2. b)

Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current ***is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same*** **heat** ***as produced by the alternating current when flowing through the same circuit for the same time.***

Analytical Method

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$.

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$= \int_0^{2\pi} \frac{i^2 \, d\theta}{(2\pi - 0)}$$

The square root of this value is $= \sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)}$

Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta \, d\theta\right)} \qquad \text{(put } i = I_m \sin \theta\text{)}$$

Now, $\cos 2\theta = 1 - 2 \sin^2 \theta \quad \therefore \ \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

$$\therefore \qquad I = \sqrt{\left(\frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta) \, d\theta\right)} = \sqrt{\left(\frac{I_m^2}{4\pi} \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{2\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{4} \quad 2} \qquad \sqrt{\frac{I_m^2}{2}} \qquad \therefore \ \mathrm{I} = \frac{I_m}{\sqrt{2}} = 0.707 \, I_m$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively. In electrical engineering work, *unless indicated otherwise, the values of the given current and voltage are always the r.m.s. values.*

It should be noted that the average heating effect produced during one cycle is

$$= I^2 R = (I_m/\sqrt{2})^2 \, R = \frac{1}{2} I_m^2 \, R$$

Average Value

The average value I_a of an alternating current is expressed *by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.*

In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. *But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.*

(*i*) Mid-ordinate Method

With reference to Fig. 11.16, $I_{av} = \dfrac{i_1 + i_2 + \ldots + i_n}{n}$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

(*ii*) Analytical Method

The standard equation of an alternating current is, $i = I_m \sin \theta$

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta \qquad \text{(putting value of } i\text{)}$$

$$= \frac{I_m}{\pi}\left|-\cos\theta\right|_0^{\pi} = \frac{I_m}{\pi}\left|+1-(-1)\right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$\therefore \qquad I_{av} = I_m/\tfrac{1}{2}\pi = 0.637\, I_m \qquad \therefore \text{ average value of current} = 0.637 \times \text{maximum value}$$

Note. R.M.S. value is always greater than average value except in the case of a rectangular wave when both are equal.

Form Factor

It is defined as the ratio, $K_f = \dfrac{\text{r.m.s. value}}{\text{average value}} = \dfrac{0.707\, I_m}{0.637\, I_m} = 1.1$. (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = \dfrac{0.707\, E_m}{0.637\, E_m} = 1.11$

As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and *vice-versa.*

Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \dfrac{\text{maximum value}}{\text{r.m.s. value}} = \dfrac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \dfrac{E_m}{E_m/\sqrt{2}} = 1.414$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

8a.

13.1. A.C. Through Resistance and Inductance

A pure resistance R and a pure inductive coil of inductance L are shown connected in series in Fig. 13.1.

Let V = r.m.s. value of the applied voltage, I = r.m.s. value of the resultant current

$V_R = IR$ –voltage drop across R (in phase with I), $V_L = I \cdot X_L$ –voltage drop across coil (ahead of I by 90°)

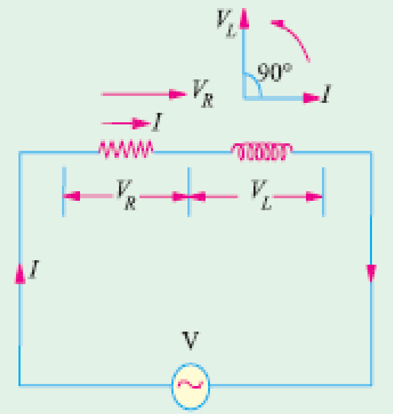


Fig. 13.1

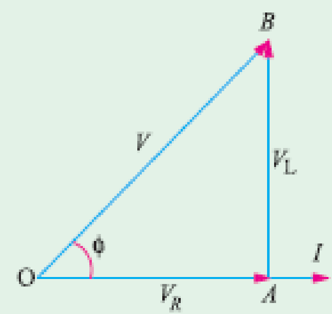


Fig. 13.2

These voltage drops are shown in voltage triangle OAB in Fig. 13.2. Vector OA represents ohmic drop V_R and AB represents inductive drop V_L. The applied voltage V is the vector sum of the two *i.e.* OB.

$$\therefore \quad V = \sqrt{(V_R^2 \quad V_L^2)} \quad \sqrt{[(IR)^2 \quad (I \cdot X_L)^2]} \quad I\sqrt{R^2 \quad X_L^2}, \frac{V}{\sqrt{(R^2 \quad X_L^2)}} \quad I$$

The quantity $\sqrt{(R^2 + X_L^2)}$ is known as the *impedance* (Z) of the circuit. As seen from the impedance triangle ABC (Fig. 13.3) $Z^2 = R^2 + X_L^2$.

i.e. $(Impedance)^2 = (resistance)^2 + (reactance)^2$

From Fig. 13.2, it is clear that the applied voltage V leads the current I by an angle φ such that

$$\tan\phi = \frac{V_L}{V_R} = \frac{I \cdot X_L}{I \cdot R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{reactance}} \quad \therefore \phi = \tan^{-1}\frac{X_L}{R}$$

The same fact is illustrated graphically in Fig. 13.4.

In other words, current I lags behind the applied voltage V by an angle φ.

Hence, if applied voltage is given by $v = V_m \sin \omega t$, then current equation is

$$i = I_m \sin(\omega t - \phi) \text{ where } I_m = V_m / Z$$

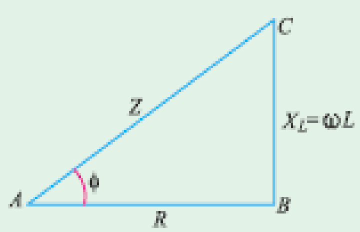


Fig. 13.3

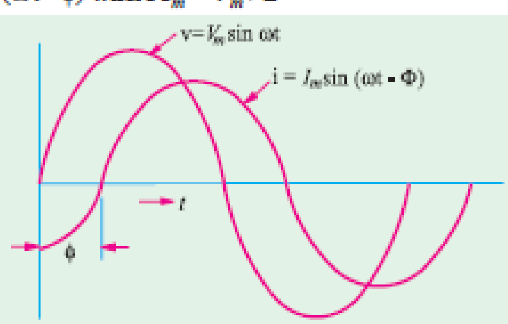


Fig. 13.4

In Fig. 13.5, I has been resolved into its two mutually perpendicular components, $I \cos \phi$ along the applied voltage V and $I \sin \phi$ in quadrature (*i.e.* perpendicular) with V.

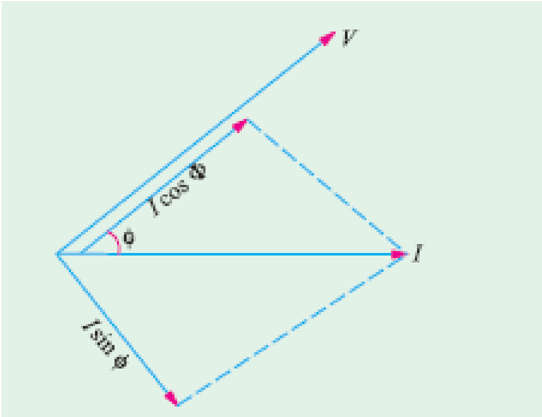


Fig. 13.5

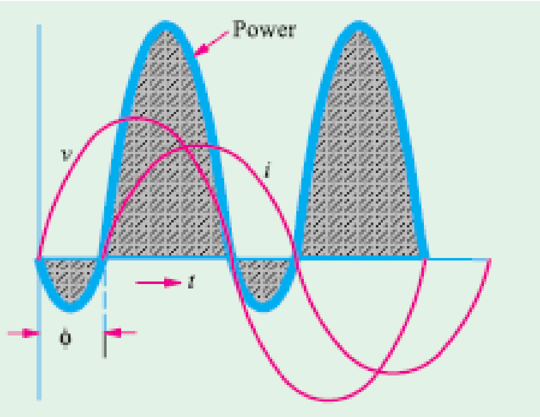


Fig. 13.6

The mean power consumed by the circuit is given by the product of V and *that component of the current I which is in phase with V.*

So $P = V \times I \cos \phi$ = r.m.s. voltage × r.m.s. current × cos φ

The term 'cos φ' is called the power factor of the circuit.

4b.

Let a series R-L circuit draw a current of I when an alternating voltage of r.m.s. value V is applied to it. Suppose that current lags behind the applied voltage by φ. The three powers drawn by the circuit are as under :

(*i*) **apparent power (*S*)**

It is given by the product of r.m.s. values of applied voltage and circuit current.

$\therefore \quad S = VI = (IZ) . I = I^2 Z$ volt-amperes (VA)

(*ii*) **active power (*P* or *W*)**

It is the power which is actually dissipated in the circuit resistance. $P = I^2 R = VI \cos \phi$ watts

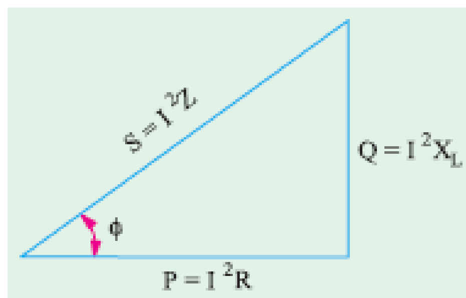


Fig. 13.11

(*iii*) **reactive power (Q)**

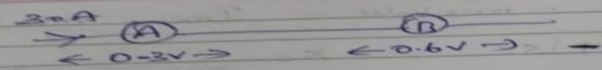
It is the power developed in the inductive reactance of the circuit.

$Q = I^2 X_L = I^2 . Z \sin \phi = I . (IZ). \sin \phi = VI \sin \phi$ volt-amperes-reactive (VAR)

These three powers are shown in the power triangle of Fig. 13.11 from where it can be seen that

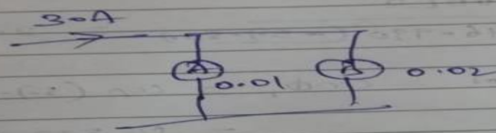
$$S^2 = P^2 + Q^2 \text{ or } S = \sqrt{P^2 + Q^2}$$

5b.



find R_A and R_B, $R_A = \frac{V}{I} = \frac{0.3}{30} = 0.01\,\Omega$

$R_B = \frac{V}{I} = \frac{0.6}{30} = 0.02\,\Omega$



$\cancel{R_{total}} =$

$I_A = \frac{30 \times 0.02}{0.01 + 0.02}$

$\boxed{I_A = 20A}$

$I_B = \frac{30 \times 0.01}{0.01 + 0.02}$

$= \underline{10A}$

$\boxed{I_B = 10A}$

1 a.

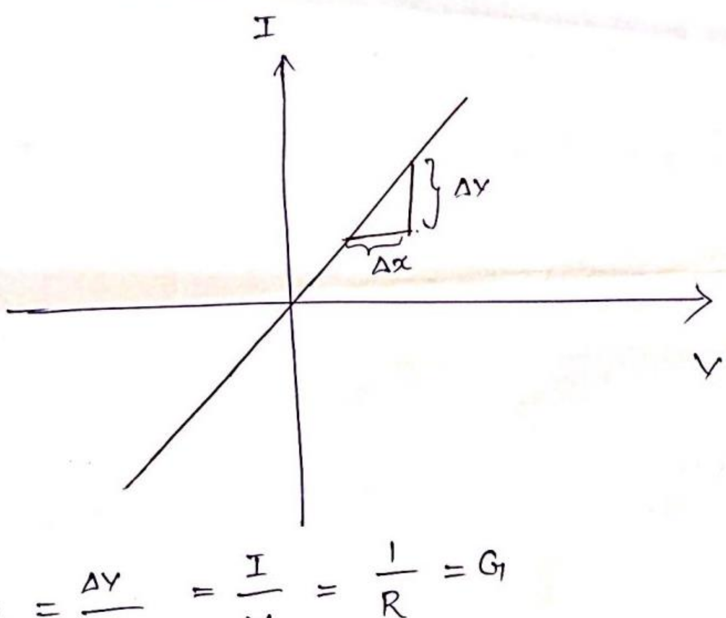
Ohm's law :-

The ratio of potential difference (v) be
any two points on a conductor to the cu
flowing between them is constant, provide
the conductor doesn't c
the temperature of the conductor

$$\frac{V}{I} = \text{constant}$$
$$= R(\Omega)$$

R - constant of proportionality
- resistance of the conductor.

Graphical representation of Ohm's law:



$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{I}{V} = \frac{1}{R} = G$$

where G is conductance (siemens) (℧).

Limitations - OHM'S LAW

1) It cannot be applied to non-linear devices like diodes, zener diodes, transistors, voltage regulator etc.

2) Ohm's law is applicable as long as tempe-rature and other physical parameters remains constant

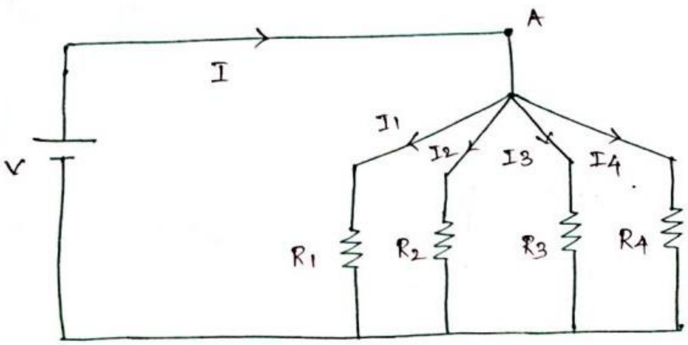
3) It cannot be applied to complicated ckts having more no of branches and emf sources.

4) Not suitable for non-metallic conductors like silicon carbide, graphite etc.

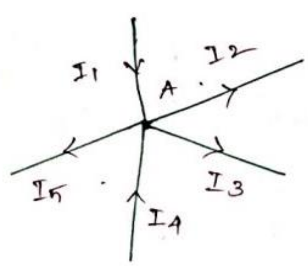
KIRCHHOFF'S CURRENT / POINT LAW: [KCL]

In any electrical network, the algebraic sum of the currents meeting at a point or junction is zero. i.e $\Sigma I = 0$.

i.e total current leaving a junction is equal to the total current entering that junction.



$$I = I_1 + I_2 + I_3 + I_4$$



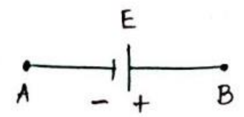
$$I_1 + I_4 = I_2 + I_3 + I_5$$

KIRCHÖFF'S VOLTAGE / MESH LAW:. [KVL]

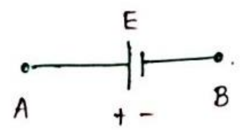
The algebraic sum of voltages [voltage drop + e.m.f] around a closed loop or circuit is zero.

$$\sum IR + \sum e.m.f = 0.$$

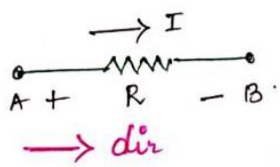
Determination of voltage sign :-



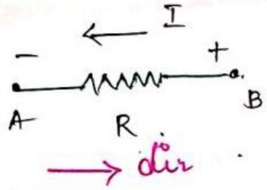
Rise in voltage +ve sign.



Fall in voltage −ve sign.



Fall in voltage −ve sign.



Rise in voltage +ve sign.

※. it should be noted that sign of voltage drop depends on the direction of current and is independent of the polarity of the emf of source in the circuit under consideration.