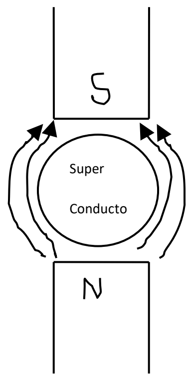
SOLUTIONS TO IAT-2 PHYSICS

1 A

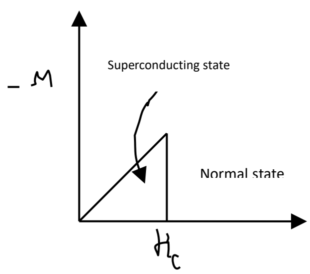
MESISSENER EFFECT

When a superconductor is placed in a magnetic field, it prevents entry of magnetic flux lines . This effect is known as **Meisseners effect.**

When a superconductor is kept in magnetic field, the surface current induced by the operation of Lenz's law. This generates opposite magnetic field and external field lines are prevented from entering the material. This effect is reversible. This is the principle of MAGLEV.



Type 1 Superconductors:



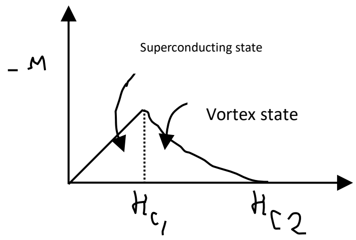
These are pure superconductors.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. The currents induced on the surface oppose the entry of flux lines . At critical magnetic field , the cooper pairs breakup and the material makes a sharp transition to normal state due to the penetration of magnetic flux lines.

These possess low critical magnetic fields. Their critical temperatures also low.They are generally pure metals.

Ex: Al, Pb

Type 2 superconductor:



These are generally alloys.

When kept in magnetic field, initially they continue to exhibit superconductivity and the negative magnetic moment increases. At lower critical magnetic field H_{C1}, the flux lines start penetrating .As the magnetic field is increased, the super conductivity coexists with magnetic field and this phase is known as mixed state(vortex state). At higher critical magnetic field H_{C2}, the penetration is complete and the material transforms to normal state. They possess higher critical magnetic fields. Their critical temperatures are high.

Ex: Nb_3Ge, $YBa_2Cu_3O_7$

TYPE I	TYPE II
Pb $40x10^3$A/m	Pb – Bi $100x10^3$A/m

1B

$$H_c = H_o[1-\left(\frac{T}{T_c}\right)^2]$$
$$= 6.5x10^{-4}\left(1-\left(\frac{4}{7.2}\right)^2\right) = 4.49X10^{-4}\text{A/m}$$

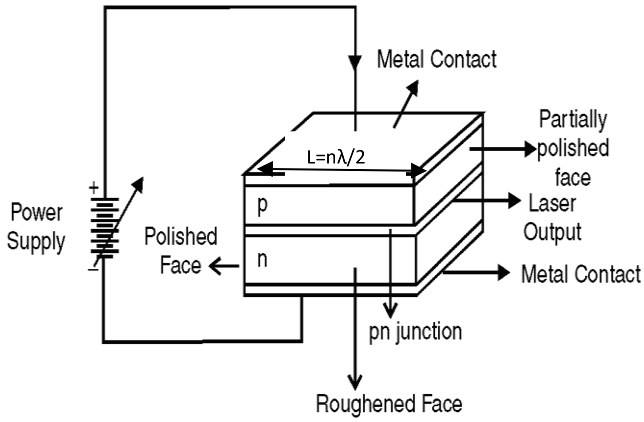
2A

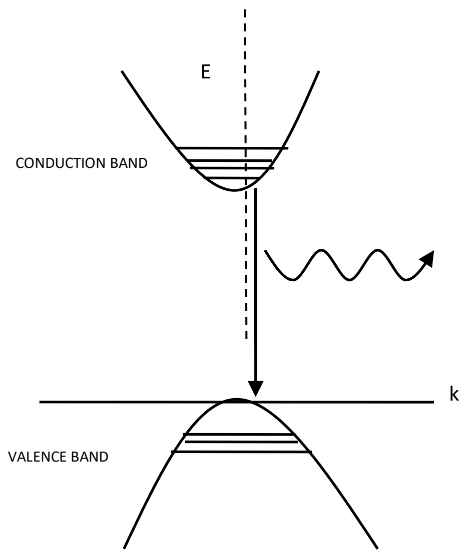
Gallium – Arsenide Semiconductor laser :

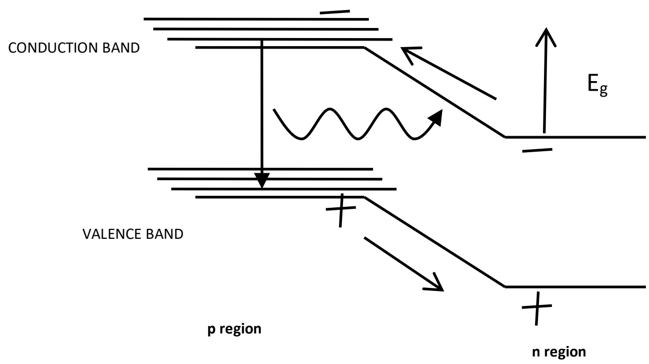
It is the only device which can be used for amplification in the infrared and optical ranges.

CONSTRUCTION

Gallium Arsenide is heavily doped with Tellurium (n side) and Zinc (P side) to a concentration of 10^{19} atoms /cm^3. Resonant cavity is formed by polishing the end faces of Junction diode. Amplification is possible if the population of the valence and conduction bands could be inverted as shown in the diagram.







WORKING

The first laser action was observed in a GaAs junction(8400Å) which is a direct gap semiconductor.

When a heavily doped junction is forward biased, electrons from n side are injected into p side causing population inversion. They combine with holes on the P side releasing photons. The junction region is the active region .The optical cavity is formed by the faces of the crystal itself which are taken on the cleavage plane and are then polished. The wavelength of the radiation depends on temperature. The wavelength of laser increases as the energy gap decreases. The frequency can be increased to the optical region by alloying with phosphor according to the relation $Ga(As)_{1-x}P_x$.

If E_g is the energy gap, then $E_g = eV_{forward} = \frac{hc}{\lambda}$

2B

$NA = n_o \sin\theta_A = \sqrt{n_{core}^2 - n_{clad}^2} = \sqrt{1.48^2 - 1.46^2} = 0.2425$

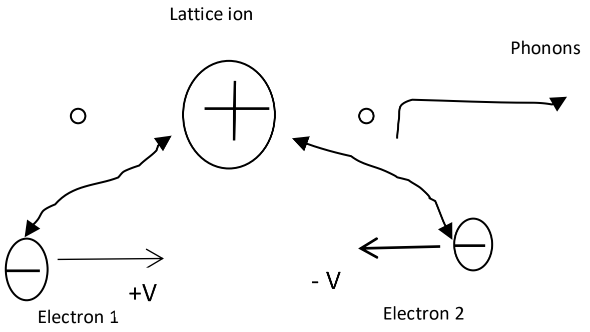
Acceptance angle $\theta_A = sin^{-1}[NA] = 14.03^o$

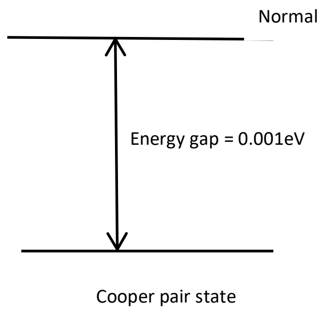
3A

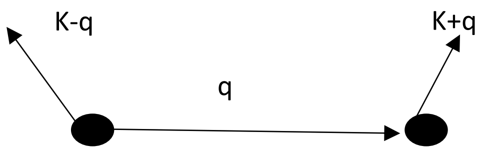
Phonons: These are lattice vibrations caused in a crystal

BCS Theory :[Bardeen , Cooper, Schrieffer]

1. When the temperature of the material is reduced below critical temperature, electrons attain lower energy state than the normal energy creating an energy gap of few milli electron volt.
2. **Positively charged lattice ion attracts a pair of electrons with equal and opposite spin and momentum , lattice vibrations known as phonons are created**. The two electrons and the lattice atom interact through a feeble attractive interaction known as electron-lattice-electron interaction constituting cooper pairs.
3. Cooper pairs interact through exchanging Phonons.
4. All the cooper pairs are in same energy state known as **Phase quantum state** and possess common wavefunction and Energy.
5. When a potential difference applied, the current is constituted by flow of cooper pairs and are not scattered as the energy required to break it up is large enough. This reduces the resistance.







6. When the temperature / magnetic field is increased beyond critical limit, cooper pairs breakup and normal state is restored.

3B

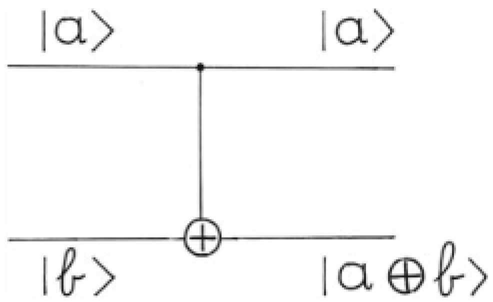
Critical current $\quad\quad$ I= $2\Pi RH$

$I = 2\ x\ 3.14\ x\ 0.0005x\ 0.02 = 6.28x10^{-5}\ A$

4A

CONTROLLED NOT GATE

a and b are two inputs. a is called control qubit and b the target qubit. Target qubit flips if and only if a= 1. If a = 0, the second qubit remains unchanged.



INPUT CONTROL BIT	INPUT TARGET BIT	OUTPUT CONTROL BIT	OUTPUT TARGET BIT
X_1	X_o	Y_1	y_o
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

MATRIX FORM

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

CNOT gate can be used to create BELL STATE (entangled state)

1. Consider two qubit $|00\rangle$
2. Apply Hadamard gate to the first qubit
   $H|0\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |1\rangle)$

   So, $|00\rangle$ transforms to $\frac{1}{\sqrt{2}}[|00\rangle + |10\rangle)$
3. Apply CNOT gate with first qubit as Control and second as target
4. For $|00\rangle$, $|0\rangle$ is Control Qubit, so the target qubit remains same $|0\rangle$
5. For $|10\rangle$, $|1\rangle$ is Control Qubit, so the target qubit changes from $|0\rangle$ to $|1\rangle$
6. The resulting entangled state is
   $\frac{1}{\sqrt{2}}[|00\rangle + |11\rangle)$ This is the Bell state

4B

Hadamard gate $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$H[H^*]^T = I$$

$$[H^*] = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[H^*]^T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H[H^*]^T = \frac{1}{2}\begin{bmatrix} (1X1) + (1X1) & (1X1) + (1X - 1) \\ (1X1) + (-1X1) & (1X1) + (-1X - 1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

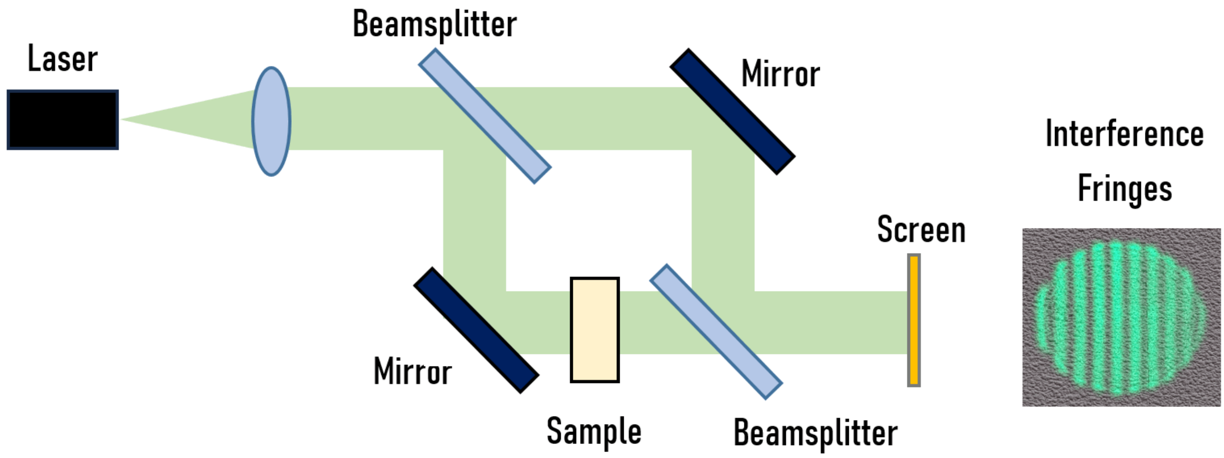
5A

Mach-Zehnder interferometer

A **Mach–Zehnder interferometer (MZI) is a device used to separate wavelengths (wavelength division multiplexing), composition of materials by measuring** phase differences between two coherent light beams.

Mach-Zehnder interferometers are simple interferometric instruments that measure the relative phase shift between two collimated light beams. This phase shift can be used to determine small displacements, the transmitted wave front error of transmissive optics, the refractive index of transparent materials, air flow in wind tunnels, and more.

Mach-Zehnder interferometers consist of a coherent light source like a laser, two beam splitters, and two mirrors (*Figure 1*). First, the light source is split into two paths using the first beam splitter. The two beams each have the same optical path length, which is the distance travelled multiplied by the refractive index of the media they travel through. Each beam reflects off of a mirror and is recombined by the second beam splitter. If there is a difference in the optical path lengths of the two beams that is less than the coherence length of the light source, interference fringes will be generated. Because the coherence length of a source can be extremely short, precision components and alignment are crucial. A sample can be measured by being placed in one of the beam paths. The resulting optical path length difference can be measured by observing the change in the interference fringes.



Fig

5B

$$\frac{N_g}{N_e} = e^{\frac{hc}{\lambda kT}}$$

$$T = 273 = 27 = 300K$$

$$\frac{N_e}{N_g} = 1.672x10^{-28} = e^{-\frac{hc}{\lambda kT}}$$

$$ln\left(\frac{N_e}{N_g}\right) = -\frac{hc}{\lambda kT} ln_e\, e$$

$$\lambda = -\frac{hc}{kT\,ln\left(\frac{N_e}{N_g}\right)} = 751.2x10^{-9}m$$

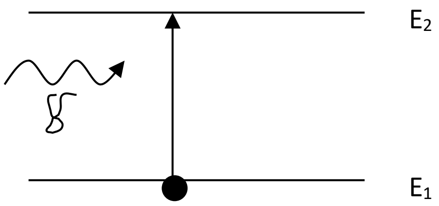
6A

Expression for energy density:

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N_1 and N_2 be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident.



$$\gamma = \frac{E_2 - E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_γ.

$$\text{Rate of absorption} = B_{12}\ N_1 U_\gamma$$

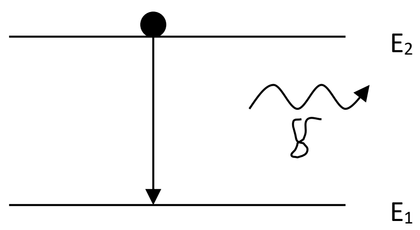
Here B_{12} is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

$$\text{Rate of spontaneous emission} = A_{21}\ N_2$$

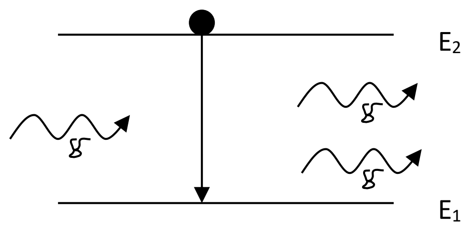
Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



$$\gamma = \frac{E_2 - E_1}{h}$$

Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_{γ}.

$$\text{Rate of stimulated emission} = B_{21}\ N_2 U_{\gamma}$$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12}\ N_1 U_\gamma \ = \ A_{21}\ N_2 \ + \ B_{21}\ N_2\ U_\gamma$$

$$U_\gamma = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

Rearranging this, we get

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}N_1}{B_{21}N_2}-1}\right]$$

From Boltzmans law ,$\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\gamma}{kT}}-1}\right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h\gamma^3}{c^3}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}-1}\right]$$

Comparing these expressions, we get

$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3}$ and$\frac{B_{12}}{B_{21}} = 1$

$$\therefore U_\gamma = \frac{A}{B}\left[\frac{1}{e^{\frac{h\gamma}{kT}}-1}\right]$$

Conclusions

1.Rate of stimulated emission is directly proportional to wavelength

2. Rate of Induced absorption is equal to rate of Stimulated emission

6B

$$no\ .photons\ per\ in\ each\ pulse\ = \frac{P\lambda t}{hc} = \frac{0.005x\lambda x8x10^{-9}}{6.62x10^{-34}x3x10^{8}}$$

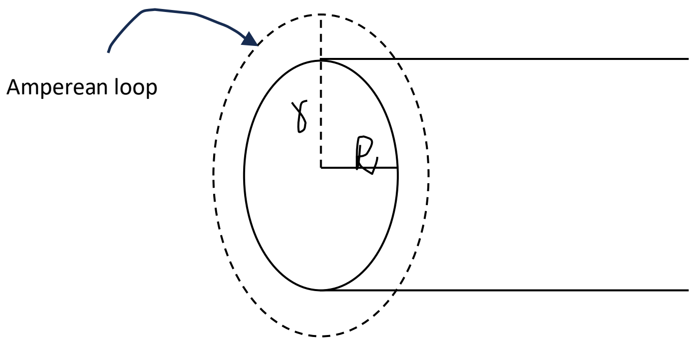
$5.61x10^8 = \frac{0.005x\lambda x8x10^{-9}}{6.62x10^{-34}x3x10^{8}}$

$$\lambda = 2.79x10^{-6}m$$

7A

Magnetic field due to straight solid cylindrical conductor:

Case 1 : B at a distance r > R



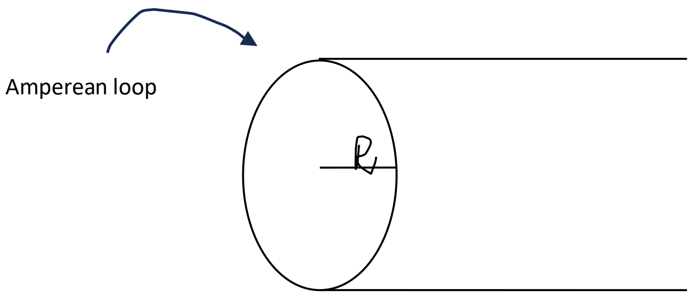
From Amperes law

$$\oint \vec{B}.\vec{dl} = \mu I_{passing\ throgh\ amperean\ loop}$$

$$B = \frac{\mu I}{2\pi r^2} \quad \because \oint dl = 2\pi r$$

$$H = \frac{I}{2\pi r}$$

Case 2: B at the surface of the conductor , r = R



$$\oint \vec{B}.\vec{dl} = \mu I_{passing\ throgh\ amperean\ loop}$$

$$B = \frac{\mu I}{2\pi R^2} \quad \because \oint dl = 2\pi R$$

$$B = \frac{\mu I}{2\pi R}$$

$\frac{B}{\mu} = H = \frac{I}{2\pi R}$

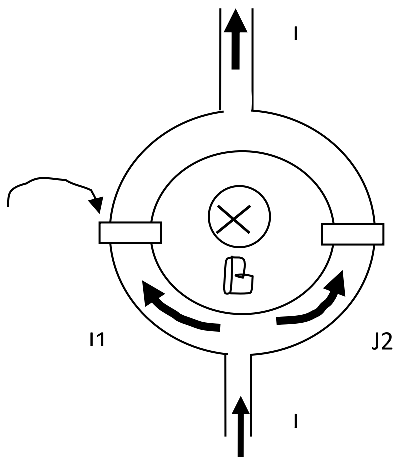
Critical Current $\quad I_C = 2\,\pi\,R\,H$

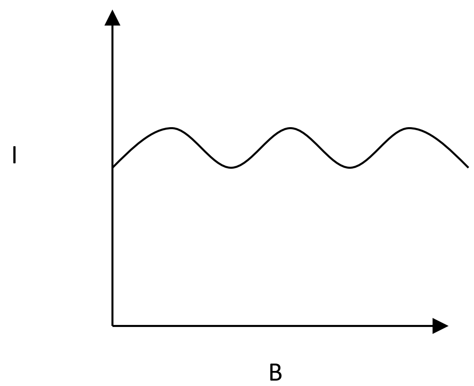
7B

DC SQUID:

It uses a pair of DC Josephson junctions. When magnetic field is applied normal to the plane of junction, current is induced in the SQUID which opposes the external magnetic field. Magnetic Flux through the SQUID is quantized .The current through the SQUID varies periodically with the external magnetic field.

$$\varphi = \frac{nhc}{2e}$$





8A

QUBIT: An atom / ion / photon / Nuclei/ etc in a lower energy level / upper energy level/ superposed state.

Quantum Superposition:

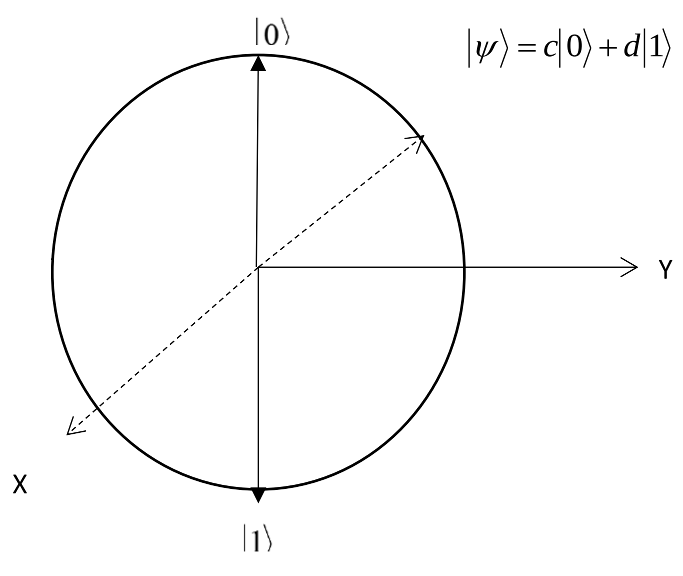
What state is the coin in while it is in the air? Is it heads or tails?
We can say that the coin is in a superposition of both heads and tails. When it lands, it has a definite state, either heads or tails. Broadly, the word “state” means any particular way that a system can possibly be described. For example, the coin can be either heads, or tails, or a combination of heads or tails while flipped in the air. All of these cases are called states of the coin system. The measurement destroys the superposition.
At any given time, a system can be described as being in a particular state. The state is related to its quantized values. For example, a tossed coin is either in a heads state or a tails state. An electron orbiting a hydrogen atom could be in the ground state or an excited state. A quantum system is special because it can be in a superposition of these definite states, i.e., both heads and tails simultaneously. It is possible for a quantum object to exist in multiple states at the same time. The outcome of a measurement is to observe some definite state with a given probability.

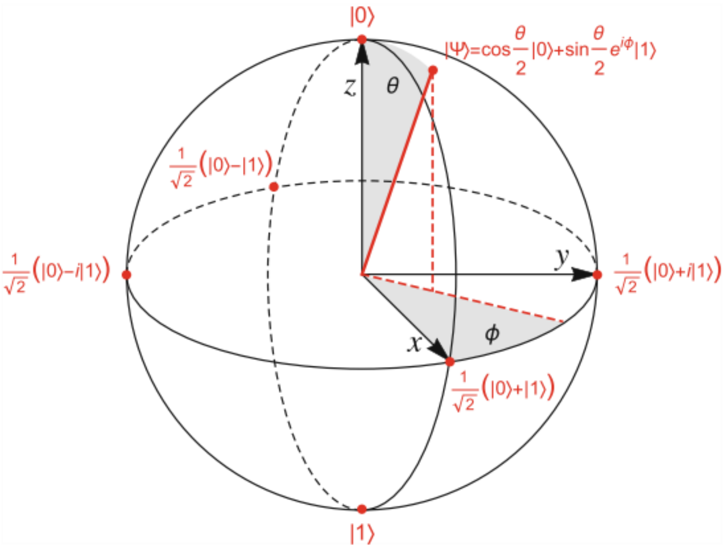
Notation to represent quantum mechanical states
Ket notation:

$$Examples:$$
$$|0\rangle \equiv \begin{bmatrix}1\\0\end{bmatrix}$$
$$|1\rangle \equiv \begin{bmatrix}0\\1\end{bmatrix}$$
$$superpoased\ state\ |\psi\rangle = c|0\rangle + d|1\rangle \equiv \begin{bmatrix}c\\d\end{bmatrix}$$

BLOCK SPHERE

It represents a sphere with all the points on its surface correspond to state vectors in Hilbert space. The vector drawn to any point on the surface from the centre represents a state. In the diagram, $|\psi\rangle = c|0\rangle + d|1\rangle$ is a superposed state. $|0\rangle$ and $|1\rangle$are represented along + Z and – Z axes.





8B

PAULI MATRICES

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$
$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

σ_x is a classical not gate. When operated on a state vector say $|0\rangle$, it flips to $|1\rangle$

$$\sigma_x|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + 1x0 \\ 1x1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\sigma_x|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + 1x1 \\ 1x0 + 0x1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + (-ix0) \\ ix1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix}$$

$$\sigma_y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + (-ix1) \\ ix0 + 0x1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix}$$

$$\sigma_z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1x1 + 0x0 \\ 0x1 + -1x0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1x0 + 0x1 \\ 0x0 + -1x1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$