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Internal Assessment Test 1 – Sept. 2018

Sub:	Discrete Mathematical Structures	Sub Code:	17CS36	Branch:	IS / CS - A, B, C
Date:	10/09/2018	Duration:	90 minutes	Max Marks:	50
		Sem / Sec:	III A and B		
Question 1 is compulsory and answer any six from Q.2 to Q.9					
		MARKS	CO	RBT	
1	Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code for the message.	[08]	CO5	L3	
2	Write a short note on Konigsberg Bridge Problem.	[07]	CO5	L2	
3	Check the following graphs for Isomorphism.	[07]	CO5	L3	
4	Prove that a tree with n vertices has n-1 edges.	[07]	CO5	L3	
5	(i) If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5, find the number of leaves in T. (ii) Check whether there exists simple graphs corresponding to the following degree sequences (a) 1, 1, 2, 3 (b) 2, 2, 4, 6	[07]	CO5	L3	

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- 6 Determine the order $|V|$ of the graph $G = (V, E)$ in the following cases [07]
- G is a cubic graph with 9 edges.
 - G is a regular graph with 15 edges.
 - G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.
- 7 Define the following with an example for each. [07]
- complete graph
 - regular graph
 - bipartite graph
- 8 Prove the following using laws of logic: [07]
- $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$
 - $[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$
- 9 Define Tautology and Contradiction. For any propositions p, q, r , verify whether the compound proposition $[(p \wedge \neg q) \rightarrow r] \rightarrow [p \rightarrow (q \vee r)]$ is a tautology. [07]

	CO5	L3
	CO5	L1
	CO1	L3
	CO1	L3

For CS branch :

Q.N. - 6 Check the validity of the following argument [07] CO1 L3

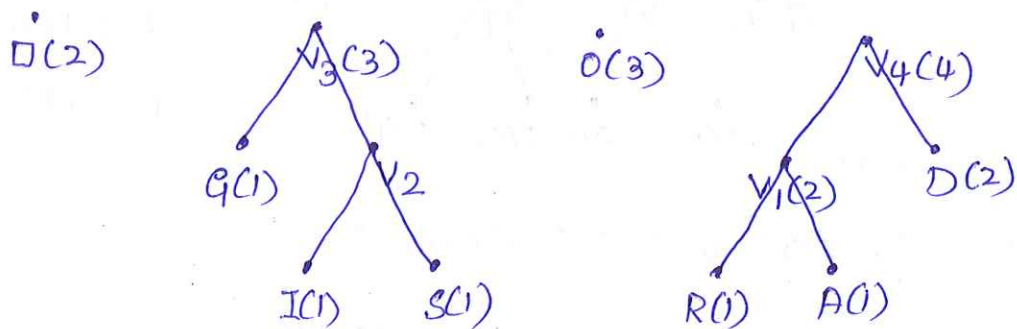
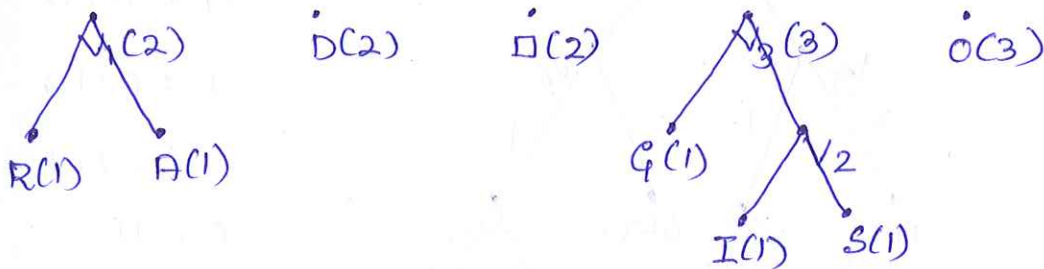
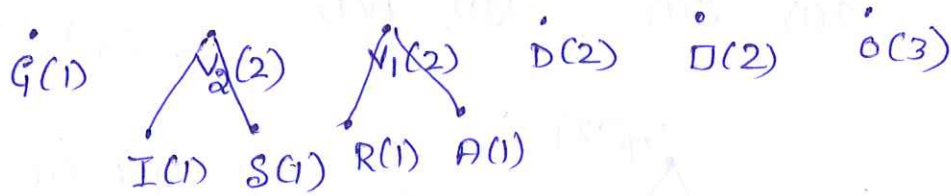
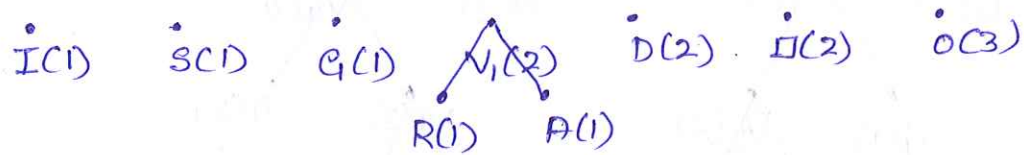
$$\begin{array}{l}
 p \wedge q \\
 p \rightarrow (r \wedge q) \\
 r \rightarrow (s \vee t) \\
 \hline
 \neg s \\
 \hline
 \therefore t
 \end{array}$$

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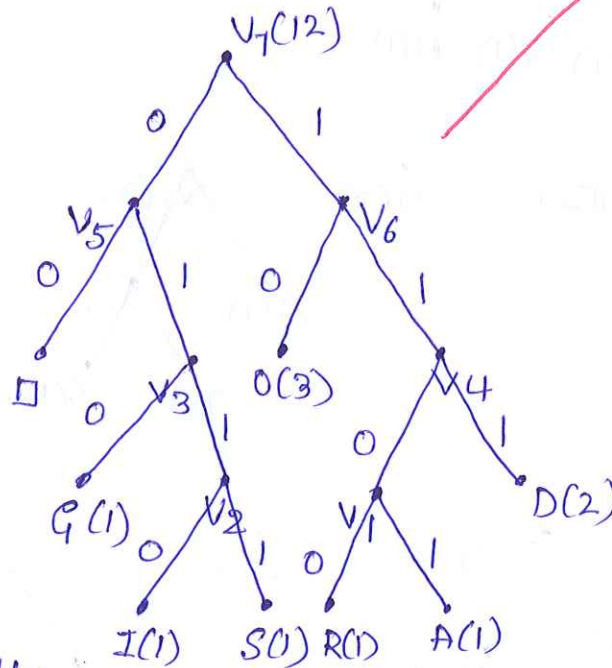
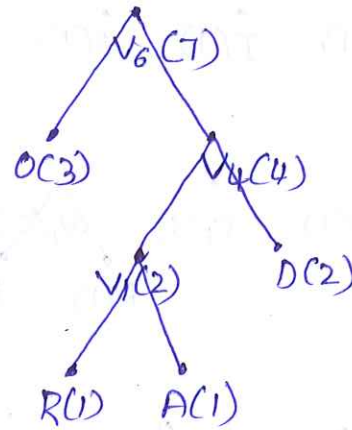
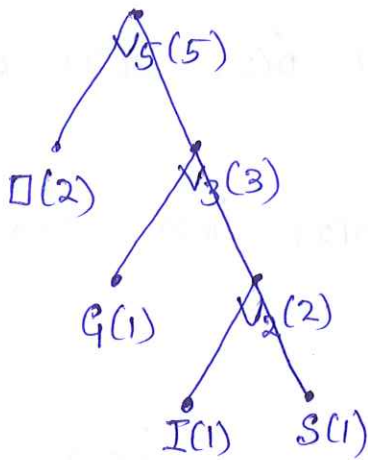
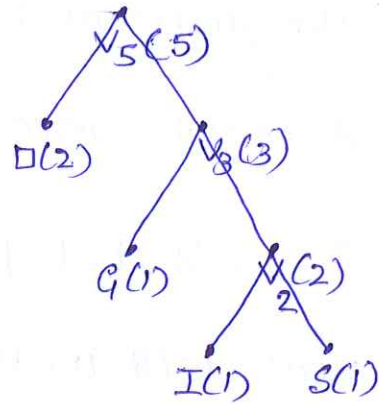
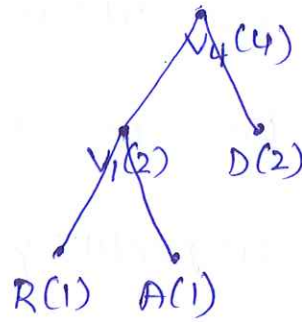
	CO5	L3
	CO5	L1
	CO1	L3
	CO1	L3

1. In the given message, the letters R, O, A, D, I, S, G. and blank space (\square) are present with the frequencies 1, 3, 1, 2, 1, 1, 1, 2 respectively. Let us write these as nodes with in non-descending order of their weights.

$R(1)$ $A(1)$ $I(1)$ $S(1)$ $G(1)$ $D(2)$ $\square(2)$ $O(3)$ — (1)



o(3)



- (5)
- ∅ : 00
 - G : 010
 - I : 0110
 - S : 0111
 - O : 10
 - D : 111
 - R : 1100
 - A : 1101

code for the

Message ~~AD~~ ROAD IS GOOD IS :

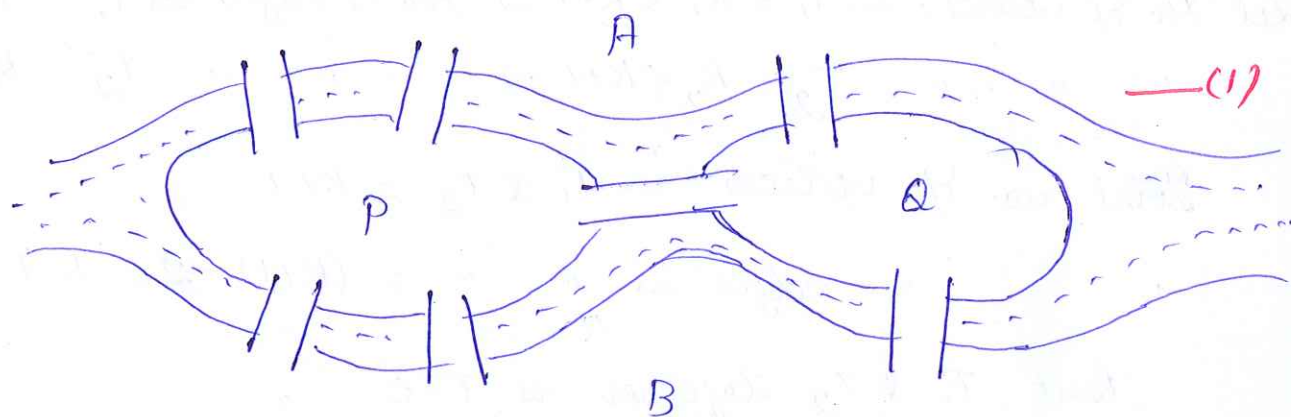
110010110111100011100111000101010111

— (1)

— (1)

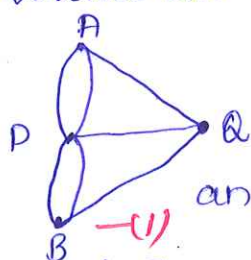
2. In 18th century, the city of Königsberg in Prussia was set on both sides of the Pregel river and included two large islands which were connected to each other and the main land by seven bridges. The problem was, by starting at any of the four land areas, can we return to that area after crossing each of the seven bridges exactly once?

Four land areas - two of these parts are the banks of the river and two are islands]



This is the starting point of development of graph theory. In 1736, Euler analyzed this problem with the help of graph and gave the solⁿ. This problem is known as Königsberg Bridge problem.

Let the land areas be denoted by A, B, P, Q. A, B are the banks of the river & P, Q are islands. Treat four land areas as four vertices and 7 bridges as 7 edges.



We note that, here $\deg(A) = 3$, $\deg(B) = 3$, $\deg(P) = 5$ & $\deg(Q) = 3$ which are not even \therefore graph doesn't have an Euler circuit.

\therefore It is not possible to walk over each of the seven bridges exactly once and return to the starting point.

4. When $n=1$ 

$n=2$ 

$n=3$



\therefore Statement is true for $n=1, 2, 3$. — (2)

Assume that the theorem is true for all trees with n vertices where $n \leq k$, $k \in \mathbb{Z}^+$ — (1)

Consider a tree with $k+1$ vertices and let e be an edge in this with end vertices u & v . Deletion of e will disconnect the graph & $T-e$ consists of exactly two components, say T_1 & T_2 . Now T_1 & T_2 are trees

Let No of vertices in $T_1 = k_1 < k+1 \Rightarrow$ No of edges in $T_1 = k_1 - 1$

" " " $T_2 = k_2 < k+1 \Rightarrow$ " " " $T_2 = k_2 - 1$

Total no. of vertices in T_1 & $T_2 = k+1$

" " " edges in " " = $(k+1) - 2 = k-1$

But T_1 & T_2 together is $T-e$

$\therefore T-e$ contains $k-1$ edges

$\therefore T$ contains exactly k edges.

\therefore The theorem is true for $n=k+1$. — (3)

Hence, by induction the theorem is true for all $n \in \mathbb{Z}^+$. — (4)

5. (i) Let N be the no. of leaves in T .

Then total no. of vertices = $N + 4 + 1 + 2 + 1 = N + 8$

$$\text{No. of edges} = (N + 8) - 1 = N + 7 \quad \text{--- (1)}$$

By Handshaking property (Statement) --- (1)

$$(N \times 1) + (4 \times 2) + (1 \times 3) + (2 \times 4) + (1 \times 5) = 2(N + 7)$$

$$\Rightarrow N = 10 \quad \text{--- (1)}$$

(ii) (a) 1, 1, 2, 3

Since the sum of the degrees of all the vertices is 7.

\therefore Graph itself doesn't exist. --- (2)

(b) 2, 2, 4, 6 - For this degree sequence the graph exists. But simple graph doesn't exist. Since the

degree of 4th vertex is greater than the total no. of vertices. --- (2)

6.

$p \wedge q$

$p \rightarrow (r \wedge q)$

$r \rightarrow (s \vee t)$

$\neg s$

\Rightarrow

p

$p \rightarrow r \wedge q$

$r \rightarrow (s \vee t)$

$\neg s$

\Rightarrow

$r \wedge q$

$r \rightarrow (s \vee t)$

$\neg s$

This is only for CS branch

6th Qn ans for IS is at the end.

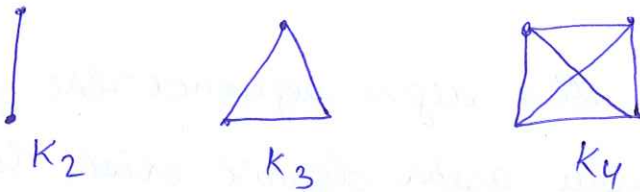
(conjunctive Simplification)

--- (2)

(modus Ponens in 1st & 2nd premises)

$$\begin{aligned} &\Rightarrow \frac{\begin{array}{l} \xi \\ \xi \rightarrow (svt) \\ \neg s \end{array}}{\frac{svt}{\neg s}} \quad (\text{cong. simplification}) \quad \text{--- (2)} \\ &\Rightarrow \frac{\frac{svt}{\neg s}}{\neg s \rightarrow t} \quad (\text{modus Ponens in 1st \& 2nd premises}) \\ &\Rightarrow \frac{\neg s \rightarrow t}{\neg s} \quad (\because p \rightarrow q \equiv \neg p \vee q) \quad \text{--- (2)} \\ &\therefore t \quad (\text{modus Ponens}) \quad \text{--- (1)} \end{aligned}$$

(7) (i) Complete graph - A simple graph of order ≥ 2 in which there is an edge b/w every pair of vertices is called a complete graph or a full graph. A complete graph with order n is denoted by K_n . --- (1)



--- (1)

(ii) Regular graph - A graph in which all the vertices are of the same degree k , called k -regular graph. --- (1)



2-regular graph

3-regular graph

--- (1)

(iii) Bipartite graph - If a simple graph G is s.t. its vertex set V is the union of two of its mutually disjoint vertex ^{sub}sets V_1 & V_2 which are s.t. each edge in G joins a vertex in V_1 & a vertex in V_2 . --- (2)



--- (1)

8(i) $p \rightarrow (q \rightarrow r)$
 $\equiv \neg p \vee (\neg q \vee r)$ ($\because p \rightarrow q \equiv \neg p \vee q$) — (1)
 $\equiv \neg(p \wedge q) \vee r$ (Associative & De Morgan's) — (1)
 $\equiv (p \wedge q) \rightarrow r$ ($\because p \rightarrow q \equiv \neg p \vee q$) — (1)

(ii) $[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$
 $\equiv [(\neg p \wedge \neg q) \wedge r] \vee [(q \vee p) \wedge r]$ (Associative & distributive) — (1)
 $\equiv [\neg(p \vee q) \vee (p \vee q)] \wedge r$ (commutative & distributive) — (1) & De Morgan's law
 $\equiv T \wedge r$ (Inverse law) — (1)
 $\equiv r$ (Identity) — (1)

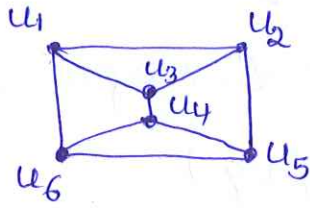
9. Tautology - A compound proposition which is true for all possible truth values of its components is called a tautology. — (1½)

If it's false for all possible truth values of its components then it is called contradiction. — (1½)

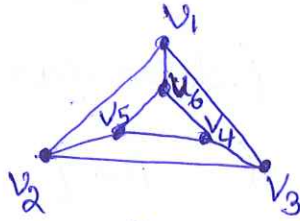
p	q	r	$\neg q$	$p \wedge \neg q$	$\textcircled{1} \rightarrow r$	$(q \vee r)$	$p \rightarrow \textcircled{2}$	$\textcircled{3} \rightarrow \textcircled{4}$	
T	T	T	F	T	T	T	T	T	\therefore It's a tautology. — (1)
T	T	F	F	T	F	T	T	T	
T	F	T	T	F	T	T	T	T	
T	F	F	T	F	F	F	F	T	
F	T	T	F	F	T	T	T	T	
F	T	F	F	F	T	T	T	T	
F	F	T	T	F	T	T	T	T	
F	F	F	T	F	T	T	T	T	

— (3)

3.



G_1



G_2

Both G_1 and G_2 have 6 vertices. — (1)

— " — have 9 edges — (1)

Both G_1 and G_2 are cubic graphs. — (1)

Vertices mapping:

$$u_i \leftrightarrow v_i \quad \text{for } i=1, 2, 3, 4, 5, 6$$

— (2)

Edges mapping:

$$\{u_1, u_2\} \leftrightarrow \{v_1, v_2\} \quad \{u_3, u_2\} \leftrightarrow \{v_3, v_2\}$$

$$\{u_2, u_5\} \leftrightarrow \{v_2, v_5\} \quad \{u_3, u_4\} \leftrightarrow \{v_3, v_4\}$$

$$\{u_5, u_6\} \leftrightarrow \{v_5, v_6\} \quad \{u_4, u_6\} \leftrightarrow \{v_4, v_6\}$$

$$\{u_6, u_1\} \leftrightarrow \{v_6, v_1\} \quad \{u_4, u_5\} \leftrightarrow \{v_4, v_5\}$$

$$\{u_1, u_3\} \leftrightarrow \{v_1, v_3\} \quad \text{— (1)}$$

In view of these, there exists a 1-1 correspondence between the vertices & edges of the 2 graphs.

\therefore They are isomorphic. — (1)

6.

This is only for IS branch

(i) Let $|V| = n$

G is a cubic graph with 9 edges

From Hand-shaking property, (Statement) — (1)

$$\sum_{i=1}^n \deg(v_i) = 2|E|$$

$$3n = 2(9)$$

$$\underline{n = 6} \quad \text{--- (1)}$$

(ii) G is a regular graph with $|E| = 15$

Let G be a k -regular graph. — (1)

From Hand-shaking prop.

$$\sum_{i=1}^n \deg(v_i) = 2|E|$$

$$nk = 2(15) = 30$$

$$k = \frac{30}{n} \quad \text{--- (1)}$$

\therefore Possible value of n are $n = 1, 2, 3, 5, 6, 10, 15, 30$ — (1)

(iii) G has 10 ~~edges~~ edges with 2 vertices of deg 4 & all other vertices of deg 3.

Wkt $\sum_{i=1}^n \deg(v_i) = 2|E|$ from H-S prop.

$$2(4) + (n-2)3 = 2(10) \Rightarrow 3n + 2 = 20 \quad \text{--- (1)}$$

$$\Rightarrow \underline{n = 6} \quad \text{--- (1)}$$

(1)

... ..

... ..

(2)

... ..

(3)

... ..

... ..

... ..

(4)

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