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CMRIT — CMR Institute of Technology, Bengaluru. Accredited with A+ Grade by NAAC

Internal Assessment Test 1 – Sept. 2018

Sub:	Discrete Mathematical Structures					Sub Code:	17CS36	Branch: IS /CS – A, B, C
Date:	10/09/2018	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	III A and B	OBE

Question 1 is compulsory and answer any six from Q.2 to Q.9

		MARKS	CO	RBT
1	Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code for the message.	[08]	CO5	L3
2	Write a short note on Konigsberg Bridge Problem.	[07]	CO5	L2
3	Check the following graphs for Isomorphism. 	[07]	CO5	L3
4	Prove that a tree with n vertices has n-1 edges.	[07]	CO5	L3
5	(i) If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5, find the number of leaves in T. (ii) Check whether there exists simple graphs corresponding to the following degree sequences (a) 1, 1, 2, 3 (b) 2, 2, 4, 6	[07]	CO5	L3

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6 Determine the order |V| of the graph G = (V, E) in the following cases [07] CO5 L3

(i) G is a cubic graph with 9 edges.
(ii) G is a regular graph with 15 edges.
(iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.

7 Define the following with an example for each. [07] CO5 L1

(i) complete graph (ii) regular graph (iii) bipartite graph

8 Prove the following using laws of logic: [07] CO1 L3

(i) $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$

(ii) $[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$

9 Define Tautology and Contradiction. For any propositions p, q, r, verify whether the compound proposition $[(p \wedge \neg q) \to r] \to [p \to (q \vee r)]$ is a tautology. [07] CO1 L3

For CS Branch:

Q.N. - 6 Check the validity of the following argument [07] CO1 L3

$p \wedge q$
$p \to (r \wedge q)$
$r \to (s \vee t)$
$\neg s$
___
$\therefore t$

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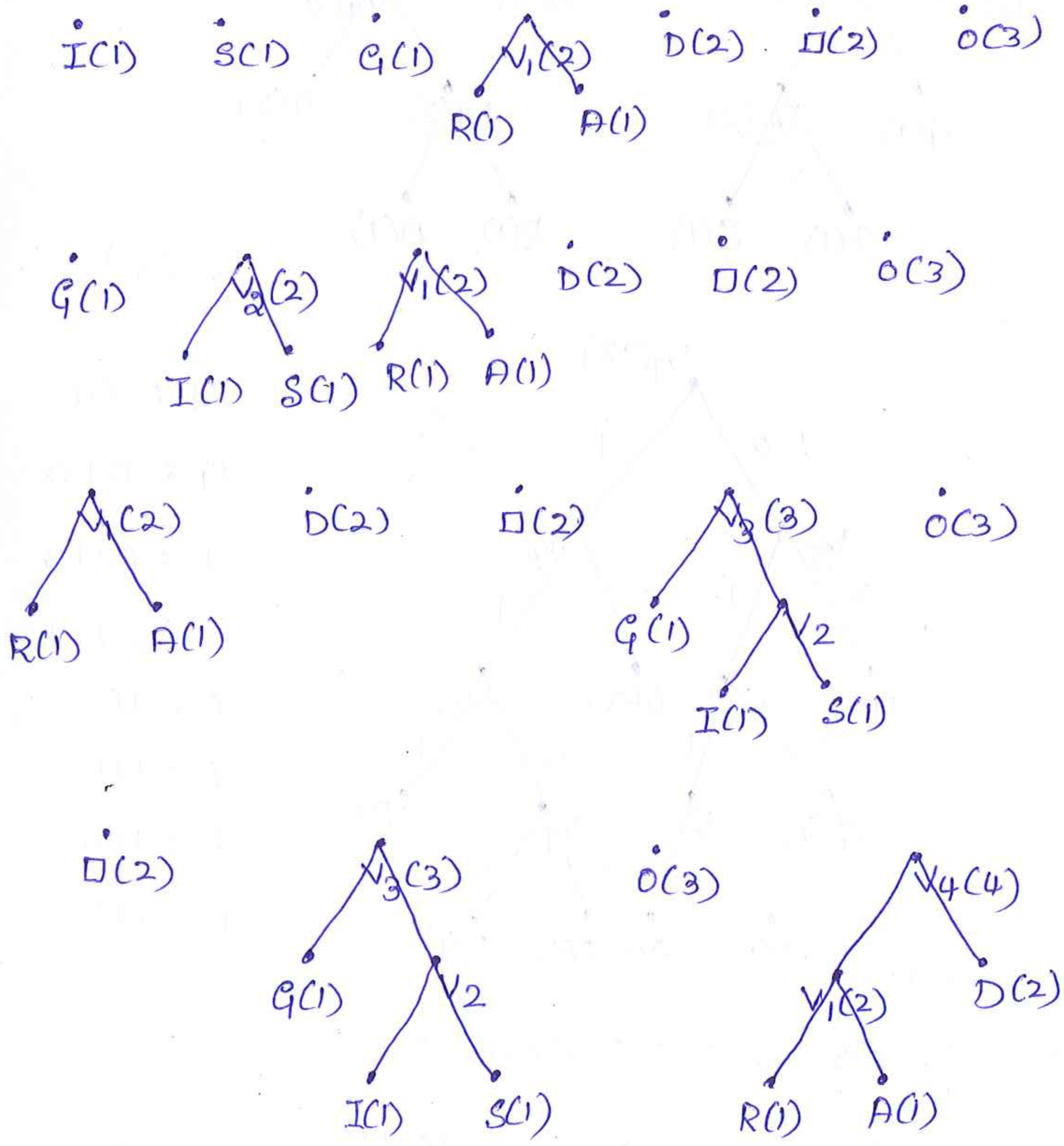
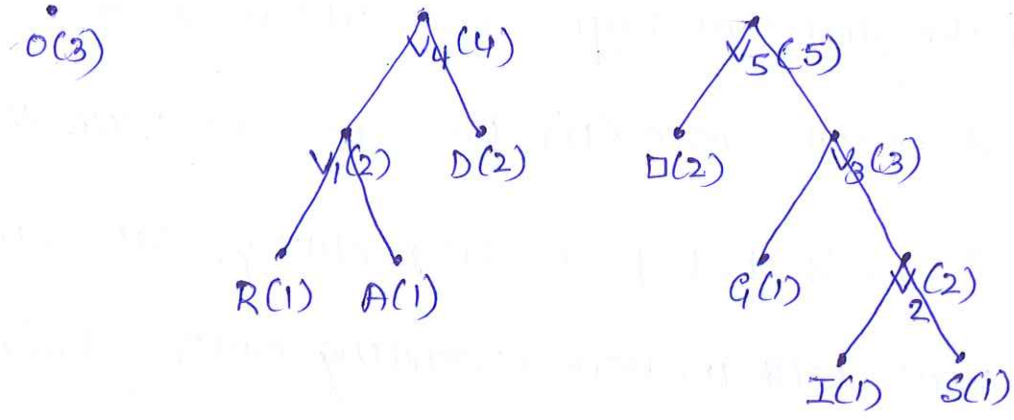
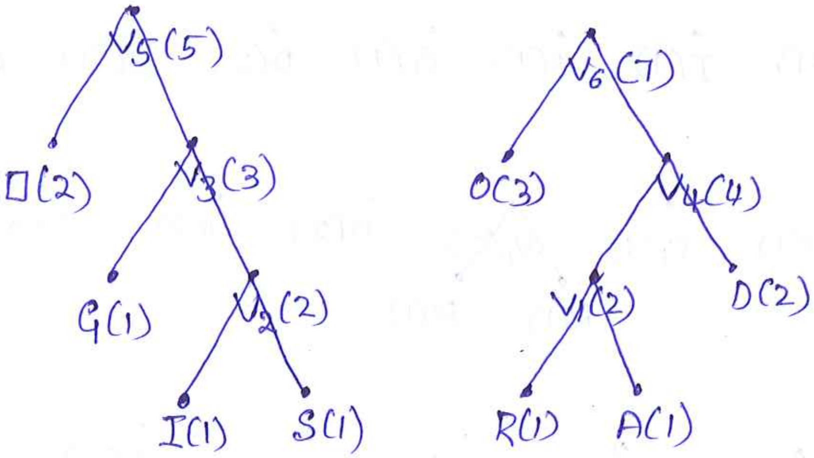
Discrete Mathematical Structures - 17CS36

1. In the given message, the letters R, O, A, D, I, S, G, and blank space (□) are present with the frequencies 1, 3, 1, 2, 1, 1, 1, 2 respectively. Let us write these as nodes ~~with~~ in non-descending order of their weights.

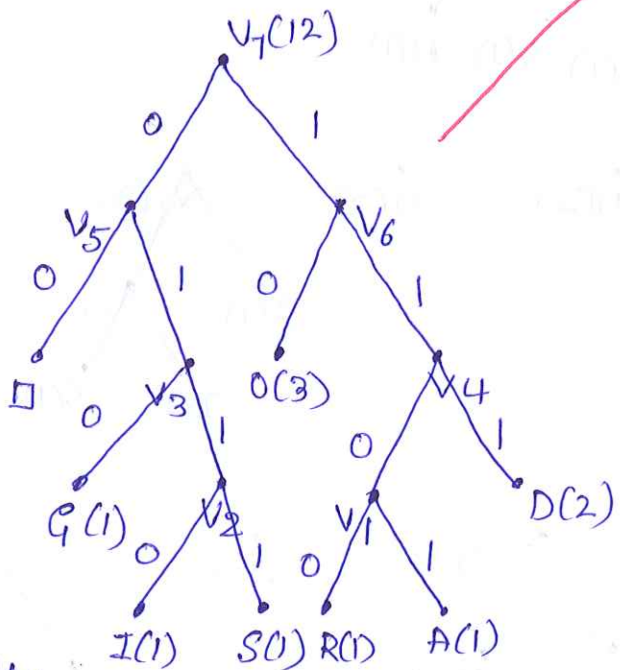
R(1) A(1) I(1) S(1) G(1) D(2) □(2) O(3) — (1)







(5)



∴ □ : 00

G : 010

I : 0110

S : 0111

O : 10

D : 111

R : 1100

A : 1101

— (1)

Code for the

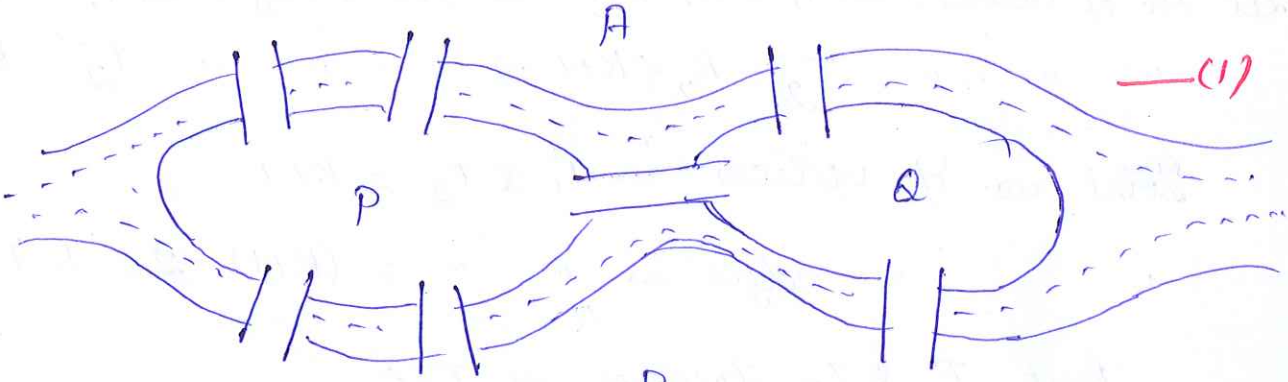
∴ Message ~~Aa~~ ROAD IS GOOD is :

11001011011110001100111000101010111

— (1)

2. In 18th century, the city of Konigsberg in Prussia was set on both sides of the Pregel river and included two large islands which were connected to each other and the main land by seven bridges. The problem was, by starting at any of the four land areas, can we return to that area after crossing each of the seven bridges exactly once?

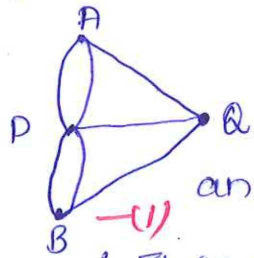
Four land areas – two of these parts are the banks of the river and two are islands]



—(1)

This is the starting point of development of graph theory. In 1736. Euler analyzed this problem with the help of graph and gave the soln. This problem is known as Konigsberg Bridge problem.

Let the land areas be denoted by A, B, P, Q. A, B are the banks of the river & P, Q are islands. Treat four land areas as four vertices and 7 bridges as 7 edges. —— (4)



—(1)

we note that, here deg (A) = 3, deg (B) = 3, deg (P) = 5 & deg (Q) = 3 which are not even. ∴ graph doesn't have an Euler circuit.

∴ It is not possible to walk over each of the seven bridges exactly once and return to the starting point. —(1)

(4.) When $n = 1$

$n = 2$

$n = 3$ 

$\therefore$ Statement is true for $n = 1, 2, 3$. —— (2)

Assume that the theorem is true for all trees with n vertices where $n \leq k$, $k \in \mathbb{Z}^+$ —— (1)

Consider a tree with $k+1$ vertices and let e be an edge in this with end vertices u & v. Deletion of e will disconnect the graph & $T-e$ consists of exactly two components, say T_1 & T_2. Now T_1 & T_2 are trees

Let No of vertices in $T_1 = k_1 < k+1 \Rightarrow$ No of edges in $T_1 = k_1 - 1$

" " " $T_2 = k_2 < k+1 \Rightarrow$ " " " $T_2 = k_2 - 1$

Total no. of vertices in T_1 & $T_2 = k+1$

" " " edges in " " $= (k+1) - 2 = k-1$

But T_1 & T_2 together is $T-e$

$\therefore$ $T-e$ contains $k-1$ edges

$\therefore$ T contains exactly k edges.

$\therefore$ The theorem is true for $n = k+1$. —— (3)

Hence, by induction the theorem is true for all $n \in \mathbb{Z}^+$. —— (1)

5. (i) Let N be the no. of leaves in T.

Then total no. of vertices $= N+4+1+2+1 = N+8$

No. of edges $= (N+8)-1 = N+7$ —— (1)

By Handshaking property (Statement) —— (1)

$$(N\times 1)+(4\times 2)+(1\times 3)+(2\times 4)+(1\times 5) = 2(N+7)$$

$\Rightarrow N = 10$ —— (1)

(ii) (a) 1, 1, 2, 3

Since the sum of the degrees of all the vertices is 7.

$\therefore$ Graph itself doesn't exist. —— (2)

(b) 2, 2, 4, 6 – For this degree sequence the graph exists. But simple graph doesn't exist. Since the degree of 4th vertex is greater than the total no. of vertices. —— (2)

6.

This is only for CS branch

6^{th} Qn ans for IS is at the end.

$$\begin{array}{l} p\wedge q \\ p\to(r\wedge q) \\ r\to(s\vee t) \\ \neg s \\ \hline \end{array}$$

(conjunctive Simplification)

$$\Rightarrow \begin{array}{l} p \\ p\to r\wedge q \\ r\to(s\vee t) \\ \neg s \\ \hline \end{array}$$

—— (2)

(modus Pones in 1st & 2nd premises

$$\Rightarrow \begin{array}{l} r\wedge q \\ r\to(s\vee t) \\ \neg s \\ \hline \end{array}$$

$\Rightarrow$ r (conj. simplification)

$r \rightarrow (s \vee t)$ —— (2)

$\neg s$

$\Rightarrow$ $s \vee t$ (Modus Pones in 1st & 2nd premises)

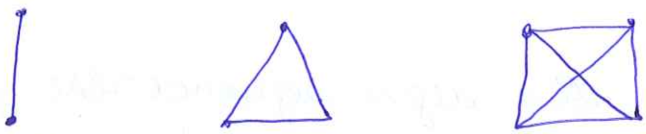
$\neg s$

$\Rightarrow$ $\neg s \rightarrow t$ ($\because p \rightarrow q \equiv \neg p \vee q$) —— (2)

$\neg s$

$\therefore t$ (Modus Pones) —— (1)

7. (i) <u>Complete graph</u> — A simple graph of order ≥ 2 in which there is an edge b/w every pair of vertices is called a complete graph or a full graph. A complete graph with order n is denoted by K_n. —— (1)



K_2 K_3 K_4 —— (1)

(ii) <u>Regular graph</u> — A graph in which all the vertices are of the same degree k, called k-regular graph —— (1)

2-regular graph 3-regular graph —— (1)

(iii) <u>Bipartite graph</u> — If a simple graph G is s.t. its vertex set V is the union of two of its mutually disjoint vertex subsets V_1 & V_2 which are s.t. each edge in G joins a vertex in V_1 & a vertex in V_2. —— (2)

V_1 V_2 V_1 V_2 —— (1)

8 (i) $p \to (q \to r)$

$\equiv \neg p \vee (\neg q \vee r)$ $\quad (\because p \to q \equiv \neg p \vee q)$ —— (1)

$\equiv \neg(p \wedge q) \vee r$ $\quad$ (Associative & De Morgan's) —— (1)

$\equiv (p \wedge q) \to r$ $\quad (\because p \to q \equiv \neg p \vee q)$ —— (1)

(ii) $[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$

$\equiv [(\neg p \wedge \neg q) \wedge r] \vee [(q \vee p) \wedge r]$ (Associative & distributive) —— (1)

$\equiv [\neg(p \vee q) \vee (p \vee q)] \wedge r$ (commutative & distributive —— (1) & De Morgan's law)

$\equiv T \wedge r$ (Inverse law) —— (1)

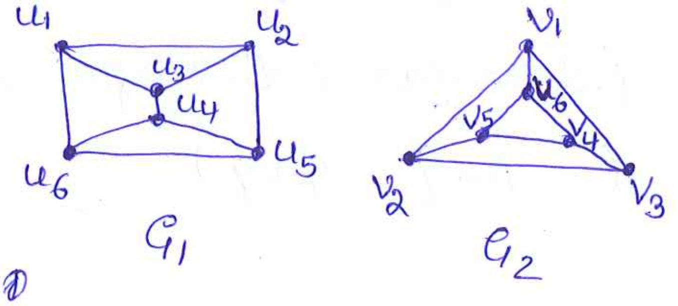
$\equiv r$ (Identity) —— (1)

9. Tautology – A compound proposition which is true for all possible truth values of its components is called a tautology. —— $(1\frac{1}{2})$

If it's false for all possible truth values of its components then it is called contradiction. —— $(1\frac{1}{2})$

p	q	r	$\neg q$	$p \wedge \neg q$ ①	① $\to r$ ③	$(q \vee r)$ ②	$p \to$ ② ④	③ $\to$ ④	
T	T	T	F	T	T	T	T	T	
T	T	F	F	T	F	T	T	T	$\therefore$
T	F	T	T	T	T	T	T	T	It's a
T	F	F	T	T	F	F	F	T	tautology.
F	T	T	F	F	T	T	T	T	—— (1)
F	T	F	F	F	T	T	T	T	
F	F	T	T	F	T	T	T	T	
F	F	F	T	F	T	T	T	T	—— (3)

3.



Both G_1 and G_2 have 6 vertices. —— (1)

—— " —— have 9 edges —— (1)

Both G_1 and G_2 are cubic graphs. —— (1)

Vertices mapping:

$u_i \longleftrightarrow v_i$ for $i = 1, 2, 3, 4, 5, 6$

—— (2)

Edges mapping:

$\{u_1, u_2\} \longleftrightarrow \{v_1, v_2\}$ $\{u_3, u_2\} \longleftrightarrow \{v_3, v_2\}$

$\{u_2, u_5\} \longleftrightarrow \{v_2, v_5\}$ $\{u_3, u_4\} \longleftrightarrow \{v_3, v_4\}$

$\{u_5, u_6\} \longleftrightarrow \{v_5, v_6\}$ $\{u_4, u_6\} \longleftrightarrow \{v_4, v_6\}$

$\{u_6, u_1\} \longleftrightarrow \{v_6, v_1\}$ $\{u_4, u_5\} \longleftrightarrow \{v_4, v_5\}$

$\{u_1, u_3\} \longleftrightarrow \{v_1, v_3\}$ —— (1)

In view of these, there exists a 1-1 correspondence between the vertices & edges of the 2 graphs.

$\therefore$ They are isomorphic. —— (1)

6. This is only for IS branch

(i) Let $|V| = n$

G is a cubic graph with 9 edges

From Hand-shaking property, (Statement) —— (1)

$$\sum_{i=1}^{n} \deg(v_i) = 2|E|$$

$$3n = 2(9)$$

$$\underline{n = 6}$$ —— (1)

(ii) G is a regular graph with $|E| = 15$

Let G be a k-regular graph. —— (1)

From Hand-shaking prop.,

$$\sum_{i=1}^{n} \deg(v_i) = 2|E|$$

$$nk = 2(15) = 30$$

$$k = \frac{30}{n}$$ —— (1)

∴ Possible value of n are $\underline{n = 1, 2, 3, 5, 6, 10, 15, 30}$ —— (1)

(iii) G has 10 ~~edges~~ edges with 2 vertices of deg 4 & all other vertices of deg 3.

Wkt $\sum_{i=1}^{n} \deg(v_i) = 2|E|$ from H-S prop.

$2(4) + (n-2)3 = 2(10)$ —(1) $\Rightarrow 3n + 2 = 20$

$\Rightarrow \underline{n = 6}$ —— (1)