OR

2 a. Find the angle between the two curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ **(07 Marks)**

b. Find the Pedal equation of the curve $r^2 = a^2\sec 2\theta$ **(06 Marks)**

c. Show that the radius of curvature at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ on the curve folium $x^3 + y^3 = 3axy$ is $\dfrac{3a}{8\sqrt{2}}$. **(07 Marks)**

Module-2

3 a. Using Maclaurin's series expand tanx upto the terms containing x^5. **(06 Marks)**

b. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$ **(07 Marks)**

c. If $u = e^x\cos y$ and $V = e^x\sin y$. Find $\dfrac{\partial(u,v)}{\partial(x,y)}$ **(07 Marks)**

OR

4 a. Evaluate : i) $\underset{x\to 0}{Lt}\left(\dfrac{a^x + b^x + c^x}{3}\right)^{1/x}$ **(06 Marks)**

b. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{z+y+z}$ **(07 Marks)**

c. Show that $f(x, y) = xy(a - x - y)$, $a > 0$ is a maximum at the point $\left(\dfrac{a}{3}, \dfrac{a}{3}\right)$. Find the maximum value. **(07 Marks)**

Module-3

5 a. Solve $\dfrac{dy}{dx} + \dfrac{y\cos y + \sin y + y}{\sin x + x\cos y + x} = 0$ **(06 Marks)**

b. A copper ball originally at 80°C cool down to 60°C in 20 minutes, if the temperature of the air being 40°C, what will be the temperature of the ball after 40 minutes from the original. **(07 Marks)**

c. Solve : $xyp^2 - (x^2 + y^2)P + xy = 0$ where $P = \dfrac{dy}{dx}$. **(07 Marks)**

OR

6 a. Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = xy^2$. **(06 Marks)**

b. Solve : $(x^2 + y^2 + x)\,dx + xydy = 0$. **(07 Marks)**

c. Find the general solution of the equation $(Px - y)(Py + x) = 2P$ by reducing into Clariut's form, taking the substitution $X = x^2$, $Y = y^2$. **(07 Marks)**

Module-4

7 a. Solve : $(4D^4 - 8D^3 - 7D^2 + 11D + 6)\,Y = 0$ **(06 Marks)**

b. Solve : $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 12x^2$ **(07 Marks)**

c. Find the solution of the Cauchy's linear equation $\dfrac{x^3d^3y}{dx^3} + \dfrac{x^2d^2y}{dx^2} = x$ **(07 Marks)**

OR

8 a. Solve : $(D^3 + 1)\,y = 5e^{2x} + 2^x$ **(06 Marks)**

b. Solve : $(D^3 + D)y = 6\cos 3x$ **(07 Marks)**

c. Using the method of variation of parameters, solve $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = \dfrac{e^{3x}}{x^2}$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 16 & 8 & -6 & -2 \end{bmatrix}$$ **(06 Marks)**

b. Solve the system of equations by Gauss elimination method
$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$ **(07 Marks)**

c. Use Power method to find the largest eigen value and the corresponding eigen vector. Carry out 5 iterations.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \text{ and } x^{(0)} = [1\ 0\ 0]^T$$ **(07 Marks)**

OR

10 a. Determine the values of λ and μ for which the system
$x + 2y + 3z = 6$
$x + 3y + 5z = 9$
$2x + 5y + \lambda z = \mu$
has i) no solution ii) unique solution iii) infinite number of solutions. **(06 Marks)**

b. Apply Gauss Jordon method to solve the equations
$2x + 5y + 7z = 52$
$2x + y - z = 0$
$x + y + z = 9$ **(07 Marks)**

c. Solve the following system of equations by Gauss – Siedel method
$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$
Perform three iterations. **(07 Marks)**

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