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Internal Assesment Test – I

Sub:	Calculus and Linear Algebra.							Code:	18MAT11
Date:	15 / 10 / 2018	Duration:	90 mins	Max Marks:	50	Sem:	I	Section :	I to M and G

Question 1 is compulsory. Answer any SIX from Questions 2 to 8.

		Marks	OBE	
			CO	RBT
1	a) Solve $(2xy + y - tany)dx + (x^2 - xtan^2y + sec^2y)dy = 0$ b) Find the rank of a matrix by reducing in to an Echelon form $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	[08]	CO4 CO6	L3 L3
2	Solve $(ylogx - 2)ydx = xdy$	[07]	CO4	L3
3	A bottle of mineral water at a room temperature of $72^\circ F$ is kept in a refrigerator where the temperature is $44^\circ F$. After half an hour, water cooled to $61^\circ F$ What is the temperature of the mineral water in another half an hour?	[07]	CO4	L3

4	A) Solve $p^2 - p(x+y) + xy = 0$ B) Find the general and singular solution of the Clairaut's equation $xp^3 - yp^2 + 1 = 0$.	[07]	CO5	L3
5	Find the largest Eigen value and the corresponding Eigen vector of the Matrix A by the power method given that $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ taking initial vector as$[1,0,0]^T$.	[07]	CO6	L3
6	Apply Gauss-Jordon method to solve the system of equations $2x + 5y + 7z = 52$ $2x + y - z = 0$ $x + y + z = 9$	[07]	CO6	L3
7	Diagonalize the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$	[07]	CO6	L3
8	A) Solve $y(2xy + 1)dx - xdy = 0$ B) Show that the family of curves $x^3 - 3xy^2 = k_1$ and $y^3 - 3x^2y = k_2$ are Orhogonal trajectories of each other.	[07]	CO4	L3

Solutions of IAT-1 (15/10/18)

Scheme of valuation

Q1 (a) $M = 2xy + y - \tan y$; $N = x^2 - x\tan^2 y + \sec^2 y$

$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2 y$; $\frac{\partial N}{\partial x} = 2x - \tan^2 y$.

$= 2x + 1 - (1 + \tan^2 y)$

$= 2x - \tan^2 y$, $\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\therefore$ Equation is exact. — $1\frac{1}{2}$

$\therefore$ the solution is given by

$\int_{y\,\text{const}} M\,dx + \int (\text{terms of } N \text{ without } x)\,dy = c$ —— ①

$\int (2xy + y - \tan y)\,dx + \int \sec^2 y\,dy = c$ —— $1\frac{1}{2}$

$\Rightarrow x^2 y + xy - x\tan y + \tan y = c$

(b) $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_2 \to 2R_2 - R_1$, $R_3 \to 2R_3 - R_1$

$A \sim \begin{bmatrix} 2 & -1 & -3 & -1 \\ 0 & 5 & 9 & -1 \\ 0 & 1 & 5 & 3 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ —— $1\frac{1}{2}$

$R_3 \to 5R_3 - R_2$, ~~$5R_4$~~

$R_4 \to 5R_4 - R_2$

$$A \sim \begin{bmatrix} 2 & -1 & -3 & -1 \\ 0 & 5 & 9 & -3 \\ 0 & 0 & 16 & 16 \\ 0 & 0 & -4 & -4 \end{bmatrix}$$ — 1

$R_4 \to 4R_4 + R_3$

$$A \sim \begin{bmatrix} 2 & -1 & -3 & -1 \\ 0 & 5 & 9 & -3 \\ 0 & 0 & 16 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1/5,\ R_2 \to R_2/5,\ R_3 \to R_3/16$ — $1\frac{1}{2}$

$$A \sim \begin{bmatrix} 1 & -1/2 & -3/2 & -1/2 \\ 0 & 1 & 9/5 & -3/5 \\ 0 & 0 & 1 & 9/8 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$ — $1/2$

$\therefore\ \rho(A) = 3$

Q2. $(y \log x - 2)y\,dx = x\,dy$

$y^2 \log x - 2y = x\frac{dy}{dx}$

$x\frac{dy}{dx} + 2y = y^2 \log x \Rightarrow y^{-2}\frac{dy}{dx} + \frac{2y^{-1}}{x} = \frac{\log x}{x}$ — 2

Given equation is Bernoulli's Equation.

let $y^{-1} = v \Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$

$-\frac{dv}{dx} + \frac{2v}{x} = \frac{\log x}{x} \Rightarrow \frac{dv}{dx} - \frac{2v}{x} = -\frac{\log x}{x}$ — 2 which is linear diff eq^n in v (2)

$$I.F. = e^{-\int \frac{2}{x} dx} = e^{-2\log x} = 1/x^2 \quad \text{——— (1)}$$

$\Rightarrow$ The solution is

$$v\left(\frac{1}{x^2}\right) = \int -\frac{\log x}{x} \cdot \frac{1}{x^2} dx + c$$

$$\frac{y^{-1}}{x^2} = -\int t e^{-2t} dt + c \quad \text{——— (2)}$$

$$= -\left[\frac{t e^{-2t}}{-2} - \frac{e^{-2t}}{4}\right] + c$$

$$\frac{1}{x^2 y} = \frac{1}{2}\left[\frac{\log x}{x^2} + \frac{1}{2x^2}\right] + c$$

Q3. By Newton's law of Cooling

$$T = t_2 + (t_1 - t_2) e^{-kt} \quad \text{——— (2)}$$

By given data, $t_1 = 72$, $t_2 = 44$.

$\therefore$ $T = 61$ at $t = 30$

$T = 9$ at $t = 60$

By applying initial condition

$$61 = 44 + (72-44) e^{-30k}$$

$$\Rightarrow 17 = 28 e^{-30k} \Rightarrow e^{30k} = \frac{28}{17} \Rightarrow 30k = \log_e\left(\frac{28}{17}\right) \Rightarrow k = 0.0166 \quad (2)$$

Again, $T = 44 + (72-44) e^{-(0.0166)(60)}$

$= 54.3$ ——— (3)

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Q4. (a) $p^2 - p(x+y) + xy = 0$

$$p = \frac{(x+y) \pm \sqrt{(x+y)^2 - 4xy}}{2} = \frac{(x+y) \pm \sqrt{(x-y)^2}}{2}$$

$$p = \frac{(x+y) \pm (x-y)}{2}$$ — 2

$p = x \quad ; \quad p = y \Rightarrow \frac{dy}{dx} = x \quad ; \quad \frac{dy}{dx} = y$

$\Rightarrow y = x\,dx \quad ; \quad \frac{dy}{y} = dx$

$\frac{y^{2}}{\cancel{x}} = \frac{x^2}{2} + c \qquad \log y = x + c \Rightarrow$ — $1\frac{1}{2}$

$\Rightarrow (2y - x^2 - c)(\log y - x - c) = 0$

(b) $xp^3 - yp^2 + 1 = 0$

$\Rightarrow yp^2 = xp^3 + 1 \Rightarrow y = px + \frac{1}{p^2}$, which is clairaut's for. — 1

$y = cx + \frac{1}{c^2}$ — 1

$0 = x - \frac{2}{c^3} \Rightarrow \frac{2}{c^3} = x \Rightarrow c = \left(\frac{2}{x}\right)^{1/3}$

$y = \left(\frac{2}{x}\right)^{1/3} x + \frac{x^{2/3}}{2^{2/3}}$ Ans — $1\frac{1}{2}$

(5) $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}, \quad X^{(0)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (3)

$$AX^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^{(1)}X^{(1)}$$ (1)

$$AX^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)}X^{(2)}$$ (1)

$$AX^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8\begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \lambda^{(3)}X^{(3)}$$ (1)

$$AX^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \begin{bmatrix} 2.93 \\ 0 \\ 2.86 \end{bmatrix} = 2.93\begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \lambda^{(4)}X^{(4)}$$ (1½)

$$AX^{(4)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \begin{bmatrix} 2.98 \\ 0 \\ 2.96 \end{bmatrix} = 2.98\begin{bmatrix} 1 \\ 0 \\ 0.99 \end{bmatrix} = \lambda^{(5)}X^{(5)}$$ (1½)

∴ The largest Eigen value is 2.98 and Eigen vector is $\begin{bmatrix} 1 & 0 & 0.99 \end{bmatrix}^T$ (1)

writing equations in Matrix form

Q6.

$$[A:B] = \left[\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 9 \end{array}\right]$$

$R_2 \to R_2 - R_1, \quad R_3 \to 2R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 0 & -4 & -8 & -52 \\ 0 & -3 & -5 & -34 \end{array}\right]$$

$R_3 \to 4R_3 - 3R_2$; $R_2 \to -R_2/4$

$$C \sim \left[\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 4 & 20 \end{array}\right]$$

3

$R_2 \to 2R_2 - R_3$, $R_3 \to R_3/4$, $R_2 \to R_2/2$

$$C \sim \left[\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$R_1 \to \dfrac{R_1 - 5R_2 - 7R_3}{2}$

$$C \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

3

$\boxed{x = 1,\ y = 3,\ z = 5}$ Answer

1

Q7. $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$

$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{vmatrix} = 0$

$-4 + \lambda - 4\lambda + \lambda^2 + 6 = 0$

$\lambda^2 - 3\lambda + 2 = 0 \Rightarrow (\lambda - 1)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2, 1$ are the eigen values.

2

Case-i) $\lambda = 1$, Eigen vector $X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 3x_2 = 0 \Rightarrow 2x_1 = 3x_2 \Rightarrow \frac{x_1}{3} = \frac{x_2}{2} \Rightarrow X_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad 1$$

Case-ii) $\lambda = 2$, Eigen vector $X_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} -3x_1 + 3x_2 = 0 \\ x_1 = x_2 \end{matrix} \qquad X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad 1$$

Let the Modal Matrix $P = [X_1 \; X_2] = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ — 1

$$P^{-1} = \frac{\operatorname{adj}(P)}{|P|} = \frac{\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}}{(1)} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \quad 1$$

$\therefore$ $$D = P^{-1}AP = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} 3 & -1 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\cancel{= [3}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad 1\tfrac{1}{2}$$

$$= \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Q8. (a) $y(2xy+1)\,dx - x\,dy = 0$

$M = 2xy^2 + y, \quad N = -x$

$\frac{\partial M}{\partial y} = 4xy + 1, \quad \frac{\partial N}{\partial x} = -1 \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ Not exact

$1\frac{1}{2}$

$\frac{(M_y - N_x)}{M} = \frac{4xy+2}{y(2xy+1)} = \frac{2}{y} = g(y)$

$\Rightarrow I.F. = e^{-\int g(y)\,dy} = e^{-\int \frac{2}{y}\,dy} = e^{-2\log y} = 1/y^2$

$1\frac{1}{2}$

Multiplying eq^n (1) by I.F.

$(2x + \frac{1}{y})\,dx - \frac{x}{y^2}\,dy = 0$ $\Rightarrow$ which is an exact now.

Now,

$\frac{\partial M}{\partial y} = -\frac{1}{y^2}, \quad \frac{\partial N}{\partial x} = -\frac{1}{y^2}$

$\therefore$ Solution is given by $\int_{y\ \text{const}} M\,dx + \int (N \text{ without } x \text{ terms})\,dy = c$

$1/2$

$\int (2x + \frac{1}{y})\,dx + \int 0\,dy = c$

$\boxed{x^2 + \frac{x}{y} = c}$

x

(b) Let $f(x,y,c) = 0 \Rightarrow x^3 - 3xy^2 = k_1$

diff eq^n (1) w.r.t. 'x'

$3x^2 - 3\left[y^2 + 2xy\frac{dy}{dx}\right] = 0$

1

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⑤

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$x^2 - [y^2 - 2xy\frac{dx}{dy}] = 0 \rightarrow (x^2-y^2)dy + 2xy\,dx = 0$ — 1

$\cancel{2yx\frac{dx}{dy} = y^2} - x^2$ $\quad M = +2xy, \; N = x^2 - y^2$

$\frac{\partial M}{\partial y} = +2x, \quad \frac{\partial N}{\partial x} = 2x$

∴ This is an exact diff. eqn.

∴ the solution.

$\int 2xy\,dx + \int(-y^2)\,dy = 0$

$x^2y - \frac{y^3}{3} = c \rightarrow \cancel{3x^2y}\; y^3 - 3x^2y = -3c$ — $1\frac{1}{2}$

$\boxed{y^3 - 3x^2y = k_2}$ is the orthogonal trajectory of given curve.

Proved.

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