

IAT-1 ODD SEM 2018-19

SCHEME

1.a. [6]

TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:

We know that the diameter of the nucleus is of the order of 10^{-14} m. If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\Delta x \leq 10^{-14} \text{ m} \quad [2]$$

Now the uncertainty in momentum is

$$\Delta p \geq \frac{h}{4\pi \Delta x}$$

$$\Delta p \geq \frac{6.62 \times 10^{-34}}{4\pi \times 10^{-14}}$$

$$\Delta p \geq 0.5 \times 10^{-19} \text{ N s}$$

Then the momentum of the electron can at least be equal to the uncertainty in momentum.

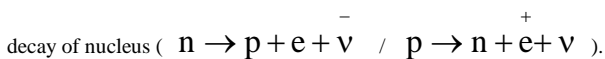
$$p \geq 0.5 \times 10^{-19} \text{ N s} \quad [2]$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities -non-relativistic-case)

$$E = \frac{p^2}{2m} = \frac{(0.5 \times 10^{-19})^2}{2 \times 9.1 \times 10^{-31}} = 1.37 \times 10^{-11} \text{ J} = 8.5 \text{ MeV}$$

[2]

The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of



Note: Two aspects of studying principle are through OBSERVER EFFECT (distortion caused by the process of measurement) and also through wave nature of matter.

1.b. [4]

$$\lambda_{\text{photon}} = \frac{hc}{E} = 6206 \times 10^{-10} \text{ m} \quad [2]$$

$$\lambda_{\text{Electron}} = \frac{h}{\sqrt{2mE}} = 8.69 \times 10^{-10} \text{ m} \quad [2]$$

~ Wavelength of Photon is 715 times greater than that of wavelength of an electron

2.a [6]

Time independent Schrödinger equation

A matter wave can be represented in complex form as

$$\Psi = A \sin kx (\cos wt + i \sin wt)$$

$$\Psi = A \sin kxe^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA \cos kxe^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A \sin kxe^{iwt} = -k^2 \Psi \dots\dots\dots (1) \quad [2]$$

From de Broglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

$$k^2 = 4\pi^2 \frac{p^2}{h^2} \dots\dots\dots (2) \quad [2]$$

Total energy of a particle

E = Kinetic energy + Potential Energy

$$E = \frac{p^2}{2m} + V$$

$$E = \frac{1}{2} m v^2 + V$$

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\pi^2 (E - V)2m}{h^2}$$

∴ From (1) [2]

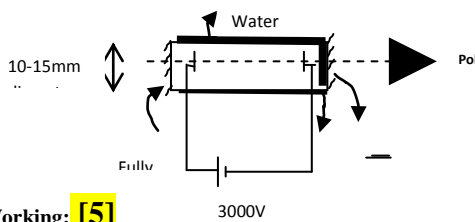
$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m (E - V)\Psi}{h^2} = 0$$

2.b. [4] Case 1 : $E_1 = \frac{1^2 h^2}{8mL^2} = 40eV$
 Case 2 : $E_1 = \frac{40}{4} = 10eV$

3.a. [7] Carbon dioxide laser

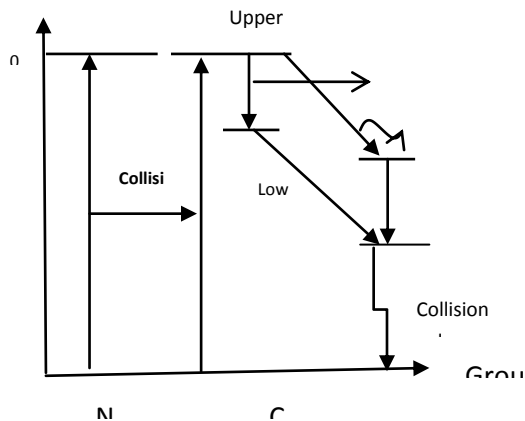
Construction [2]

- 1.Active medium – Mixture of CO₂, N₂ and He in the ratio 1:2:8. Nitrogen absorbs energy from the pumping source efficiently. Helium gas conducts away the heat and also catalyses collisional deexcitation of CO₂ molecules.
- 2.The discharge tube consists of a glass tube of 10-15mm diameter with a coaxial water cooling jacket.
- 3.Partially reflecting and fully reflecting mirrors are mounted at the ends of the tube.
- 4.Optical pumping is achieved by electric discharge caused by applying potential difference of over 1000V.



Working: [5]

- 1.CO₂ is a linear molecule and has three modes of vibration –Symmetric stretching (100), Asymmetric stretching (001) and bending (010).
2. Asymmetric stretching (001) is the upper laser level which is a metastable state. (100) and (020) are the lower lasing states
- 3.During electric discharge, the electrons released due to ionisation excite N₂ molecules to its first vibrational level which is close to upper lasing level of CO₂.
- 4.N₂ molecules undergo collisions with CO₂ molecules and excite them to (001). This results in population inversion.
- 5.Lasing transition occurs between (001) and (100) emitting at 10.6μm and (001) to (020) emitting at 9.6μm
6. CO₂ molecules deexcite to ground state through collisions with Helium atom.

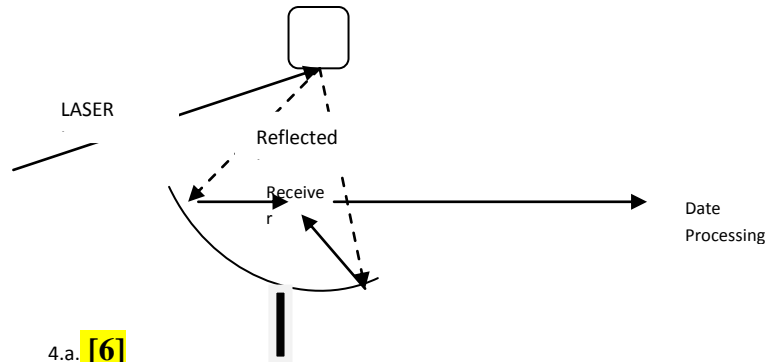


3.b. [3]

LASER RANGE FINDER

Laser rangefinders have numerous applications such as measuring of rooms and buildings in the construction sector, to determine the depth of snow in inaccessible areas, Cloud base height for atmospheric study, air pollutant distribution, attitude characterization of space debris, trajectory of aircraft, satellites. Laser technology is more cost effective.

The laser rangefinder uses a laser signal is transmitted and returned from a target. The time delay between transmission and receipt of the signal is used to determine the distance to the target based on the speed of light. The receiver consists of reflector, photodetector and amplifier.



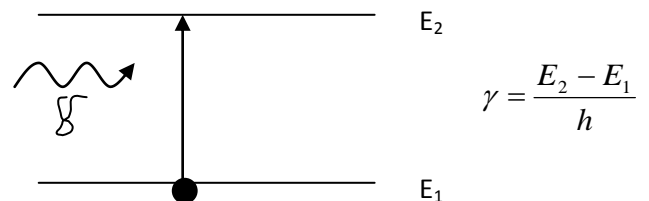
4.a. [6]

Exp Expression for energy density:

Induced absorption: [1]

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E₁ and E₂ be the energy levels in an atom and N₁ and N₂ be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident..



Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_γ.

$$\text{Rate of absorption} = B_{12} N_1 U_\gamma$$

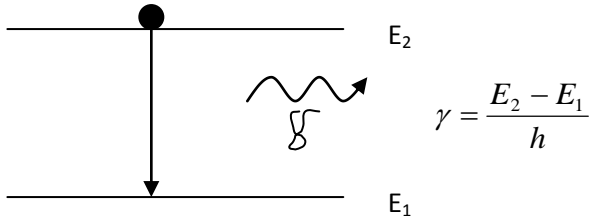
Here B₁₂ is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission: [1]

It is a process in which atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

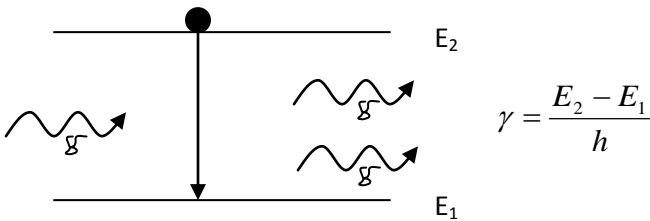
$$\text{Rate of spontaneous absorption} = A_{21} N_2$$

Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



Stimulated emission: [1]

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_γ .

$$\text{Rate of stimulated emission} = B_{21} N_2 U_\gamma$$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12} N_1 U_\gamma = A_{21} N_2 + B_{21} N_2 U_\gamma$$

$$U_\gamma = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Rearranging this, we get

$$U_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right]$$

From Boltzmann's law, $\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h \nu^3}{c^3} \left[\frac{1}{e^{\left[\frac{h\nu}{kT}\right]} - 1} \right]$$

Comparing these expressions, we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1 \quad [3]$$

4.b. [4]

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}} = e^{\frac{1.54 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}}$$

From Boltzmann's law,

$$\frac{N_1}{N_2} = 6.9 \times 10^{25}$$

5.a. [7]

EXPRESSION FOR FERMI ENERGY

From Fermi-Dirac theory

$$n = \int_0^{E_F} g(E) \cdot f(E) \cdot dE = \int_0^{E_F} \frac{4\pi(2m)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}} dE \quad [3]$$

$$= \frac{4\pi(2m)^{\frac{3}{2}}}{h^3} \frac{E_F^{\frac{3}{2}}}{\frac{3}{2}}$$

$$E_F^{\frac{3}{2}} = \frac{h^3 3n}{8\pi(2m)^{\frac{3}{2}}}$$

$$E_F = \frac{h^2}{8m} \left[\frac{3n}{\pi} \right]^{\frac{2}{3}} \quad [4]$$

5.b. [3]

$$E_F = \frac{h^2}{8m} \left[\frac{3n}{\pi} \right]^{\frac{2}{3}} = 8.002 \times 10^{-18} \text{ J} = 50.01 \text{ eV}$$

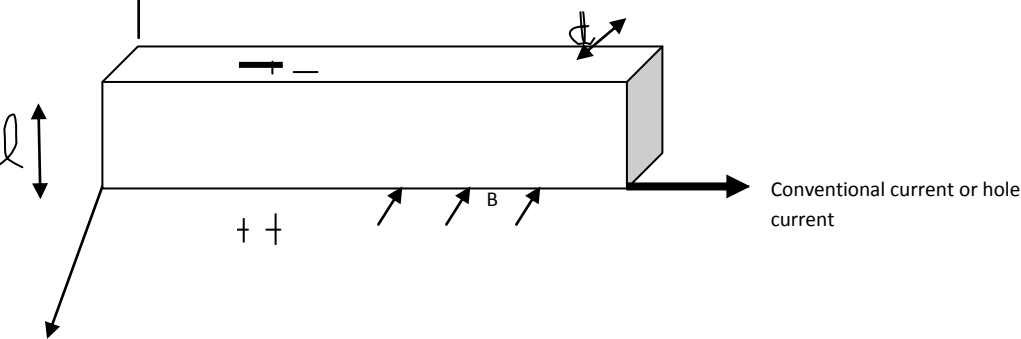
$$v_f = \sqrt{2Ef/m} = 4.19 \times 10^6 \text{ m/s}$$

$$T_f = E_f/k = 5.79 \times 10^5 \text{ K}$$

6.a [7]

Hall effect: When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.

Here B is along -X, V is along -Y axis
 Lorentz force = $-e(-\hat{j} \times X - \hat{i}) = +\hat{k}$
 So the electron is deflected along + Z axis



Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is

applied along -Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. This develops a potential V_H -Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

[4]

Expression for Hall Coefficient:

At equilibrium, Lorentz force is equal to force due to applied electric field

$$Bev = -e E_H$$

$$\text{Hall Field } E_H = Bv$$

$$\text{Current density } J = -n_e ev$$

$$v = \frac{J}{n_e e}$$

$$E_H = B \frac{-J}{n_e e}$$

$$\text{Hence } \frac{E_H}{JB} = -\frac{1}{n_e e} = R_H \quad [3]$$

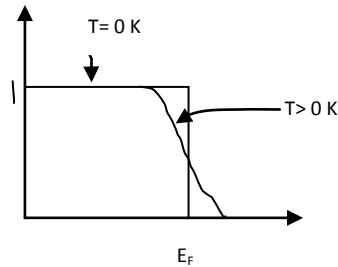
$$V_H = E_H l = -R_H JB l$$

6.b. [3]

To show that energy levels below Fermi energy are completely occupied:

For $E < E_F$, at $T = 0$,

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = 1$$



To show that energy levels above Fermi energy are empty:

For $E > E_F$, at $T=0$

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = 0$$

At ordinary temperatures, for $E = E_F$,

$$f(E) = \frac{1}{2}$$

7.a. [7]

Different polarization mechanisms: [3]

There are 4 mechanisms.

1. Electronic polarization
2. Ionic polarization
3. Orientation polarization
4. Space charge polarization

Electronic polarization: These are generally seen in the case of covalent compounds.

When a covalent compound is placed in electric field, displacement of electron cloud takes place relative to the nucleus. This displacement creates a dipole which develops dipole moment.

$$\text{Electronic polarisability } \alpha_e = \frac{\epsilon_o (\epsilon_r - 1)}{N}$$

N is number of dipoles per unit volume

It is independent of temperature.

Ionic polarization:

This is exhibited by ionic compounds.

When ionic compounds are kept in an electric field, displacement of positive and negative ions occurs developing a dipole moment.

$$\text{Ionic polarisability } \alpha_{i=} = \frac{\epsilon_o (\epsilon_r - 1)}{N_i} \approx 0.1 \alpha_e$$

Orientation polarization:

Polar molecules exhibit this mechanism.

When polar molecules are kept in an electric field, already existing dipoles tend to align in the direction of applied electric field. This increases the dipole moment.

$$\text{Orientation polarization } \alpha_o = \frac{\mu^2}{kT}$$

Space charge polarization:

This polarization exists in materials possessing different phases due to difference in temperatures. In such materials charge carriers drift and accommodate in certain regions of higher conductivity (electrodes) causing dipole moment. It occurs in ferrites and semiconductors. Its magnitude is very small compared to other mechanisms.

CLAUSIUS – MOSOTTI RELATION: [4]

This expression relates dielectric constant of an insulator (ϵ) to the polarization of individual atoms (α) comprising it.

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

where N is the number of atoms per unit volume

α is the polarisability of the atom

ϵ_r is the relative permittivity of the medium

ϵ_0 is the permittivity of free space.

Proof:

If there are N atoms per unit volume, the electric dipole moment per unit volume – known as polarization is given by

$$P = N\alpha E_i$$

By the definition of polarization P, it can be shown that

$$P = \epsilon_0 E_a (\epsilon_r - 1) = N\alpha E_i$$

$$\epsilon_0 \epsilon_r E_a - \epsilon_0 E_a = N\alpha E_i$$

$$\epsilon_r = 1 + \frac{N\alpha E_i}{\epsilon_0 E_a} \dots\dots\dots(1)$$

The internal field at an atom in a cubic structure ($\gamma = 1/3$) is of the form

$$E_i = E_a + \frac{P}{3\epsilon_0} = E_a + \frac{N\alpha E_i}{3\epsilon_0}$$

$$\frac{E_i}{E_a} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\epsilon_0}\right)\right]}$$

Substituting for $\frac{E_i}{E_a}$ in equation (1)

$$\begin{aligned} \epsilon_r &= 1 + \frac{N\alpha}{\epsilon_0} \left[\frac{1}{\left(1 - \frac{N\alpha}{3\epsilon_0}\right)} \right] = \frac{\epsilon_0 \left[1 - \frac{N\alpha}{3\epsilon_0}\right] + \frac{N\alpha\epsilon_0}{\epsilon_0}}{\epsilon_0 \left[1 - \frac{N\alpha}{3\epsilon_0}\right]} \\ &= \frac{1 + \frac{2}{3} \left(\frac{N\alpha}{\epsilon_0}\right)}{1 - \frac{1}{3} \left[\frac{N\alpha}{\epsilon_0}\right]} \end{aligned}$$

$$\left[\frac{\epsilon_r - 1}{\epsilon_r + 2} \right] = \frac{1 + (2/3) \frac{N\alpha}{\epsilon_0} - 1}{1 - (1/3) \frac{N\alpha}{\epsilon_0}} = \frac{N\alpha}{3\epsilon_0} \frac{1 - (1/3) \frac{N\alpha}{\epsilon_0} + 2}{1 - (1/3) \frac{N\alpha}{\epsilon_0}}$$

7.b. [3]

$$\begin{aligned} P &= \epsilon_0 E_a (\epsilon_r - 1) \\ \epsilon_r &= 5.8 \end{aligned}$$

8.a. [6]

Expression for Fermi Level in Intrinsic Semiconductor

Electron density in conduction band is given by

$$n_e = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{\frac{3}{2}} e^{-\frac{E_c - E_f}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{\frac{3}{2}} e^{-\frac{E_f - E_v}{kT}} \quad [3]$$

For an intrinsic semiconductor, $n_e = n_h$

$$2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{\frac{3}{2}} e^{-\frac{E_c - E_f}{kT}} = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{\frac{3}{2}} e^{-\frac{E_f - E_v}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*} \right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2} \ln \left(\frac{m_e^*}{m_h^*} \right) = \frac{-2E_f + E_v + E_c}{kT} \quad [3]$$

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4} kT \ln \left(\frac{m_e^*}{m_h^*} \right)$$

8.b. [4]

$$\sigma = \frac{1}{\rho} = ne(\mu_e + \mu_h)$$

$$n = 1.39 \times 10^{19} / m^3$$