290

	USN	CMRIT

Internal Assessment Test – II January - 2026

Sub:	**Mathematics-1 for CSE Stream**							Code:	1BMATS101
Date:	05-01-2026	Duration:	90 mins	Max Marks:	50	Sem:	I	SEC	**A to H (PHY CYCLE)**

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	Find range, Null space and also Verify the Rank-nullity Theorem for the linear transformation $T:R^3 \to R^3$ defined by $T(x,y,z)=(x+2y-z,\ y+z,\ x+y-2z)$.	[08]	CO3	L2
2	Prove that cylindrical coordinate system is orthogonal.	[07]	CO1	L3
3	$u=f(x-y, y-z, z-x)$ show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$	[07]	CO1	L2

Internal assessment Test - 02

1. given $T: R^3 \to R^3$

$T(x,y,z) = (x+2y-z, y+z, x+y-2z)$

$V_3(R) = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T[1,0,0] = [1, 0, 1]$

$T[0,1,0] = [2, 1, 1]$

$T[0,0,1] = [-1, 1, -2]$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_1$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$r(A) = 2$

$n(A) = 1$

Rank nullity theorem

$$\boxed{\begin{array}{l} r(A) + n(A) = \dim(T) \\ 2+1 = 3 \\ \therefore \text{ Rank nullity theorem verified} \end{array}}$$

$R(T) = L(S)$

$R(T) = \{ (1,0,1), (0,1,-1) \}$

$R(T) = \{ x_1(1,0,1) + x_2(0,1,-1) \}$

$R(T) = \{ (x_1, 0, x_1) + (0, x_2, -x_2) \}$

$$\boxed{R(T) = \{ x_1, x_2, x_1 - x_2 \}}$$

$N(T) = \{ (x,y,z) \in R^3 \mid T(x,y,z) = 0 \}$

$T(x,y,z) = 0$

$(x+2y-z, y+z, x+y-2z) = 0$

$x+2y-z = 0$ — ①

$y+z = 0$ — ② $\Rightarrow$ $\cancel{y} = +z = -y$

$x+y-2z = 0$ — ③ put $y = t$

$z = -t$

eqn: ① $\Rightarrow$ $x + 2(t) - (-t) = 0$

$x + 2t + t = 0$

$x + 3t = 0$

$x = -3t$

$$\begin{bmatrix} x = -3t \\ y = t \\ z = -t \end{bmatrix}$$

$N(T) = \{ (-3t, t, -t) \mid t \in R \}$

$\boxed{N(T) = \{ t(-3,1,-1) \}}$ // $[N(T) = \{ \text{span}(-3,1,-1) \}]$

2. To prove cylindrical coordinate is orthogonal

$$\boxed{\begin{matrix} h_1 = 1 & h_2 = \rho & h_3 = 1 \\ x = \rho\cos\phi & y = \rho\sin\phi & z = z \end{matrix}}$$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = \rho\cos\phi\,\hat{i} + \rho\sin\phi\,\hat{j} + z\hat{k}$

$\hat{e_1} = \frac{1}{h_1}\left[\frac{\partial \vec{r}}{\partial \rho}\right]$

$\hat{e_1} = \frac{1}{1}\left[\cos\phi\,\hat{i} + \sin\phi\,\hat{j}\right]$

$\boxed{\hat{e_1} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}}$

$\hat{e_2} = \frac{1}{h_2}\left[\frac{\partial \vec{r}}{\partial \phi}\right]$

$\hat{e_2} = \frac{1}{\rho}\left[-\rho\sin\phi\,\hat{i} + \rho\cos\phi\,\hat{j}\right]$

$\boxed{\hat{e_2} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}}$

$\hat{e_3} = \frac{1}{h_3}\left[\frac{\partial \vec{r}}{\partial z}\right]$

$\hat{e_3} = \frac{1}{1}\left[\hat{k}\right]$

$\boxed{\hat{e_3} = \hat{k}}$

$\hat{e_1}\cdot\hat{e_2} = \left[\cos\phi\,\hat{i} + \sin\phi\,\hat{j}\right]\left[-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}\right]$

$= \left[-\sin\phi\cos\phi + \sin\phi\cos\phi\right]$

$= 0$

$\hat{e_2}\cdot\hat{e_3} = [-\sin\phi\,\hat{i}+\cos\phi\,\hat{j}][\hat{k}]$

$= 0$

$\hat{e_3}\cdot\hat{e_1} = [\hat{k}][\cos\phi\,\hat{i}+\sin\phi\,\hat{j}]$

$= 0$

$\therefore$ The cylindrical coordinate system is orthogonal

3. $u = f(x-y,\, y-z,\, z-x)$

let

$$\boxed{\begin{aligned} r &= x-y \\ s &= y-z \\ t &= z-x \end{aligned}}$$

To prove: $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial x}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial y}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial z}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial z}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$$

$\frac{\partial r}{\partial x} = 1 \qquad \frac{\partial s}{\partial x} = 0 \qquad \frac{\partial t}{\partial x} = -1$

$\frac{\partial r}{\partial y} = -1 \qquad \frac{\partial s}{\partial y} = 1 \qquad \frac{\partial t}{\partial y} = 0$

$$\frac{\partial r}{\partial z} = 0 \qquad \frac{\partial s}{\partial z} = -1 \qquad \frac{\partial t}{\partial z} = 1$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} - \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial u}{\partial r} + \frac{\partial u}{\partial s}$$

$$\frac{\partial u}{\partial z} = -\frac{\partial u}{\partial s} + \frac{\partial u}{\partial t}$$

now

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \left[\cancel{\frac{\partial u}{\partial r}} - \cancel{\frac{\partial u}{\partial t}} - \cancel{\frac{\partial u}{\partial r}} + \cancel{\frac{\partial u}{\partial s}} - \cancel{\frac{\partial u}{\partial s}} + \cancel{\frac{\partial u}{\partial t}}\right]$$

$$= 0$$

$$= RHS$$

$$\therefore \boxed{\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0}$$

5. $\boxed{A = c_1 B + c_2 C + c_3 D}$

$$A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

now

$$\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = c_1 \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} c_1 & c_1 \\ 0 & -c_1 \end{bmatrix} + \begin{bmatrix} c_2 & c_2 \\ -c_2 & 0 \end{bmatrix} + \begin{bmatrix} c_3 & -c_3 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} c_1+c_2+c_3 & c_1+c_2-c_3 \\ -c_2 & -c_1 \end{bmatrix} \quad - (1)$$

$c_1+c_2+c_3 = 3$

$c_1+c_2-c_3 = -1$

$-c_2 = 1 \Rightarrow c_2 = -1$

$-c_1 = -2 \Rightarrow c_1 = 2$

Then

$c_1+c_2-c_3 = -1$

$-1+2-c_3 = -1$

$+1-c_3 = -1$

$-c_3 = -1-1 = -2$

$c_3 = 2$

$\therefore \boxed{c_1 = 2 \quad c_2 = -1 \quad c_3 = 2}$

Subsituting in eqn: (1)

$$\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2-1+2 & 2-1-2 \\ -(-1) & -(2) \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}}$$

$\therefore$ The given matrix is the linear combination

6. $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -2 & -1 \end{bmatrix}$

$R_3 \to R_3 + R_2$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$R_2 \to \frac{R_2}{2}$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$

Dimension of $R(A) = 2$

Basis of $R(A) = \{ (1,1,0)\ (0,1,1/2) \}$

Dimension of $C(A) = 2$

Basis of $C(A) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$

$\boxed{AX = 0}$

$x_1 + x_2 = 0$ — ①

$x_2 + \frac{x_3}{2} = 0$ — ②

$x_3 = t$

then $x_2 + \frac{t}{2} = 0$

$x = -\frac{t}{2}$

eqn1: ① $\Rightarrow$ $x_1 = -x_2$

$x_1 = -\left(-\frac{t}{2}\right)$

$x_1 = \frac{t}{2}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t/2 \\ -t/2 \\ t \end{bmatrix}$$

$$X = t\begin{bmatrix} 1/2 \\ -1/2 \\ 1 \end{bmatrix} = t\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$ [∵ multiplied by 2]

Dimension of $X(A) = 1$

Basis of $X(A) = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

1. given $T: R^1 \to R^3$

$T(x) = (x, x^2, x^3)$ — ①

consider

$\alpha = x_1$

$T(\alpha) = T(x_1)$

$T(\alpha) = (x_1, x_1^2, x_1^3)$

conditions

$T(\alpha) + T(\beta) = T(\alpha + \beta)$

$T(c\alpha) = c\,T(\alpha)$

$c\alpha = cx_1$

$T(c\alpha) = T(cx_1)$

$= (cx_1, (cx_1)^2, (cx_1)^3)$

$= c\,T(\alpha)$

$\beta = x_2$

$T(\beta) = T(x_2)$

$T(\beta) = (x_2, x_2^2, x_2^3)$

$T(\alpha) + T(\beta) = T(x_1) + T(x_2)$

$= (x_1, x_1^2, x_1^3) + (x_2, x_2^2, x_2^3)$

$\boxed{T(\alpha) + T(\beta) = (x_1 + x_2,\ x_1^2 + x_2^2,\ x_1^3 + x_2^3)}$ — ②

consider

$\alpha + \beta = x_1 + x_2$

$T(\alpha+\beta) = T(x_1 + x_2)$

$\boxed{T(\alpha+\beta) = (x_1 + x_2,\ (x_1+x_2)^2,\ (x_1+x_2)^3)}$ — ③

on comparing eqn: ① with ②

$T(\alpha) + T(\beta) \neq T(\alpha+\beta)$ $\left[\because \begin{array}{l} x_1^2 + x_2^2 \neq (x_1+x_2)^2 \\ x_1^3 + x_2^3 \neq (x_1+x_2)^3 \end{array} \right]$

$\therefore$ The transformation is not linear

[since it does not satisfy one of the conditions ie
$T(\alpha) + T(\beta) = T(\alpha+\beta)$ it is not linear.]

8. given

$W = \{(x,y,z) \mid x - 3y + 4z = 0\}$

① consider

$\alpha : (x_1, y_1, z_1) \Rightarrow x_1 - 3y_1 + 4z_1 = 0$

$\beta : (x_2, y_2, z_2) \Rightarrow x_2 - 3y_2 + 4z_2 = 0$

now

$c_1\alpha + c_2\beta = c_1(x_1, y_1, z_1) + c_2(x_2, y_2, z_2)$

$= (c_1x_1, c_1y_1, c_1z_1) + (c_2x_2, c_2y_2, c_2z_2)$

$= (\underbrace{c_1x_1 + c_2x_2}_{x}, \underbrace{c_1y_1 + c_2y_2}_{y}, \underbrace{c_1z_1 + c_2z_2}_{z})$

$x - 3y + 4z = 0$

$(c_1x_1 + c_2x_2) - 3(c_1y_1 + c_2y_2) + 4(c_1z_1 + c_2z_2) = 0$

$c_1x_1 + c_2x_2 - 3c_1y_1 - 3c_2y_2 + 4c_1z_1 + 4c_2z_2 = 0$

$c_1(x_1 - 3y_1 + 4z_1) + c_2(x_2 - 3y_2 + 4z_2) = 0$

$c_1(0) + c_2(0) = 0$

$0 = 0$

$\therefore \boxed{c_1\alpha + c_1\beta \in W}$

② consider

$c_1\alpha = c_1(x_1, y_1, z_1)$

$c_1 \alpha = (c_1 x_1, c_1 y_1, c_1 z_1)$

$x - 3y + 4z = 0$

$c_1 x_1 - 3c_1 y_1 + 4c_1 z_1 = 0$

$c_1(x_1 - 3y_1 + 4z_1) = 0$

$c_1(0) = 0$

$\boxed{\therefore c_1 \alpha \in W}$

Hence the vector space is a subspace of R_3

4. given

$f(x,y) = x^3 + y^3 - 3x - 12y + 20$

$\frac{\partial f}{\partial x} = 0$ $\qquad$ $\frac{\partial f}{\partial y} = 0$

$3x^2 - 3 = 0$ $\qquad$ $3y^2 - 12 = 0$

$3x^2 = 3$ $\qquad$ $3y^2 = \cancel{4}12$

$x^2 = 1$ $\qquad$ $y^2 = 4$

$\boxed{x = \pm 1}$ $\qquad$ $\boxed{y = \pm 2}$

$\boxed{\text{The points are } (1,2)\ (1,-2)\ (-1,2)\ (-1,-2)}$

$A = \frac{\partial^2 f}{\partial x^2}$ $\qquad$ $B = \frac{\partial f}{\partial y \partial x}$ $\qquad$ $C = \frac{\partial^2 f}{\partial y^2}$

$A = \frac{\partial}{\partial x}(3x^2 - 3)$ $\qquad$ $B = \frac{\partial}{\partial y}(3x^2 - 3)$ $\qquad$ $C = \frac{\partial}{\partial y}(3y^2 - 12)$

$\boxed{A = 6x}$ $\qquad$ $\boxed{B = 0}$ $\qquad$ $\boxed{C = 6y}$

	$(1,2)$	$(1,-2)$	$(-1,2)$	$(-1,-2)$
$A = 6x$	$6>0$	$6>0$	$-6<0$	$-6<0$
$B = 0$	0	0	0	0
$C = 6y$	12	-12	12	-12
$AC-B^2$	$72>0$	$-72<0$	$-72<0$	$72>0$
	minimum	saddle	saddle	maximum

The $f(x,y)$ is minimum at $(1,2)$

$$f(1,2) = (1)^3 + (2)^3 - 3(1) - 12(2) + 20$$

$$= 1 + 8 - 3 - 24 + 20$$

$$= 29 - 27$$

$$\boxed{= 2}$$

$f(x,y)$ is maximum at $(-1,-2)$

$$f(-1,-2) = (-1)^3 + (-2)^3 - 3(-1) - 12(-2) + 20$$

$$= -1 - 8 + 3 + 24 + 20$$

$$= -9 + \cancel{3}\,47$$

$$\boxed{= 38}$$