CBCS SCHEME

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21MAT31

Third Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Transform Calculus Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of
i) $\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^3$ ii) t.Cosat **(07 Marks)**

b. Find the Laplace transform of the following rectifier f(t) = E sin wt, $0 < t < \frac{\pi}{W}$ having a period $\frac{\pi}{W}$. **(06 Marks)**

c. Using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ **(07 Marks)**

OR

2 a. Using unit step function. Find the Laplace transform of $f(t)=\begin{cases}\text{Cost}, & 0<t<\pi\\ 1, & \pi<t<2\pi\\ \sin t, & t>2\pi\end{cases}$ **(07 Marks)**

b. Find the inverse Laplace transform $\frac{4s+5}{(s+1)^2(s+2)}$ **(06 Marks)**

c. Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$. Using Laplace transform. **(07 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x)=\frac{\pi-x}{2}$ in $0 < x < 2\pi$. Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots..=\frac{\pi}{4}$ **(07 Marks)**

b. Find a Cosine half range series for $f(x) = (x-1)^2$, $0 \le x \le 1$ **(06 Marks)**

c. Obtain the constant term and the first cosine and sine terms in the Fourier series:

x°	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

(07 Marks)

OR

4 a. Obtain the Fourier series expansion for the function $f(x) = x - x^2$ in $(-\pi < x < \pi)$ **(07 Marks)**

b. Obtain the sine half range series of $f(x)=1-\left(\frac{x}{\pi}\right)$ in $0 < x < \pi$ **(06 Marks)**

c. Obtain the constant term and the first two coefficient in the Fourier cosine series for y using the following table

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(07 Marks)

Module-3

5 a. Find the complex Fourier transform of $f(x)=\begin{cases}1 & \text{for } |x|\le a\\ 0 & \text{for } |x|>0\end{cases}$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$ **(07 Marks)**

b. Find the Fourier sine and cosine transform of $f(x) = e^{-\alpha x}$, $\alpha > 0$ **(06 Marks)**

c. Find the Z-transform of sin (3n + 5) **(07 Marks)**

OR

6 a. Find the Fourier cosine transform of the function

$$f(x)=\begin{cases}4x, & 0<x<1\\ 4-x, & 1<x<4\\ 0 & x>4\end{cases}$$ **(07 Marks)**

b. Find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$ **(06 Marks)**

c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transform. **(07 Marks)**

Module-4

7 a. Classify the following partial differential equation :
i) $u_{xx} + 4u_{xy} + 4u_{yy} - u_x + 2u_y = 0$
ii) $x^2u_{xx} + (1-y^2)u_{yy} = 0,\ -1 < y < 1$
iii) $(1+x^2)u_{xx} + (5+2x^2)u_{xx} + (4+x^2)u_{tt} = 0$
iv) $y^2u_{xx} - 2yu_{xy} + u_{yy} - u_y = 8y$ **(10 Marks)**

b. Solve the wave equation $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$ subject to u(0, t) = 0, u(4, t) = 0, $u_t(x, 0) = 0$ and u(x, 0) = x(4 – x) by taking h = 1, k = 0.5 upto four steps. **(10 Marks)**

OR

8 a. Find the numerical solution of the parabolic equation $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ when $u(0, t) = 0 = u(4, t)$ and $u(x, 0) = x(4 - x)$ by taking $h = 1$. Find the value upto $t = 5$. **(10 Marks)**

b. Solve Laplaces equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in the following Fig.Q.8(b).

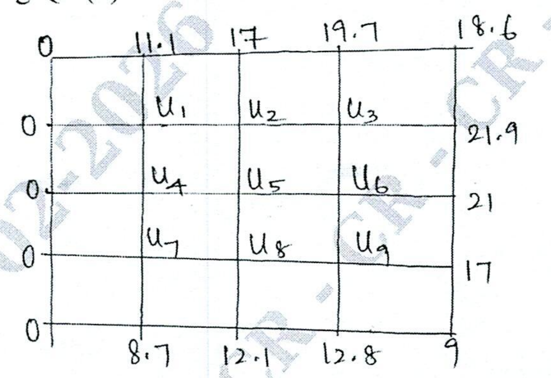

Fig.Q.8(b)

(10 Marks)

Module-5

9 a. Use Runge-Kutta method to find $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute $y(0.2)$ and $y'(0.2)$. **(07 Marks)**

b. Find the extremal of the functional $\int_a^b \left(x^2y'^2 + 2y^2 + 2xy\right)dx$ **(06 Marks)**

c. Derive Euler's Equation. **(07 Marks)**

OR

10 a. Apply Milne's method to compute $y(0.4)$, given that $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

(07 Marks)

b. Find the extremal of the functional $\int_{x_1}^{x_2} (y' + x^2y'^2)dx$ **(06 Marks)**

c. Prove that shortest distance between two points in a plane ins a straight line. **(07 Marks)**

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