CBCS SCHEME

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18MAT21

Second Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the angle between normal's to the surface $xy = z^2$ at the point (4, 1, 2) and (3, 3, –3). **(06 Marks)**

b. If $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$ find div $\vec{F}$ and curl $\vec{F}$ at (1, 2, 3). **(07 Marks)**

c. Find 'a' and 'b' such that $\vec{F} = (axy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (bxz^2 - y)\vec{k}$ is irrotational. Also find a scalar ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. Using Stoke's theorem evaluate $\int \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a$ and $y = b$. **(06 Marks)**

b. Using divergence theorem evaluate $\int \vec{A} \cdot \hat{n}\, ds$ where $\vec{A} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(07 Marks)**

c. Find work done in moving a particle once around ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ in plane $z = 0$ where $\vec{F} = (3x - 2y)\vec{i} + (2x + 8y)\vec{j} + y^2\vec{k}$. **(07 Marks)**

Module-2

3 a. Solve $(D^2 + 4)y = x^2 + \cos 2x + e^{2x}$. **(06 Marks)**

b. Solve $y'' + a^2y = \sec a\, x$ by the method of variation of parameter. **(07 Marks)**

c. The current i and the charge q in a series circuit containing an inductance 'L' and capacitance 'C', emf E, Express q and i interms of 't' given that L, C, E are constants and i and q are zero at the initial stage. **(07 Marks)**

OR

4 a. Solve $(D^3 - 7D + 6)y = 1 - x + x^2$. **(06 Marks)**

b. Solve : $(2x + 3)^2 y'' + 6(2x + 3)y' + 6y = \log(2x + 3)$. **(07 Marks)**

c. The differential equation of the displacement x(t) of a spring at the upper end and weight at its lower end is given by $10\frac{d^2x}{dt^2} + \frac{dx}{dt} + 200x = 0$. The weight is pulled down 0.25 cm below the equilibrium position and then released. Find the expression of the displacement of the weight from its equilibrium at any time 't' during first upward motion. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating arbitrary constants from : $\log(az - 1) = x + ay + b$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} + 9x^2y^2 = \cos(2x - y)$ given $z = 0$ when $y = 0$ and $z_y = 0$ when $x = 0$. **(07 Marks)**

c. Derive one dimensional wave equation in standard form. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating arbitrary function from $xyz = f(x^2 + y^2 + z^2)$. **(06 Marks)**

b. Solve : $(mz - ny)p + (nx - \ell z)q = (\ell y - mx)$. **(07 Marks)**

c. Solve one dimensional heat equation, using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$. **(05 Marks)**

b. Solve Bessel's differential equation leading to $J_n(x)$. **(10 Marks)**

c. Express $x^3 + 2x^2 - x = 3$ interms of Legendre polynomials. **(05 Marks)**

OR

8 a. Find the nature of the series :

$$\left[\frac{2^2}{1^2}-\frac{2}{1}\right]^{-1}+\left[\frac{3^3}{2^3}-\frac{3}{2}\right]^{-2}+\left[\frac{4^4}{3^4}-\frac{4}{3}\right]^{-3}+\text{-----}$$ **(05 Marks)**

b. Show that

i) $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$

ii) $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$ **(10 Marks)**

c. State Rodrigue's formula for Legendre polynomials and obtain the expression for $P_3(x)$ from it. Find $\int_{-1}^{1} x^2 P_3(x)\,dx$. **(05 Marks)**

Module-5

9 a. Find the interpolating polynomial f(x) satisfying f(0) = 0, f(2) = 4, f(4) = 56, f(6) = 204, f(8) = 496, f(10) = 980. Hence find f(3) and f(5). **(06 Marks)**

b. Find the real root of the equation $x e^x = 2$. Correct to 4 decimal places using Newton's Raphson's method. **(07 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule taking 6 equal intervals and hence deduce an approximate value for $\log_e 2$. **(07 Marks)**

OR

10 a. The following table gives normal weights of babies during first eight months of life.

Age (in months)	0	2	5	8
Weight (in pounds)	6	10	12	16

Estimate the weight of the baby at the age of seven months using Lagrang's interpolation formula. **(06 Marks)**

b. Compute the real root of $x\log x_{10}-1.2=0$ by the method of false position carry out three iterations. **(07 Marks)**

c. Evaluate the integral $\int_0^6 \frac{dx}{1+x^2}$ by using the Weddle's rule with h = 0.5. Compare the result with actual value. **(07 Marks)**

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