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MATDIP401

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions.***
2. Mathematics formulae handbook is allowed.

1 a. Find the angle between the vectors $\vec{a} = 5\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. **(06 Marks)**

b. Find the value of λ so that the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{j} + \lambda\hat{k}$ are coplanar. **(06 Marks)**

c. Prove that $\left[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}\right] = 0$. **(08 Marks)**

2 a. Find the sine of an angle between $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(06 Marks)**

b. Determine the value of a so that $\vec{A} = 2\hat{i} - a\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular. **(06 Marks)**

c. Show that the four points whose position vectors are $3\hat{i} - 2\hat{j} + 4\hat{k}$, $6\hat{i} + 3\hat{j} + \hat{k}$, $5\hat{i} + 7\hat{j} + 3\hat{k}$ and $2\hat{i} + 2\hat{j} + 6\hat{k}$ are coplanar. **(08 Marks)**

3 a. Find the component of velocity and acceleration at t = 2 on the curve $r = (t^2 + 1)i + (4t - 3)j + (2t^2 - 6t)k$ in the direction of $i + 2j + 2k$. **(06 Marks)**

b. Find the directional derivative of $f(x, y, z) = xy^3 + yz^3$ at the point (2, –1, 1) in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. **(06 Marks)**

c. Find the constants a, b, c so that the vector field $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. **(08 Marks)**

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4 a. Show that $\vec{F} = (-x^2 + yz)\vec{i} + (4y - z^2x)\vec{j} + (2xz - 4z)\vec{k}$ is Solenoidal. **(06 Marks)**

b. Show that $\vec{F} = (2xy^2 + yz)\vec{i} + (2x^2y + xz + 2yz^2)\vec{j} + (2y^2z + xy)\vec{k}$ is a conservative force field (irrotational). **(06 Marks)**

c. Find $\text{div}\vec{F}$ and $\text{Curl}\,\vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(08 Marks)**

5 a. Solve : $(D^2 + 6D + 9)\,y = 0$, where $D = \frac{d}{dx}$. **(06 Marks)**

b. Solve : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = e^{-2x}$. **(06 Marks)**

c. Solve : $\frac{d^2y}{dx^2} - 4y = \cos x$. **(08 Marks)**

6 a. Solve : $(D^3 - 2D + 4D - 8)\,y = 0$, where $D = \frac{d}{dx}$. **(06 Marks)**

b. Solve : $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = e^x$. **(06 Marks)**

c. Solve : $(D^2 + 3D + 2)\,y = \sin 2x$, where $D = \frac{d}{dx}$. **(08 Marks)**

7 a. Find the rank of the matrix :

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$$

By reducing to echelon form. **(06 Marks)**

b. Solve :

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

by Gauss elimination method. **(06 Marks)**

c. Test for consistency and solve :

$$x + 2y + 3z = 14$$
$$4x + 5y + 7z = 35$$
$$3x + 3y + 4z = 21.$$

(08 Marks)

8 a. Find the rank of the matrix by elementary row transformation

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}.$$

(06 Marks)

b. Solve :

$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$
$$x + y + z = 9$$

by Gauss Elimination Method. **(06 Marks)**

c. Find the Eigen values and one Eigen vector of the matrix :

$$\begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}.$$

(08 Marks)

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