



CBCS SCHEME

USN

18MAT41

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026 Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. Use of statistical tables allowed.

Module-1

- 1 a. Derive the Cauchy-Riemann equations in Cartesian form. (06 Marks)
- b. Find the constant values a, b, c and d given :
 $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$
 is an analytic function. (07 Marks)
- c. Find the analytic function $f(z)$ whose real part is $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. (07 Marks)

OR

- 2 a. Derive Cauchy-Riemann equations in polar form. (06 Marks)
- b. Show that $f(z) = z^3$ is an analytic function and find its derivative. (07 Marks)
- c. If $f(z)$ is an analytic function, then show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. (07 Marks)

Module-2

- 3 a. Discuss the conformal transformation of e^z . (06 Marks)
- b. Find the bilinear transformation of which maps the points $z = -i, 0, i$ onto the points $w = -1, i, 1$. (07 Marks)
- c. Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is a circle $|z| = 2$. (07 Marks)

OR

- 4 a. State and prove Cauchy's integral formula. (06 Marks)
- b. Find the bilinear transformation of which maps the points $z = \infty, i, 0$ on to the points $w = 0, -i, \infty$. (07 Marks)
- c. Evaluate: $\oint_C \frac{e^z}{z^2 + 5z + 6} dz$ where C is a circle : i) $|z| = 4$ ii) $|z| = \frac{5}{2}$. (07 Marks)

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Module-3

- 5 a. The probability density function of a finite variable x is given by

| | | | | | | | |
|------|----|----|----|----|----|----|---|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| p(x) | K | 2K | 3K | 4K | 3K | 2K | K |

Find : i) Value of K ii) $P(x \leq 1)$ iii) $P(x > 1)$ iv) $P(-1 < x \leq 2)$. (06 Marks)

- b. If 20% bolts produced by a machine are defective. Calculate the probability that out of a simplified selected at random of 7 bolts, not more than one bolt will be defective. (07 Marks)
- c. A sample of 100 dry battery cells tested to find the length of life produce the following results.
 Mean = 12 hours and S.D = 3 hours
 Assuming the data is normally distributed. What is the percentage of battery cells are expected to have a life :
 i) More than 15 hours
 ii) Less than 6 hours
 iii) Between 10 hours and 14 hours
 Given :
 $P(0 < z < 1) = 0.3414$
 $P(0 < z < 2) = 0.4772$
 $P(0 < z < 0.66) = 0.2454$. (07 Marks)

OR

- 6 a. A random variable x has the probability density function is given by
 $f(x) = kx^2 \quad -3 \leq x \leq 3$
 $= 0 \quad \text{otherwise}$
 Find : i) Value of K ii) $P(1 \leq x \leq 2)$ iii) $P(x \leq 2)$ iv) $P(x > 1)$. (06 Marks)
- b. In a certain factory producing cycle tyre, these is small chance of 1 in 500 tyres to be defective. The tyres are supplied in a lots of 10. Calculate the approximate number of lots containing.
 i) No defective tyres
 ii) One defective tyres
 iii) Two defective tyres.
 in the consignment of 10,000 lots. (07 Marks)
- c. In a test on 2000 electric bulbs, it was found that life of a particular make was normally distributed with average life of 2040 hours and S.D of 60 hours. Estimate number of bulbs likely to bush.
 i) More than 2150 hours
 ii) Less than 1950 hours
 iii) Between 1920 hours and 2160 hours
 Given :
 $P(0 < z < 1.83) = 0.4664$
 $P(0 < z < 1.50) = 0.4332$
 $P(0 < z < 2) = 0.4772$. (07 Marks)

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Module-4

7 a. Fit the curve $y = ax^b$ from the following data :

| | | | | | |
|---|-----|---|-----|---|------|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 0.5 | 2 | 4.5 | 8 | 12.1 |

(06 Marks)

b. Prove that $\tan \theta = \left(\frac{1-r^2}{r} \right) \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$, where θ is the angle between two regression lines.

(07 Marks)

c. Find Mean of x, mean of y, coefficient of regression and S.D of y from the lines of regression :

$$4x - 5y + 33 = 0$$

$$20x - 9y - 107 = 0$$

and variance of x = 9.

(07 Marks)

OR

8 a. Fit a second degree parabola $y = a + bx + cx^2$ form the following data :

| | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|
| x | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| y | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

(06 Marks)

b. Find coefficient of correlation from the following data :

| | | | | | | | |
|---|---|---|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

(07 Marks)

c. Find the rank correlation for the following data showing ranks of 10 students in the two subjects :

| | | | | | | | | | | |
|-----------|---|---|----|---|---|----|---|---|---|---|
| Physics | 3 | 8 | 9 | 2 | 7 | 10 | 4 | 6 | 1 | 5 |
| Chemistry | 5 | 9 | 10 | 1 | 8 | 7 | 3 | 4 | 2 | 6 |

(07 Marks)

Module-5

9 a. The joint distribution of two random variables x and y is as follows :

| | | | | |
|---|---|---------------|---------------|---------------|
| | Y | -4 | 2 | 7 |
| X | | | | |
| 1 | | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| 5 | | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

Determine :

- i) Covariance of X and Y
- ii) Correlation of X and Y.

(06 Marks)

b. A coin is tossed 400 times turns up head 216 times. Discuss whether the coin is biased.

(07 Marks)

c. A certain stimulus administered each of 12 patients resulted in the following change in blood pressure 5, 2, 8, -1, 3, 0, -1, 1, 5, 0, 4. Can it be considered that stimulus will increase the blood pressure?

(Given table value of t = 2.20 for degree of freedom V = 11 at 5% level of significance).

(07 Marks)

OR

10 a. Explain Hypothesis, errors and level of significance.

(06 Marks)

b. In a city 'A' 20% of random sample 900 school boys had a certain slight physical defect. In another city 'B' 18.5% of a random sample of 1600 school boys had the same physical defect. Is there a difference between two city school boys with respect to physical defect?

(07 Marks)

c. The following table gives the number of aircraft accidents that occurs at various days of the week. Find whether the circuit accidents are uniformly distributed over the week days.

| | | | | | | | |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| Days of week | MON | TUE | WED | THR | FRI | SAT | SUN |
| Accidents | 14 | 10 | 8 | 12 | 11 | 9 | 4 |

Apply Chi-square test.

Given table value of $\chi^2 = 12.59$ for degree of freedom $\gamma = 6$ at 5% level of significance).

(07 Marks)
