CBCS SCHEME

USN 1CR25MC049

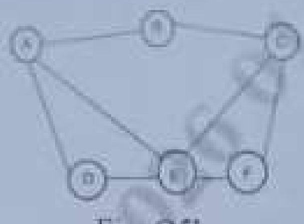
MMC102

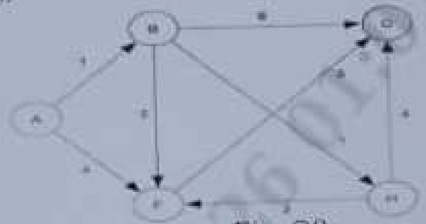
First Semester MCA Degree Examination, Dec.2025/Jan.2026
Discrete Mathematics and Graph Theory

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	What is a set ? With an example explain the following set operations. i. union ii. intersection iii. compliment iv. relative compliment v. symmetric difference	10	L1	CO1
	b.	State and prove the following laws of set theory. i. Distributive laws ii. DeMorgan's laws	10	L1	CO1
		OR			
Q.2	a.	A survey of 500 television viewers of a sports channel produced the following information.: 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three kinds of the game. (i) How many viewers watch all the three kinds of the game? (ii) How many viewers watch exactly one of the sports?	10	L2	CO1
	b.	Define eigen value and eigen vector of matrix. Find the eigen value and eigen vector of $\begin{bmatrix} 1 & 4 \\ -4 & -7 \end{bmatrix}$	10	L2	CO1
		Module – 2			
Q.3	a.	Define tautology . Find whether the following compound propositions are tautologies or not. i. $((p \vee q) \wedge ((p \to r) \wedge (q \to r))) \to r$ ii. $((p \to q) \vee (p \to r)) \leftrightarrow (p \to (q \vee r))$	10	L2	CO2
	b.	For any propositions p, q and r prove the following.(≡ stands for logically equivalent propositions) i. $((p \to q) \wedge (p \to r)) \equiv (p \to (q \wedge r))$ ii. $((p \to q) \to r) \equiv ((p \wedge \neg r) \to \neg q)$	10	L2	CO2
		OR			
Q.4	a.	Write the converse, inverse and contrapositive of the statement "If a quadrilateral is a parallelogram then its diagonal bisects each other".	06	L2	CO2
	b.	Write the negation of the statement "If it rains, then I do not drive the car".	06	L2	CO2
	c.	Verify whether 't' is a valid conclusion from the premises : $p \to q$, $q \to r$, $r \to s$, $\neg s$ and $p \vee t$.	08	L2	CO2

		Module – 3			
Q.5	**a.**	With an example explain the following. i. regular graph ii. complete graph iii. Bipartite graph iv. walk in graph v. paths in a graph	**10**	L2	**CO3**
	b.	What is handshaking property? Verify the handshaking property to the following graph.  Fig. Q5b	**05**	L2	**CO3**
	c.	List the in-degree and out-degree of all the vertices of the following graph.  Fig. Q5c	**05**	L2	**CO3**
		OR			
Q.6	**a.**	What are isomorphic graphs?. Verify whether the following graphs are isomorphic or not.  Fig. Q6a	**10**	L3	**CO3**
	b.	With an example explain the following. i. subgraphs ii. finite graph iii. infinite graph iv. circuit v. null graph	**10**	L2	**CO3**
		Module – 4			
Q.7	**a.**	With an example explain the following. i. Eulerian circuit ii. Hamiltonian circuit	**10**	L3	**CO4**
	b.	With an example explain the following operations on graphs. i. union ii. intersection. iii. compliment iv. ring sum	**10**	L2	**CO4**

		OR			
Q.8	a.	Explain Konigsberg seven bridge problem.	05	2	4
	b.	With the help of a graph explain Travelling Salesman Problem.	05	2	4
	c.	Apply Dijkstra's algorithm to find the shortest distance fromnode'A' to remaining nodes.  Fig. Q8c	10	3	4
		Module – 5			
Q.9	a.	What is graph coloring? What is chromatic number of a graph? With the help of an example find the chromatic number of complete graph and bipartite graph.	10	2	5
	b.	What is chromatic polynomial of a graph? Find the chromatic polynomial of the complete graph with 4 vertices.	10	3	5
		OR			
Q.10	a.	Explain five color theorem.	10	3	5
	b.	With an example explain greedy coloring algorithm.	10	3	5

* * * * *

Discrete Mathematics & Graph Theory

Q1.

a) Set - A set is a collection of well-defined objects.

Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$.
Universal set $U = \{1, 2, 3, 4, 5, 6\}$

i. Union ($A \cup B$)
The Union of two sets contain all elements from both sets. $A \cup B = \{1, 2, 3, 4, 5\}$

ii. Intersection ($A \cap B$)
The intersection contains only the common elements in both sets.

$$A \cap B = \{3\}$$

iii. Complement (A')
The complement of a set contains all elements in the universal set that are not in A.

$$A' = U - A = \{4, 5, 6\}$$

iv. Relative complement ($A-B$)
Elements that are in A not in B.

$$A - B = \{1, 2\}$$

Similarly, $B - A = \{4, 5\}$

v. Symmetric Difference ($A \Delta B$)
Set of all Elements that are in A or B but not in both.

$$A \Delta B = (A \cup B) - (A \cap B)$$

16\. i. Distributive Laws

For any sets A, B, C:

a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Let $x \in A \cap (B \cup C)$
$\Rightarrow x \in A$ and $x \in B \cup C$
$\Rightarrow x \in A$ and $x \in B$ or $x \in C$

$\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \in C$
$\Rightarrow x \in (A \cap B)$ or $x \in (A \cap C)$
$\Rightarrow x \in (A \cap B) \cup (A \cap C)$
$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ — ①

Now let $x \in (A \cap B) \cup (A \cap C)$
$\Rightarrow x \in A \cap B$ or $x \in A \cap C$
$\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \in C$

$\Rightarrow x \in A$ and $x \in B$ or $x \in C$
$\Rightarrow x \in A$ and $x \in B \cup C$

$\Rightarrow x \in A \cap (B \cup C)$
$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ — ②

From ① & ②

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

ii) DeMorgan's Laws

To prove

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

Consider RHS $= \bar{A} \cap \bar{B}$

$= \{x / x \in \bar{A} \text{ and } x \in \bar{B}\}$

$= \{x / x \notin A \text{ and } x \notin B\}$

$= \{x / x \notin A \cup B\}$

$= \{x / x \in \overline{A \cup B}\} = \overline{A \cup B} = \text{LHS}$

To prove

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

Consider $\bar{A} \cup \bar{B} = \{x / x \in \bar{A} \text{ or } x \in \bar{B}\}$

$= \{x / x \notin A \text{ or } x \notin B\}$

$= \{x / x \notin A \cap B\}$

$= \{x / x \in \overline{A \cap B}\} = \overline{A \cap B}$

$\therefore \bar{A} \cup \bar{B} = \overline{A \cap B}$

Q2 a, Let S be the set of all television viewers of a sports channel. Let A, B, C be the set of all viewers who watch Cricket, Hockey, Football respectively.

Given $|S| = 500$, $|A| = 285$, $|B| = 195$, $|C| = 115$

$|A\cap C| = 45$, $|A\cap B| = 70$, $|B\cap C| = 50$, $|\overline{A\cup B\cup C}| = 50$

(i) $|A\cap B\cap C| = ?$

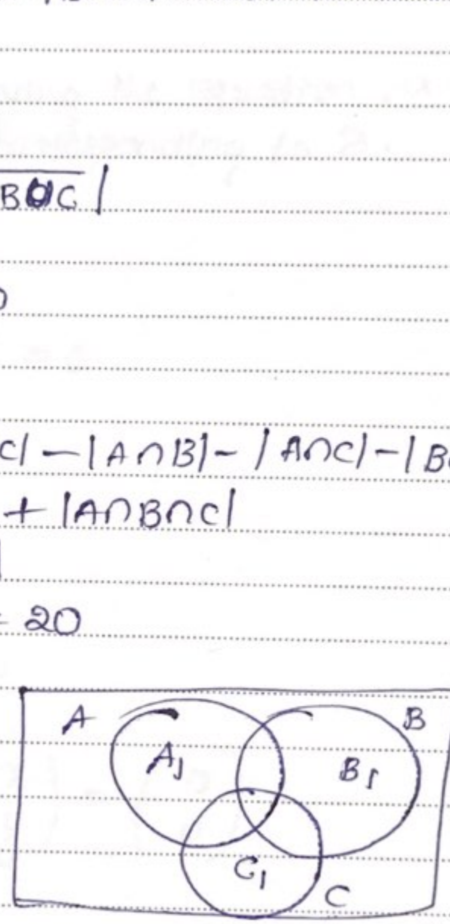
$$|A\cup B\cup C| = |S| - |\overline{A\cup B\cup C}|$$
$$= 500 - 50$$
$$= 450$$

wkt, $|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$

$450 = 430 + |A\cap B\cap C|$

$\Rightarrow |A\cap B\cap C| = 450 - 430 = 20$

(ii) Let A_1, B_1, C_1 be the set of all students who watch only cricket, only Hockey & Only Football.



$|A_1| = |A| - |A\cap B| - |A\cap C| + |A\cap B\cap C|$
$= 285 - 70 - 45 + 20$
$= 190$

IIIly

$|B_1| = |B| - |A\cap B| - |B\cap C| + |A\cap B\cap C|$
$= 95$

& $|C_1| = |C| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$
$= 40$

Experiment No. Page No.:

No of viewers who watch exactly one of the sports =

$$190 + 95 + 40 = 326$$

26. A scalar λ is called an eigen value of a square matrix A if there exists a non-zero vector X such that

$$AX = \lambda X$$

A non-zero vector X satisfying the equation $AX = \lambda X$ is called the eigen vector corresponding to λ.

Consider $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 4 \\ -4 & -7-\lambda \end{vmatrix} = 0$$

$$\lambda^2 + 6\lambda + 9 = 0$$

$$\lambda = -3, -3$$

Consider $(A - \lambda I)X = 0$

when $\lambda = -3$

$$\begin{pmatrix} 4 & 4 \\ -4 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$4x + 4y = 0$$

$$x = -y \Rightarrow (1, -1)$$

Eigen value : $\lambda = -3$ (double root)

Eigen vector : Any scalar multiple of $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Q3 (a) A compound proposition which is always true regardless of truth values of its components is called a Tautology.

(i) (i) $\underbrace{\{(p\vee q)\wedge((p\rightarrow r)\wedge(q\rightarrow r))\}}_{①}\rightarrow r$

p	q	r	$(p\vee q)$	$(p\rightarrow r)$	$(q\rightarrow r)$	$(p\rightarrow r)\wedge(q\rightarrow r)$	①	① $\rightarrow r$
0	0	0	0	1	1	1	0	1
0	0	1	0	1	1	1	0	1
0	1	0	1	1	0	0	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

Since all the entries of last column are 'one's, given compound proposition is a tautology.

(ii) $\{(p\rightarrow q)\vee(p\rightarrow r)\}\leftrightarrow\{p\rightarrow(q\vee r)\}$

Let $x: (p\rightarrow q)\vee(p\rightarrow r)$ & $y: p\rightarrow(q\vee r)$

p	q	r	$p\rightarrow q$	$(p\rightarrow r)$	x	$q\vee r$	y	$x\leftrightarrow y$
0	0	0	1	1	1	0	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	1	1	1	1
1	1	1	1	1	1	1	1	1

Since all the entries of last column are '1's, given compound proposition is a tautology.

3 b) i) $(p \to q) \wedge (p \to r) \equiv (p \to (q \wedge r))$

LHS $= (p \to q) \wedge (p \to r)$ wkt $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee r)$

$\Leftrightarrow \neg p \vee (q \wedge r)$ Distributive law

$\Leftrightarrow p \to (q \wedge r)$ using $\neg p \vee q \Leftrightarrow p \to q$

$=$ RHS

ii) $(p \to q) \to r \Leftrightarrow (p \wedge \neg r) \to \neg q$

LHS $= (p \to q) \to r$

$(\neg p \vee q) \to r$

$\neg(\neg p \vee q) \vee r$

$(p \wedge \neg q) \vee r$

RHS $= (p \wedge \neg r) \to \neg q$

$\neg(p \wedge \neg r) \vee \neg q$

$\neg p \vee r \vee \neg q$

Q4 a) Let p: A quadrilateral is a parallelogram.

q: Diagonals of the ~~parallelogram~~ quadrilateral bisect each other.

Given $p \to q$

Converse: $q \to p$

ie If the diagonals of a ~~parallelogram~~ quadrilateral bisect each other then it is a parallelogram.

Inverse: $\neg p \to \neg q$

ie If the quadrilateral is not a parallelogram then its diagonals do not bisect each other.

Contrapositive: $\neg q \to \neg p$

ie. If the diagonals of a quadrilateral do not bisect each other then it is not a parallelogram.

4 b) Let p: It rains
q: I drive the car.

Given $p \rightarrow \neg q$

Negation: $\neg(p \rightarrow \neg q)$
$\Leftrightarrow p \wedge \neg(\neg q)$
$\Leftrightarrow p \wedge q$

ie., It rains and I drive the car.

4c)

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ r \rightarrow s \\ \neg s \\ \underline{p \vee t} \\ \therefore t \end{array} \Rightarrow \begin{array}{l} p \rightarrow r \\ r \rightarrow s \\ \neg s \\ \underline{p \vee t} \\ \therefore t \end{array}$$ Rule of syllogism

$$\Rightarrow \begin{array}{l} p \rightarrow s \\ \neg s \\ \underline{p \vee t} \\ \therefore t \end{array}$$ Rule of syllogism

$$\Rightarrow \begin{array}{l} \neg p \\ \underline{p \vee t} \\ \therefore t \end{array}$$ Modus Tollens

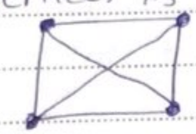
$\neg p \rightarrow t \Leftrightarrow p \vee t$

$$\Rightarrow \begin{array}{l} \neg p \\ \underline{\neg p \rightarrow t} \\ \therefore t \end{array}$$

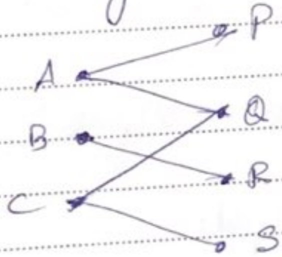
This is a valid argument using Modus Ponens.

5 a) (i) Regular Graph - A graph in which degree of all vertices is same is called a regular graph. Eg: 

(ii) Complete Graph - A simple graph with minimum 2 vertices in which there is an edge between every pair of vertices is called a complete graph & a complete graph with n vertices is denoted by K_n.

Eg:  K_4

(iii) Bipartite graph:
Suppose a simple graph G is such that its vertex set V is the union of two of its mutually disjoint nonempty subsets V_1 & V_2 which are such that each edge in G joins a vertex in V_1 & a vertex in V_2. then G is called a bipartite graph.

Eg:- 

Bipartites:
$V_1 = \{A, B, C\}$
$V_2 = \{P, Q, R, S\}$

(iv) walk in graph:
Let G be a graph with atleast one edge. Consider a finite, alternating sequence of vertices & edges of the form.
$v_i e_j v_{i+1} e_{j+1} \cdots e_k v_m$ which begins and ends with vertices and which is such that each edge in the sequence is incident on the vertices preceding & foll^g it in the sequence, such sequence is called a walk in G.

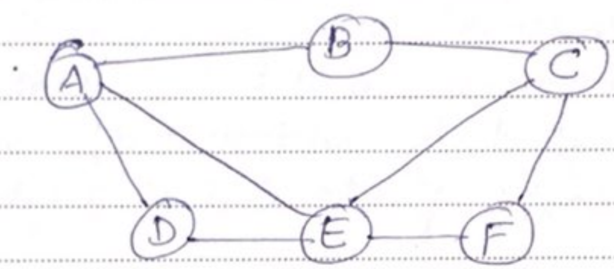
(v) If in an open walk no edge and no vertex appears more than once then it is called a path.

5b) Handshaking Property:

The sum of the degrees of all the vertices in a graph is an even number and this number is twice the number of edges in the graph.

In a graph $G(V, E)$,

$$\sum deg(v) = 2|E|$$



LHS $= \sum deg(v) = deg(A) + deg(B) + deg(C) + deg(D) + deg(E) + deg(D)$

$= 3 + 2 + 3 + 2 + 4 + 2$

$= 16$, even no

RHS $= 2|E| = 2(8) = 16$

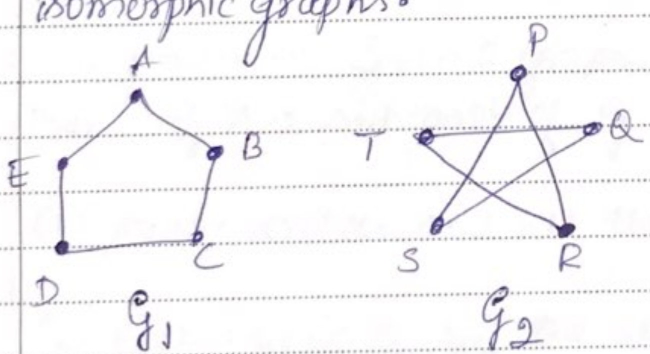
LHS = RHS = even no

Hence, verified.

5 c)

~~deg~~ Vertex	In-degree	Out-degree
0	2	0
1	0	2
2	2	2
3	1	0
4	0	1
	5	5

6 a) Isomorphism: Consider two graphs $G = (V, E)$ & $G' = (V', E')$. Suppose there exists a func^n $f: V \to V'$ such that (i) f is a one-to-one correspondence & (ii) for all vertices A, B of G, $\{A, B\}$ is an edge of G if & only if $\{f(A), f(B)\}$ is an edge of G'. Then f is called an isomorphism between G & G', and we say that G & G' are isomorphic graphs.



Both G_1 & G_2 have 5 vertices & 5 edges.

Both G_1 & G_2 are 2-regular graphs.

Vertex mapping:

$E \leftrightarrow T$
$A \leftrightarrow Q$
$B \leftrightarrow S$
$C \leftrightarrow P$
$D \leftrightarrow R$

Edge mapping:
$\{A, B\} \leftrightarrow \{Q, S\}$
$\{B, C\} \leftrightarrow \{S, P\}$
$\{C, D\} \leftrightarrow \{P, R\}$
$\{D, E\} \leftrightarrow \{R, T\}$
$\{E, A\} \leftrightarrow \{T, Q\}$

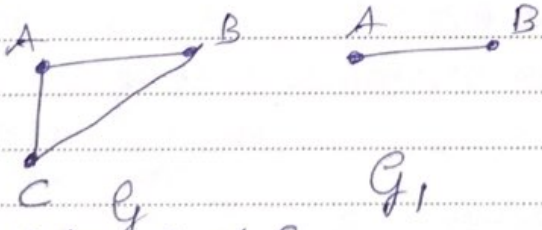
$\therefore$ There exists a 1-1 mapping b/w the vertices & edges of G_1 & G_2. Hence, they are isomorphic.

(i)
6 b. Subgraphs – Given 2 graphs G and G_1, we say that G_1 is a subgraph of G if

(1) All the vertices and all the edges of G_1 are in G.

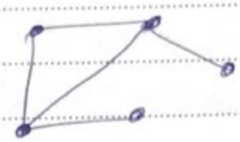
(2) Each edge of G_1 has the same end vertices in G as in G_1.

Eg



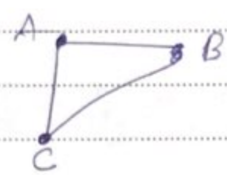
G_1 is a subgraph of G.

(ii) Finite graph : A graph with finite no of graphs is called a finite graph. Eg:



(iii) Infinite Graph : A graph with infinite no of vertices is called an infinite graph.

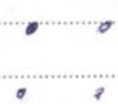
(iv) Circuit – A closed ~~graph~~ walk in which no edge is repeated is called a circuit.



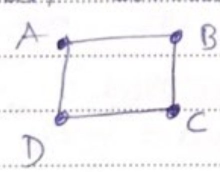
ABCA is a circuit.

(v) Null graph – A graph that has only vertices & no edges is called a null graph.

Eg:

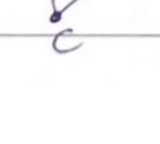


7a) Eulerian Circuit – A closed walk in which each edge in the graph is traversed exactly once is called an Eulerian Circuit.



Eg: ABCDA

Hamiltonian Circuit – A closed walk in which each vertex in the graph is traversed exactly once is calld a Hamiltonian circuit.



Eg: ABCA

7b. Consider two graphs $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$

(i) Then the graph whose vertex set is $V_1 \cup V_2$ & edge set is $E_1 \cup E_2$ is called the union of G_1 & G_2. It is denoted by

$$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2).$$

(ii) III^{ly} if $V_1 \cap V_2 \neq \phi$, the graph whose vertex set is $V_1 \cap V_2$ & the edge set is $E_1 \cap E_2$ is called the intersection of G_1 & G_2; it is denoted by $G_1 \cap G_2$.

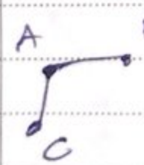
$$G_1 \cap G_2 = (V_1 \cap V_2, E_1 \cap E_2).$$

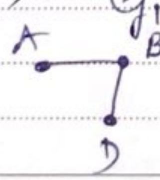
(iii) Compliment: Given a graph G & a subgraph G_1 of G, the subgraph of G obtained by deleting from G all the edges that belong to G_1 is called the complement of G_1 in G and is denoted by $G - G_1$.

(iv) Ring sum.

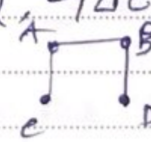
A graph with vertex set $V_1 \cup V_2$ & edge set is $E_1 \Delta E_2$ (symmetric difference of E_1 & E_2) is called the ring sum of G_1 & G_2.

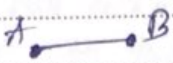
Thus, $G_1 \Delta G_2 = (V_1 \cup V_2, E_1 \Delta E_2)$.

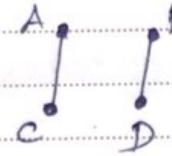




$G_1 \cup G_2$:



$G_1 \cap G_2 =$ 

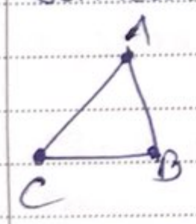
$G_1 \Delta G_2 =$ 

9 a) Given planar or non-planar graph G, if we assign colours to its vertices in such a way that no two adjacent vertices have the same colour, then we say that, the graph G is properly coloured.

A graph G which is k-colorable, but not (k-1) colorable is called a k-chromatic graph.

If a graph G is k-chromatic, then k is called the chromatic no of G & is usually denoted by $\chi(G)$.

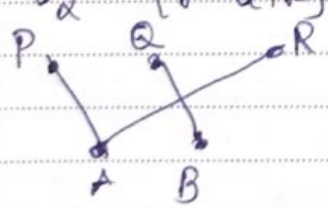
Consider a complete graph with 3 vertices, K_3.



3 colours are required to color this graph, say $A \xrightarrow{} C_1$, $B \to C_2$, $C \to C_3$.

So, chromatic no is 3.

Consider a bipartite graph with bipartites $V_1 = \{A, B\}$ $V_2 = \{P, Q, R\}$



A & B are not adjacent.

So, we can assign the same colour C_1.

P, Q, R are not adjacent. So, we can assign the same colour C_2 to them.

So, chromatic no = 2.

9 b) The no of different ways of properly coloring a graph G with λ no of colors is called a chromatic polynomial & is denoted by $P(G, \lambda)$.

$$P(K_n, \lambda) = 0 \text{ if } \lambda < n$$
$$= 1 \text{ if } \lambda = n$$
$$= \lambda(\lambda-1)(\lambda-2)\cdots(\lambda-n+1) \text{ if } \lambda > n.$$

$$\therefore P(K_4, \lambda) = 0 \text{ if } \lambda < n$$
$$= 1 \text{ if } \lambda = n$$
$$= \lambda(\lambda-1)(\lambda-2)(\lambda-3) \text{ if } \lambda > n.$$

10 a) It states: "Every planar graph can be colored using atmost five colors such that no two adjacent regions share the same color.

The proof used mathematical induction.

Step1: In any planar graph there is always atleast one vertex with degree ≤ 5.

Step2: Remove that vertex from the graph.
Then ~~remove that~~ color the remaining graph using 5 colors.

Step3: Add back the vertex.
That vertex has atmost 5 neighbours.
If fewer than 5 colors are used among neighbour

assign a free color.

Step 4: A clever argument : rearranges colors.
This ensures one color becomes available.

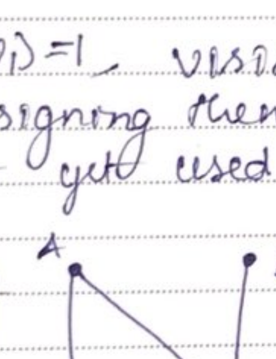
So, the vertices can always be colored with one of the colors.

10 b) Greedy coloring algorithm:

It is an easy way to colour a graph. The idea is to no the vertices and then starting

$c(v_1) = 1$, visit the remaining vertices in order, assigning them the lowest numbered colour not yet used for a neighbor.

Eg



Let us assign the colour C_1 to A. We can't assign C_1 to C & D as C and D are adjacent to A.

Now, let us assign color C_2 to B. Since B and D not adjacent to D, we can assign C_2 to D also. Assign a new color C_3 to C as C is adjacent to both A and B.

$A \rightarrow C_1$, $B \rightarrow C_2$, $C \rightarrow C_3$, $D \rightarrow C_2$.

Greedy coloring algorithm is used to color the above graph.