CBCS SCHEME

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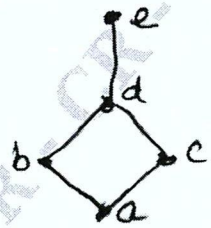
BCS405A

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

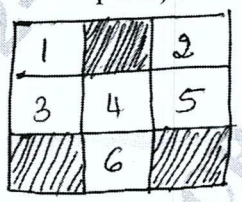
		Module – 1	M	L	C
Q.1	a.	Define a logically equivalent. Show that $[(p \vee q) \rightarrow r] \leftrightarrow [\neg r \rightarrow \neg (p \vee q)]$ for primitive statements p, q, r is logically equivalence.	6	L1	CO1
	b.	Establish the validity of the following argument using the rules of inference: $\{(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg q \vee \neg s)\} \rightarrow \neg (p \wedge r)$	7	L2	CO1
	c.	i) Give an indirect proof of the statement: "The product of two even integers is an even integer". ii) Provide a proof by contradiction of the following statement: "For every integer n, if n^2 is odd, then n is odd".	7	L3	CO1
		OR			
Q.2	a.	Define a tautology. Prove that, for an propositions p, q, r the compound proposition. $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$ is tautology.	6	L1	CO1
	b.	Using law of logic, prove the result. $[(p \vee q) \wedge (p \vee \neg q)] \vee q \Leftrightarrow p \vee q$	7	L2	CO1
	c.	Find whether the following argument is valid: If a triangle has two equal sides, then it is isoceles. If a triangle is isoceles, then it has two equal angles. A certain triangle PQR does not have two equal angles. ∴ The triangle PQR does not have two equal sides.	7	L3	CO1
		Module – 2			
Q.3	a.	By mathematical induction, prove that $(n!) \geq 2^{n-1}$ for all integers $n \geq 1$.	6	L2	CO2
	b.	Find an in explicit form for the following: i) $a_1 = 4, a_n = a_{n-1} + n$ for $n \geq 2$ ii) $a_1 = 7, a_n = 2a_{n-1} + n$ for $n \geq 2$	7	L2	CO2
	c.	Find how many distinct four-digit integers one can make from the digits 1, 3, 3, 7, 7, 8	7	L3	CO2

		OR			
Q.4	**a.**	Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$.	**6**	**L2**	**CO2**
	b.	i) How many arrangements are there for all letters in the word SOCIOLOGICAL? ii) In how many of these arrangements all the vowels are adjacent?	**7**	**L3**	**CO2**
	c.	Determine the coefficient of i) xyz^2 in the expansion of $(2x - y - z)^4$ ii) $x^2y^2z^3$ in the expansion of $(3x - 2y - 4z)^7$	**7**	**L2**	**CO2**
		Module – 3			
Q.5	**a.**	Let A = {1, 2, 3, 4, 5, 6} and B = {6, 7, 8, 9, 10}. If a function $f : A \rightarrow B$ is defined by f = {(1, 7), (2, 7), (3, 8), (4, 6), (5, 9), (6, 9)} determine $f^{-1}(6)$ and $f^{-1}(9)$. If $B_1 = \{7, 8\}$ and $B_2 = \{8, 9, 10\}$, find $f^{-1}(B_1)$ and $f^{-1}(B_2)$.	**6**	**L2**	**CO2**
	b.	State Pigeon-hole principle. ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that atleast two of these points are such the distance between them is less than 1/2 cm.	**7**	**L3**	**CO2**
	c.	Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$ if $(gof)(x) = 9x^2 - 9x + 3$, determine a & b.	**7**	**L2**	**CO2**
		OR			
Q.6	**a.**	Let f, g, h be functions from z to z defined by $f(x) = x - 1$, $g(x) = 3x$, $h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$ Determine (f o (g o h)) (x) and ((f o g) o h) (x) and verify that f o (g o h) = (f o g) o h.	**6**	**L2**	**CO3**
	b.	For A = {a, b, c, d, e}, the Hasse diagram for the poset (A, R) is as shown in Fig.Q.6(b). i) Determine the relation matrix for R. ii) Construct the diagraph for R.  Fig.Q.6(b) CMRIT LIBRARY BANGALORE - 560 037	**7**	**L3**	**CO3**
	c.	Let A = {1, 2, 3, 4} and let R be the relation on A defined by xRy if and only if "x divides y", i) Write down R as a set of ordered pairs ii) Draw the diagraph of R iii) Determine the in-degrees and out-degrees of the vertices in the digraph.	**7**	**L3**	**CO3**

Module – 4

Q.7	a.	In how many ways 5 number of a's, 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block.	6	L2	CO4
	b.	Find the number of permutations of the letters a, b, c, ….. x, y, z in which none of the patterns spin, game, path or net occurs.	7	L3	CO4
	c.	Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$, given that $a_0 = 5$, $a_1 = 12$.	7	L2	CO4

OR

Q.8	a.	Find the number of derangements of 1, 2, 3, 4. Mention them.	6	L2	CO4
	b.	By using the expansion formula, find the rook polynomial for the board c shown below (madeup of unshaded parts) : 	7	L3	CO4
	c.	Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$	7	L2	CO4

Module – 5

Q.9	a.	Let $w_4 = \{1, -1, i, -i\}$, the set of all fourth roots of unity. Show that w_4 is an abelian group under x.	6	L2	CO5
	b.	Define Klein-4-group. If A = {e, a, b, c} then show that this is a Klein-4 group.	7	L2	CO5
	c.	State and prove Lagrange's theorem.	7	L2	CO5

OR

Q.10	a.	Show that the group (G, ∗) whose multiplication table is as given below is cyclic.	6	L	CO

∗	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

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	b.	Let G = S_4, the symmetric group of degree 4. For $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$. Find the subgroup H = <α>. Also, determine the number of left cosets of H in G.	7	L3	CO5
	c.	If * is an operation on z defined by x * y = x + y + 1, prove that (z, *) is an abelian group.	7	L2	CO5

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