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| Sub | ENGINEERING MATHEMATICS III I Internal Test | | | | | | | Code | 17MAT31 |
|---|---|---|---|---|---|---|---|---|---|
| Date | 07 / 09 / 2018 | Duration | 90 mins | Max Marks | 50 | Sem | III | Branch | CS B,C |

| | Question 1 is compulsory. Answer any SIX questions from the rest. | Marks | OBE | |
|---|---|---|---|---|
| 1. | The following table gives the variations of a periodic current A over a period T. Show that there is a constant part of 0.75 amp in the current A, and obtain the amplitude of the first harmonic. | 08 | C301.1 | L3 |
| 2. | For the function $f(x) = x(L-x)$, obtain the half range cosine series and sine series over the interval (0,L) | 07 | C301.1 | L3 |
| 3. | Find the Fourier expansion for the function $f(x) = \begin{cases} 2-x, & 0<x<4 \\ x-6, & 4<x<8 \end{cases}$ | 07 | C301.1 | L3 |

| t(sec) | 0 | T/6 | T/3 | T/2 | 2T/3 | 5T/6 | T |
|---|---|---|---|---|---|---|---|
| A(amp) | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |


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| | | | | |
|---|---|---|---|---|
| 4. | Obtain the Fourier series for the function $i(x)$ given by $i(x)=\begin{cases}1+\dfrac{2x}{\pi}, -\pi \le x \le 0,\\ 1-\dfrac{2x}{\pi}, 0 \le x \le \pi\end{cases}$ Deduce that $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.........=\dfrac{\pi^2}{8}$. Sketch the graph of the function(triangular wave form) in $-\pi < x < \pi$. | 07 | C301.1 | L3 |
| 5. | Find the Fourier transform of $f(x)=\begin{cases}1-\lvert x\rvert & \lvert x\rvert \le 1\\ 0 & \lvert x\rvert > 1\end{cases}$. Hence deduce that $\int_0^\infty \dfrac{\sin^2 x}{x^2}dx=\dfrac{\pi}{2}$ | 07 | C301.2 | L3 |
| 6. | Find the Fourier transform of $\exp(-a^2x^2)$ . Show that $f(x)=\exp\left(-x^2/2\right)$ is self reciprocal w.r.to Fourier transform. | 07 | C301.2 | L3 |
| 7. | Find the Fourier cosine transform of $f(x)=\dfrac{e^{-ax}}{x}, a>0, x\neq 0$ | 07 | C301.2 | L3 |
| 8. | If $0<n<1$, prove that $F_s(x^{n-1})=\sqrt{\dfrac{2}{\pi}}\dfrac{\Gamma n}{\alpha^n}\sin\left(\dfrac{n\pi}{2}\right)$. Hence deduce that $f(x)=\dfrac{1}{\sqrt{x}}$ is self reciprocal with respect to Fourier sine transform. | 07 | C301.2 | L3 |

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1. $A = A(t)$ is periodic with period $T$ and is defined over $(0, T)$

| $t$ | $\theta = \frac{2\pi}{T} t$ | $\cos\theta$ | $\sin\theta$ | $A$ | $A\cos\theta$ | $A\sin\theta$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1.98 | 1.98 | 0 |
| $T/6$ | $\pi/3$ | 0.5 | 0.867 | 1.30 | 0.65 | 1.1258 |
| $T/3$ | $2\pi/3$ | -0.5 | 0.867 | 1.05 | -0.525 | 0.9093 |
| $T/2$ | $\pi$ | -1 | 0 | 1.30 | -1.3 | 0 |
| $2T/3$ | $4\pi/3$ | -0.5 | -0.867 | -0.88 | 0.44 | 0.7621 |
| $5T/6$ | $5\pi/3$ | 0.5 | -0.867 | -0.25 | -0.125 | 0.2165 |
| | | | | 4.5 | 1.12 | 3.0137 |

3

$$a_0 = 2[[A]] = 2\left(\frac{4.5}{6}\right) = 1.5$$

$$a_1 = 2[[A\cos\theta]] = 2\left(\frac{1.12}{6}\right) = 0.3733$$

$$b_1 = 2[[A\sin\theta]] = 2\left(\frac{3.0137}{6}\right) = 1.0046$$ 1

FE $A = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta$

$$= \frac{1.5}{2} + 0.3733\cos\theta + 1.0046\sin\theta$$ 1

Amplitude of I harmonic is $\sqrt{a_1^2 + b_1^2}$

$$\sqrt{(0.3733)^2 + (1.0046)^2} = 1.0717$$ 1

2. $f(x) = x(L-x)$ over $(0, L)$

cosine series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{L}\int_0^L x(L-x)\,dx$$

$$= \frac{2}{L}\left[L\frac{x^2}{2} - \frac{x^3}{3}\right]_0^L = \frac{L^2}{3}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

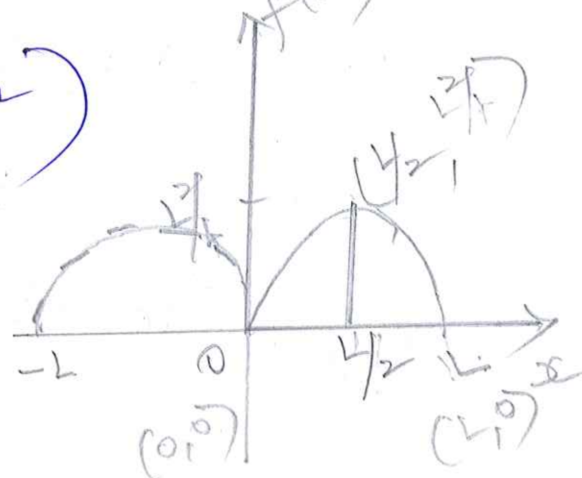
$$= \frac{2}{L}\int_0^L (Lx - x^2)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{2}{L}\Bigg[(Lx - x^2)\frac{L}{n\pi}\sin\left(\frac{n\pi x}{L}\right) - (L-2x)\left(-\frac{L}{n\pi}\right)^2\cos\left(\frac{n\pi x}{L}\right)$$

$$+ (-2)\frac{-L^3}{n^3\pi^3}\sin\left(\frac{n\pi x}{L}\right)\Bigg]_0^L$$

$$= \frac{2}{L}\left[L\frac{-L^2}{n^2\pi^2}\cos(n\pi) + L\frac{-L^2}{n^2\pi^2}\right]$$

$$= -\frac{2L^2}{n^2\pi^2}\left(1 + (-1)^n\right)$$



(5)

$$f(x) = \frac{L^2}{6} - \frac{2L^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1+(-1)^n}{n^2} \cos\left(\frac{n\pi x}{L}\right)$$

(4)

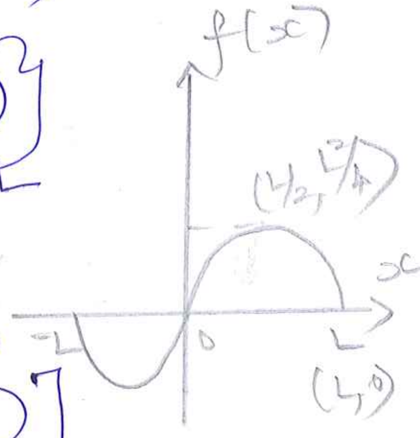
sine series

$$f(x) = \sum b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_0^L (Lx - x^2) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L} \Bigg[ (Lx - x^2)\left(-\frac{L}{n\pi} \cos\left(\frac{n\pi x}{L}\right)\right) - (L-2x)\left\{-\frac{L^2}{n^2\pi^2} \sin\left(\frac{n\pi x}{L}\right)\right\} + (-2)\left\{\frac{L^3}{n^3\pi^3} \cos\left(\frac{n\pi x}{L}\right)\right\}\Bigg]_0^L$$




$$= \frac{2}{L}\left[\frac{-2L^3}{n^3\pi^3}(\cos n\pi - 1)\right]$$

$$= \frac{-4L^2}{n^3\pi^3}\left((-1)^n - 1\right) = \frac{4L^2}{n^3\pi^3} 1-(-1)^n$$

sine series

$$f(x) = \sum_{n=1}^{\infty} \frac{4L^2}{n^3\pi^3} 1-(-1)^n \sin\left(\frac{n\pi x}{L}\right)$$

$$= \frac{4L^2}{\pi^3} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^3} \sin\left(\frac{n\pi x}{L}\right)$$

(3)

3. $f(x) = \begin{cases} 2-x, & 0 < x < 4 \\ x-6, & 4 < x < 8 \end{cases}$

$f(x)$ is defined in $(0, 2L) - (0, 8)$

$2L = 8 \quad L = 4$

<u>FS</u> $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\dfrac{n\pi x}{L}\right) + b_n \sin\left(\dfrac{n\pi x}{L}\right) \right)$ 1

$a_0 = \dfrac{1}{L} \int_0^{2L} f(x)\,dx = \dfrac{1}{4} \int_0^8 f(x)\,dx$

$= \dfrac{1}{4} \left[ \int_0^4 (2-x)\,dx + \int_4^8 (x-6)\,dx \right]$

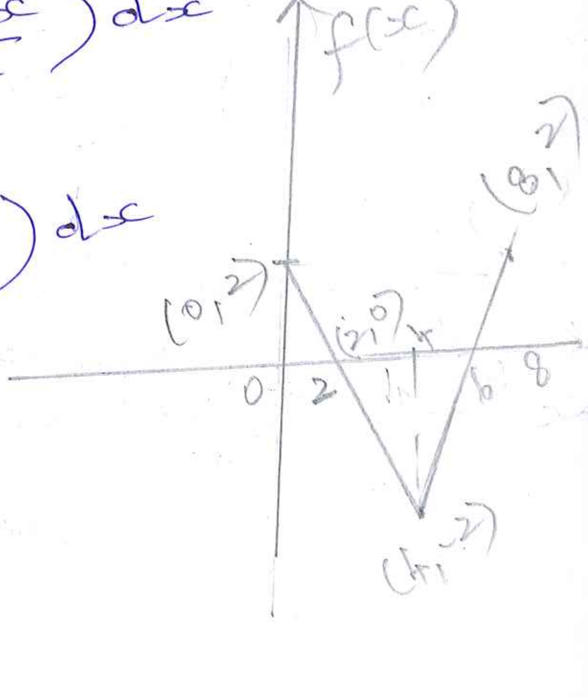
$= \dfrac{1}{4} \left[ \left(2x - \dfrac{x^2}{2}\right)_0^4 + \left(\dfrac{x^2}{2} - 6x\right)_4^8 \right]$

$= \dfrac{1}{4} \left[ (8-8) + (32-48) - (8-24) \right] = 0$ 1

$\boxed{a_0 = 0}$

$a_n = \dfrac{1}{L} \int_0^{2L} f(x) \cos\left(\dfrac{n\pi x}{L}\right) dx$

$= \dfrac{1}{4} \int_0^8 f(x) \cos\left(\dfrac{n\pi x}{4}\right) dx$



$$a_n = \frac{1}{4}\left[\int_0^4 (2-x)\cos\left(\frac{n\pi x}{4}\right)dx + \int_4^8 (x-6)\cos\left(\frac{n\pi x}{4}\right)dx\right]$$

$$= \frac{1}{4}\left[(2-x)\frac{\sin\left(\frac{n\pi x}{4}\right)}{n\pi/4} - (-1)\frac{-\cos\left(\frac{n\pi x}{4}\right)}{(n\pi/4)^2}\right]_0^4$$

$$+ \frac{1}{4}\left[(x-6)\frac{\sin\left(\frac{n\pi x}{4}\right)}{n\pi/4} - 1.\frac{-\cos\left(\frac{n\pi x}{4}\right)}{(n\pi/4)^2}\right]_4^8$$

$$= \frac{1}{4}\left[-\left(\frac{4}{n\pi}\right)^2(\cos n\pi - 1) + \frac{4}{n^2\pi^2}(\cos 2n\pi - \cos n\pi)\right]$$

$$a_n = \frac{8}{n^2\pi^2}(1-\cos n\pi) = \frac{8}{n^2\pi^2}(1-(-1)^n)$$

2

$$b_n = \frac{1}{4}\int_0^8 f(x)\sin\left(\frac{n\pi x}{4}\right)dx$$

$$= \frac{1}{4}\left[\int_0^4 (2-x)\sin\left(\frac{n\pi x}{4}\right)dx + \int_4^8 (x-6).\sin\left(\frac{n\pi x}{4}\right)dx\right]$$

$$b_n = \frac{1}{4}\left[(2-x)\frac{-4}{n\pi}\cos\left(\frac{n\pi}{4}\right)x - \left(\frac{4}{n\pi}\right)^2 \sin\left(\frac{n\pi x}{4}\right)\right]_0^4$$

$$+ \frac{1}{4}\left[(x-6)\frac{-4}{n\pi}\cos\left(\frac{n\pi x}{4}\right) + \left(\frac{4}{n\pi}\right)^2 \sin\left(\frac{n\pi x}{4}\right)\right]_4^8$$

$$b_n = \frac{1}{4}\left[(-2)\frac{(-4)}{n\pi}\cos(n\pi) - 2\left(\frac{-4}{n\pi}\right) + 2\left(\frac{-4}{n\pi}\right) + 2\left(\frac{-4}{n\pi}\cos(n\pi)\right)\right] = 0$$ 2

FE $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{4}\right) + b_n \sin\left(\frac{n\pi x}{4}\right)\right)$$ 1

$$= \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}\cos\left(\frac{n\pi x}{4}\right)$$

4.

(9)

4. $$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

Deduce that $\frac{\pi^2}{8} = \sum \frac{1}{(2n-1)^2}$

$$f(-x) = 1 - \frac{2x}{\pi}$$
$$1 + \frac{2x}{\pi} = f(x)$$

$f(x)$ is even in $(-\pi, \pi)$

FE of even func

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

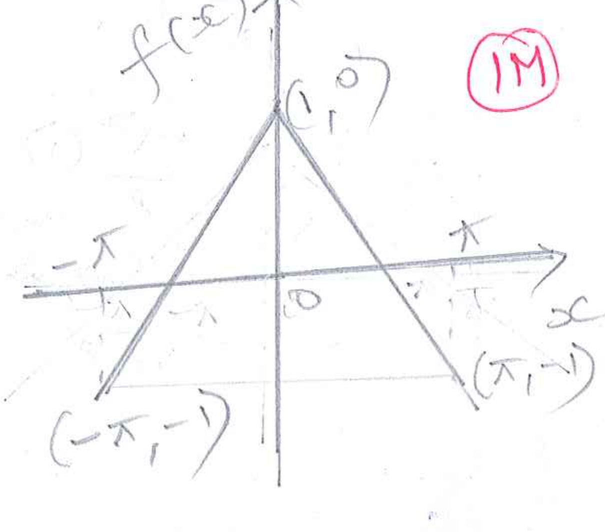
$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \left(1 - \frac{2x}{\pi}\right) dx$$

$$= \frac{2}{\pi} \left[ x - \frac{2}{\pi}\left(\frac{x^2}{2}\right) \right]_0^{\pi} = \frac{2}{\pi}\left[\pi - \pi\right]$$
$$= 0$$

$$\boxed{a_0 = 0}$$

1M



$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)\cos(nx)\,dx$$

$$= \frac{2}{\pi}\left[\left(1-\frac{2x}{\pi}\right)\frac{\sin(nx)}{n} - \left(\frac{-2}{\pi}\right)\frac{-\cos(nx)}{n^2}\right]_{x=0}^{\pi}$$

$$= \frac{2}{\pi}\left[0 - \frac{2}{\pi}\frac{1}{n^2}\left(\cos n\pi - 1\right)\right]$$

$$a_n = \frac{4}{\pi^2}\frac{1}{n^2}\left(1-(-1)^n\right) \qquad n \in Z^+$$

4M

$n$ is even $\quad a_n = 0$

$n$ is odd $\quad a_n = \frac{4}{\pi^2}\frac{2}{n^2}$

FE $\quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx)$

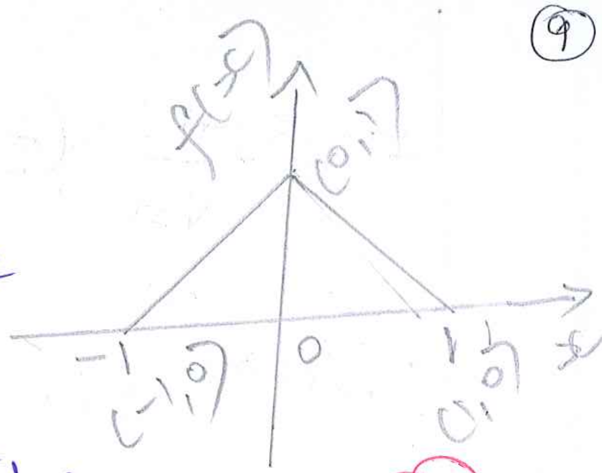
$$f(x) = \sum_{n=1,3,5\ldots}^{\infty}\frac{8}{\pi^2 n^2}\cos(nx)$$

$$\frac{\pi^2}{8}f(x) = \sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n^2}\cos(nx)$$

$x=0$ $\quad \frac{\pi^2}{8}(1) = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$$

2M




5. $$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{i\alpha x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} \{1-|x|\}\, e^{i\alpha x}\, dx$$ 1M

$$= \frac{1}{\sqrt{2\pi}} \left[ \int_{-1}^{0} \underset{u}{(1+x)}\, \underset{v}{e^{i\alpha x}}\, dx + \int_{0}^{1} (1-x)\, e^{i\alpha x}\, dx \right]$$

$$\frac{1}{\sqrt{2\pi}} \left[ (1+x)\frac{e^{i\alpha x}}{i\alpha} - 1.\frac{e^{i\alpha x}}{(i\alpha)^2} \right]_{-1}^{0}$$

$$+ \frac{1}{\sqrt{2\pi}} \left[ (1-x)\frac{e^{i\alpha x}}{i\alpha} - (-1)\frac{e^{i\alpha x}}{(i\alpha)^2} \right]_{0}^{1}$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{1}{i\alpha}\left(1 - e^{-i\alpha}(0)\right) + \frac{1}{\alpha^2}\left(1 - e^{-i\alpha}\right) \right]$$

$$+ \frac{1}{\sqrt{2\pi}} \left[ 0 - \frac{1}{i\alpha} - \frac{1}{\alpha^2}\left(e^{i\alpha} - 1\right) \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \left( \frac{1}{i\alpha} - \frac{1}{i\alpha} \right) + \frac{1}{\alpha^2}\left(1 - e^{-i\alpha} - e^{i\alpha} + 1\right) \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left( \frac{2 - \overline{e^{i\alpha} + e^{-i\alpha}}}{\alpha^2} \right)$$

$$F(\alpha)=\frac{1}{\sqrt{2\pi}}\left(\frac{2-2\cos\alpha}{\alpha^2}\right)=\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos\alpha}{\alpha^2}\right)$$

Inverse $$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}F(\alpha)\,e^{-i\alpha x}\,d\alpha$$

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos\alpha}{\alpha^2}\right)\left(\cos(\alpha x)-i\sin(\alpha x)\right)d\alpha$$

$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\left\{\frac{\cos\alpha x-\cos\alpha\cos\alpha x-i\sin\alpha x+i\sin\alpha x\cos\alpha}{\alpha^2}\right\}d\alpha$$

$$f(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\cos\alpha x-\cos\alpha\cos\alpha x}{\alpha^2}\,d\alpha$$

$$f(x)=\frac{2}{\pi}\int_{0}^{\infty}\left(\frac{1-\cos\alpha}{\alpha^2}\right)\cos(\alpha x)\,d\alpha$$ 4M

$x=0$

$$f(0)=\frac{2}{\pi}\int_{0}^{\infty}\frac{1-\cos\alpha}{\alpha^2}\,d\alpha \qquad \because f(0)=1$$

$$\frac{\pi}{2}=\int_{0}^{\infty}\frac{2\sin^2(\alpha/2)}{\alpha^2}\,d\alpha$$

$\alpha = 2t \quad d\alpha = 2\,dt$

as $\alpha \to 0$ to $\infty$ $\quad t \to 0$ to $\infty$

$$\frac{\pi}{2} = 2\int_0^\infty \frac{\sin^2 t}{(2t)^2}\, 2\,dt$$

2M

$$\int_0^\infty \frac{\sin^2 t}{t^2}\, dt = \pi/2$$

6. $$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} e^{i\alpha x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(a^2x^2 - i\alpha x)}\, dx$$

2M

$$a^2x^2 - i\alpha x = \left[(ax)^2 - i\alpha x + \left(\frac{i\alpha}{2a}\right)^2\right] - \left(\frac{i\alpha}{2a}\right)^2$$

$$= \left(ax - \frac{i\alpha}{2a}\right)^2 - \left(\frac{i\alpha}{2a}\right)^2$$

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left[\left(ax - \frac{i\alpha}{2a}\right)^2 + \frac{\alpha^2}{4a^2}\right]}\, d\alpha$$

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} e^{-\alpha^2/4a^2} \int_{-\infty}^{\infty} e^{-\left(ax - \frac{i\alpha}{2a}\right)^2} dx$$

$$t = ax - \frac{i\alpha}{2a} \quad dt = a\,dx$$

as $x \to -\infty$ to $\infty$ $\quad t \to -\infty$ to $\infty$

$$= \frac{1}{\sqrt{2\pi}} e^{-\alpha^2/4a^2} \int_{t=-\infty}^{\infty} \underbrace{e^{-t^2}}_{\text{even fn}} \frac{1}{a}\, dt$$

$$= \frac{1}{a} \frac{1}{\sqrt{2\pi}} e^{-\left(\alpha^2/4a^2\right)} 2\int_{t=0}^{\infty} e^{-t^2} dt$$

$$= \sqrt{\frac{2}{\pi}} \frac{1}{a} e^{-\left(\alpha^2/4a^2\right)} \frac{\sqrt{\pi}}{2}$$

$$= \frac{1}{\sqrt{2}\,a} e^{-\left(\alpha^2/4a^2\right)}$$ 4M

$$F\left(e^{-a^2x^2}\right) = \frac{1}{\sqrt{2}\,a} e^{-\left(\alpha^2/4a^2\right)}$$

$$a^2 = 1/2 \quad F\left(e^{-x^2/2}\right) = \frac{1}{\cancel{\sqrt{2}}\ 1/\cancel{\sqrt{2}}} e^{-\left(\alpha^2/2\right)}$$ 1M

$f(x) = e^{-\left(x^2/2\right)}$ is self reciprocal w.r.to F.T

8. We k.t $\Gamma n = \int_0^\infty e^{-t} t^{n-1} dt$

1M

$t = ax \quad dt = a\,dx$

$$= \int_{x=0}^{\infty} e^{-ax} (ax)^{n-1} a\,dx$$

$$= a^n \int_{x=0}^{\infty} e^{-ax} x^{n-1} dx$$

$a = i\alpha$

$$\Gamma n = (i\alpha)^n \int_{x=0}^{\infty} e^{-i\alpha x} x^{n-1} dx$$

$$\frac{\Gamma n}{\alpha^n} = i^n \int_{x=0}^{\infty} x^{n-1} e^{-i\alpha x} dx$$

$$i^n = \left(e^{i\pi/2}\right)^n = e^{ni\pi/2} \qquad i^{-n} = e^{-(in\pi/2)}$$

$$\frac{\Gamma n}{\alpha^n} e^{-in\pi/2} = \int_0^\infty x^{n-1} e^{-i\alpha x} dx$$

$$\sqrt{\frac{2}{\pi}} \frac{\Gamma n}{\alpha^n} \left(\cos\left(\frac{n\pi}{2}\right) - i\sin\left(\frac{n\pi}{2}\right)\right) = \sqrt{\frac{2}{\pi}} \int_0^\infty x^{n-1} \left(\cos(\alpha x) - i\sin(\alpha x)\right) dx$$

$$\sqrt{\frac{2}{\pi}}\ \frac{\Gamma n}{\alpha^n}\left[\cos\left(\frac{n\pi}{2}\right) - i\sin\left(\frac{n\pi}{2}\right)\right]$$

$$= F_c(\alpha) - i F_s(\alpha)$$

Equating R.P & I.P

$$F_c(\alpha) = \sqrt{\frac{2}{\pi}}\ \frac{\Gamma n}{\alpha^n}\cos\left(\frac{n\pi}{2}\right)$$

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}}\ \frac{\Gamma n}{\alpha^n}\sin\left(\frac{n\pi}{2}\right)$$ 5M

or

$$F_s(x^{n-1})$$

$n = 1/2$ $$F_s(x^{-1/2}) = \sqrt{\frac{2}{\pi}}\ \frac{\Gamma 1/2}{\sqrt{\alpha}}\sin\left(\frac{\pi}{4}\right)$$

$$= \sqrt{\frac{2}{\pi}}\ \frac{\sqrt{\pi}}{\sqrt{\alpha}}\ \frac{1}{\sqrt{2}}$$ 1M

$$F_s(x^{-1/2}) = \alpha^{-1/2}$$

7. $$f(x) = \frac{e^{-ax}}{x} \quad a > 0 \quad x \neq 0$$

$$F_c(\alpha) = \sqrt{\frac{2}{\pi}}\int_{x=0}^{\infty}\frac{e^{-ax}}{x}\cos(\alpha x)\,dx$$ 1M

diff w.r.to $\alpha$

$$\frac{d}{d\alpha} F_c(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{e^{-ax}}{x} (-x \sin \alpha x)\, dx$$

$$= -\sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \sin(\alpha x)\, dx$$

$$= -\sqrt{\frac{2}{\pi}} \left[ \frac{e^{-ax}}{(-a)^2+\alpha^2} (-a \sin \alpha x - \alpha \cos \alpha x) \right]_0^\infty$$

$$= -\sqrt{\frac{2}{\pi}} \frac{1}{a^2+\alpha^2} (0-(-\alpha))$$

5M

$$F_c'(\alpha) = -\sqrt{\frac{2}{\pi}} \frac{\alpha}{a^2+\alpha^2}$$

Intg $\quad F_c(\alpha) = -\sqrt{\frac{2}{\pi}} \frac{1}{2} \log(a^2+\alpha^2)$

$$= \frac{1}{\sqrt{2\pi}} \log\left(\frac{1}{a^2+\alpha^2}\right) \;//$$

1M