CBCS SCHEME

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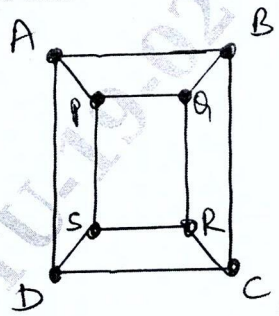
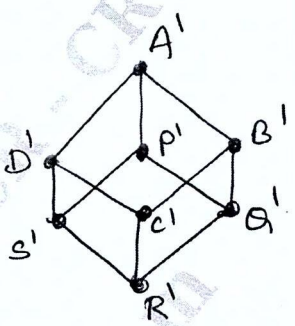
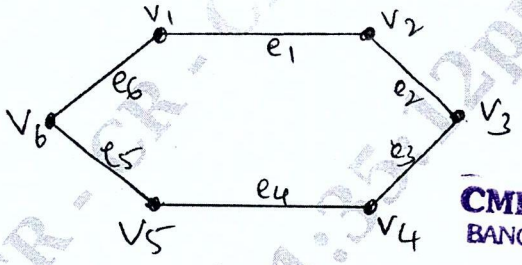
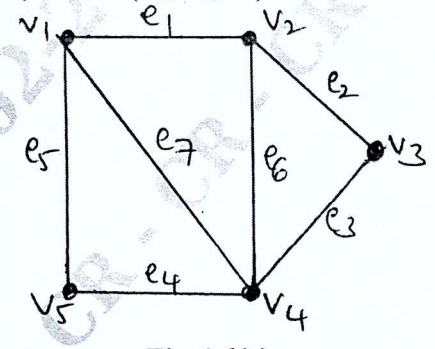
BCS405B

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026
Graph Theory

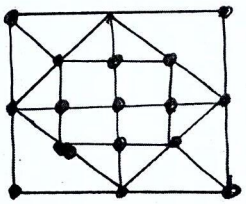
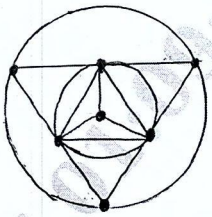
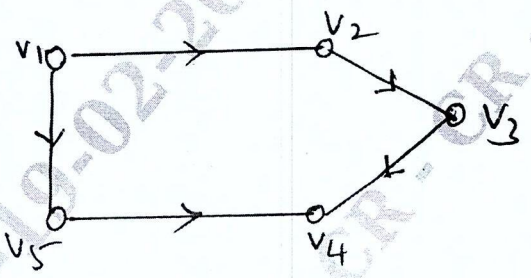
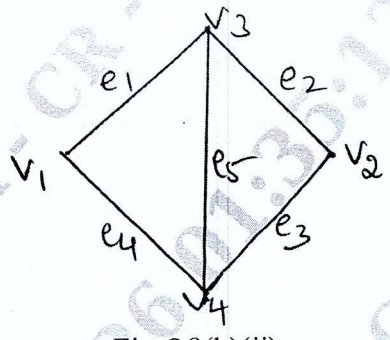
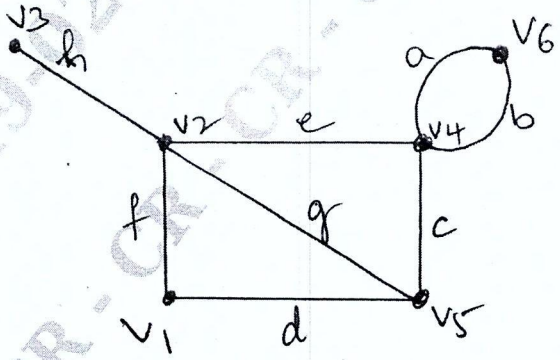
Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define : i) Bipartite Graph ii) Isomorphism	06	L1	CO1
	b.	Verify whether the two graphs shown in Fig.Q1(b)(i) and Fig.Q1(b)(ii) are isomorphic or not.  Fig.Q1(b)(i)  Fig.Q1(b)(ii)	07	L2	CO1
	c.	Show that in a complete graph of n vertices the degree of every vertex is $(n-1)$ and that the total number of edges is $\frac{n(n-1)}{2}$.	07	L3	CO1
		OR			
Q.2	a.	Define : (i) Subgraph (ii) Walk	06	L1	CO1
	b.	Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ number of edges.	07	L3	CO1
	c.	If G is a simple graph with no cycles, prove that G has atleast one pendent vertex.	07	L3	CO1
		Module – 2			
Q.3	a.	Prove that the complete graph K_n, where $n \geq 3$, is a Hamiltonian graph.	06	L3	CO2
	b.	If a connected graph G can be decomposed into edge-disjoint cycles, then show that G has an Euler circuit.	07	L3	CO2
	c.	Let D be a digraph with an odd number of vertices. If each vertex of D has an odd out-degree, prove that D has an odd number of vertices with odd in-degrees.	07	L3	CO2

		OR			
Q.4	**a.**	Define : (i) Hamiltonian cycle (ii) Euler circuit	**06**	**L1**	**CO2**
	b.	Let G be a simple graph with n vertices and m edges where m is atleast 3. If $m \geq \frac{1}{2}(n-1)(n-2)+2$, prove that G is a Hamiltonian graph. Is the converse true?	**07**	**L3**	**CO2**
	c.	Show that a connected graph with exactly two vertices of odd degree has an Euler Trial.	**07**	**L3**	**CO2**
		Module – 3			
Q.5	**a.**	Define : (i) Rooted Trees (ii) Spanning Trees	**06**	**L1**	**CO3**
	b.	Prove that if in a graph G, there is one and only one path between every pair of vertices, then G is a tree.	**07**	**L2**	**CO3**
	c.	Distinguish between Edge-connectivity and Vertex connectivity.	**07**	**L2**	**CO3**
		OR			
Q.6	**a.**	Find the edge connectivity and the vertex connectivity of the graph shown in below Fig.Q6(a).  Fig.Q6(a) CMRIT LIBRARY BANGALORE - 560 037	**06**	**L2**	**CO1**
	b.	Prove that a graph is connected if and only if it has a spanning tree.	**07**	**L3**	**CO3**
	c.	For the graph shown in below Fig.Q6(c), find the vertex partitions caused by the following cut-sets. i) $C_1 = \{e_2, e_3\}$ ii) $C_2 = \{e_1, e_3, e_6\}$  Fig.Q6(c)	**07**	**L2**	**CO1**
		Module – 4			
Q.7	**a.**	Define Planar Graph. Show that a graph of order 5 and size 8 is a planar graph.	**06**	**L2**	**CO4**
	b.	Prove that the Peterson graph is non-planar.	**07**	**L3**	**CO4**
	c.	Prove that a connected planar graph G with n vertices and m edges has exactly m – n + 2 regions.	**07**	**L3**	**CO4**

OR

Q.8	**a.**	Verify Euler's formula for the following planar graphs in Fig.Q8(a)(i) and Fig.Q8(a)(ii).  Fig.Q8(a)(i)  Fig.Q8(a)(ii)	**06**	**L2**	**CO1**
	b.	i) Write down the adjacency matrix for the graph G in Fig.Q8(b)(i).  Fig.Q8(b)(i) ii) Consider the undirected graph G as shown in below Fig.Q8(b)(ii). Find the incidence matrix.  Fig.Q8(b)(ii)	**07**	**L2**	**CO4**
	c.	Write down the path matrix and circuit matrix for the given graph in Fig.Q8(c).  Fig.Q8(c)	**07**	**L2**	**CO4**

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Module – 5					
Q.9	**a.**	Prove that a graph G is 2-chromatic if and only if it is a non-null bipartite graph.	**10**	**L3**	**CO5**
	b.	Find the chromatic polynomial for the following : i) Graph $K_{2,n}$ ii) Cycle C_4 of length 4	**10**	**L2**	**CO5**
OR					
Q.10	**a.**	State and prove Five Colour Theorem.	**10**	**L3**	**CO5**
	b.	Three boys b_1, b_2, b_3 and four girls g_1, g_2, g_3 and g_4 are such that : i) b_1 is a cousin of g_1, g_3, g_4 ii) b_2 is a cousin of g_2 and g_4 iii) b_3 is a cousin of g_2 and g_3 Can everyone of the boys marry a girl who is one of his cousins? If so, find five possible sets of such samples.	**10**	**L3**	**CO5**

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