USN

CMRIT

Internal Assessment Test – 1

Sub:	Engineering Mathematics - III				Sub Code:	17MAT31	Branch:	CS,EEE, CV	
Date:	07.09.2018	Duration:	90 min's	Max Marks: 50	Sem / Sec:	III/ CS-A, EEE-A,CV-A		OBE	

Question 1 is compulsory. Answer SIX questions from Question 2 to 8.

		MARKS	CO	RBT
1.	The following table gives the variation of periodic current over a period.	[08]	CO1	L1

t sec.	0	T/6	T/3	T/2	2T/3	5T/6	T
Aamp.	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.

2. Find the Fourier series expansion of the function $(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$ [07] CO1 L3

Sketch the graph of triangle wave function from -4 to 4.

3.	Use the Regula – falsi method to obtain a root of the equation $2x - \log_{10} x = 7$ which lies between 3.5 and 4. Carryout 3 iterations.	[07]	CO6	L3
4.	Find the Fourier series expansion of $f(x) = \begin{cases} -\pi, & -\pi \le x < 0 \\ x, & 0 < x \le \pi \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.	[07]	CO1	L3
5.	Use Newton's Raphson method to find a root of the equation $x^3 - 3x + 1 = 0$ correct to 3 decimal places that lies between 0 and 0.5. Perform 4 itterations.	[07]	CO6	L3
6.	Obtain the half range Fourier sine series for e^x in $0 < x < 1$.	[07]	CO1	L3
7.	Use Newton-Raphson method to find a root of the equation $\tan x - x = 0$ near $x = 4.5$. Carry out two iterations.	[07]	CO6	L3
8.	Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.	[07]	CO1	L3

Q.1 here $N = 6$, $0 \le t \le T$

Compare $(0, T)$ with $(0, 2l)$

$$\therefore \boxed{l = T/2}$$

The Fourier series expansion of $f(x)$ upto 1^{st} harmonic is

$$f(x) = \frac{a_0}{2} + a_1 \cos\left(\frac{2n\pi t}{T}\right) + b_1 \sin\left(\frac{2n\pi t}{T}\right) \quad \text{— (1)}$$

1

take $\theta = \dfrac{2n\pi t}{T}$

t	$\theta = \frac{2n\pi t}{T}$	$y = A$ (amp)	$\cos\theta$	$\sin\theta$	$y\cos\theta$	$y\sin\theta$
0	0	1.98	1	0	1.98	0
$T/6$	60°	1.30	0.5	0.866	0.650	1.1258
$T/3$	120°	1.05	-0.5	0.866	-0.525	0.9093
$T/2$	180°	1.30	-1	0	-1.30	0
$2T/3$	240°	-0.88	-0.5	-0.866	0.440	0.7621
$\frac{5T}{6}$	300°	-0.25	0.5	-0.866	-0.125	0.2165
		4.5			1.12	3.0137

3

$$a_0 = \frac{2}{N} \Sigma y = \frac{2}{6}(4.5) = 1.5$$

$$a_1 = \frac{2}{N} \Sigma (y\cos\theta) = \frac{2}{6}(1.12) = 0.373 \quad \text{— 1}$$

$$b_1 = \frac{2}{N} \Sigma (y\sin\theta) = \frac{2}{6}(3.0137) = 1.005 \quad \text{— 1}$$

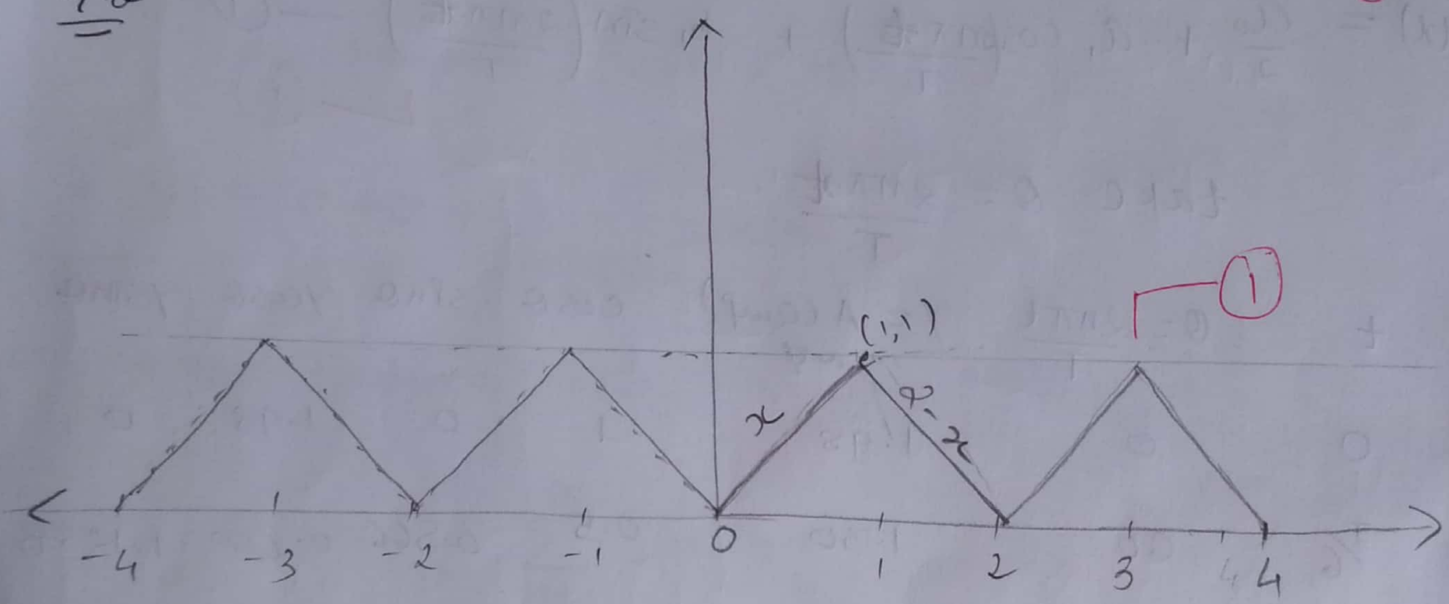
Thus the direct current part in the variable

Current $= \frac{a_0}{2} = 0.75$ — 1

and Amplitude of the first harmonic $= \sqrt{a_1^2 + b_1^2}$

$\boxed{= 1.072}$ — 1

Q.2



— 1

The Fourier series expansion of $f(x)$ is.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

— 1 — 1

$\Rightarrow \quad 2l = 2 \Rightarrow l = 1.$

$$f(x) = \begin{cases} x & ; \ 0 \le x \le 1 \\ 2-x & ; \ 1 \le x \le 2 \end{cases}$$

— 0.5

$f_1(x) = x$

$f_1(2-x) = 2-x = f_2(x) \quad \therefore f(x)$ is an even fun.

$\therefore \boxed{b_n = 0}$, $\forall n$ — 0.5

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{1}\int_0^1 x\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1$$

$$\boxed{a_0 = 1}$$ — 1

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= 2\int_0^1 x\cos(n\pi x)\,dx$$

$$= 2\left[x\left\{\frac{\sin(n\pi x)}{n\pi}\right\} - (1)\left\{-\frac{\cos(n\pi x)}{(n\pi)^2}\right\}\right]_0^1$$

$$= 2\left[\left\{0 + \frac{(-1)^n}{(n\pi)^2}\right\} - \left\{0 + \frac{1}{(n\pi)^2}\right\}\right]$$

1

$$= \frac{2}{(n\pi)^2}\left[(-1)^n - 1\right]$$

$$a_n = \begin{cases} 0 & ; n\text{-even} \\ \dfrac{-4}{n^2\pi^2} & ; n\text{-odd} \end{cases}$$ — 1

$\therefore$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos nx + b_n \sin nx\right]$$

$$\boxed{f(x) = \frac{1}{2} + \sum_{\substack{n=1 \\ n\text{-odd}}}^{\infty} \frac{-4}{n^2\pi^2} \cos nx + 0.}$$

— (1)

Q.3

$$f(x) = 0$$

$$2x - \log_{10} x - 7 = 0.$$

take $a = 3.5$, $b = 4$

$$f(3.5) = -0.54406$$

$$f(4) = 0.3979.$$

Ist Iteration

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$ — (1)

$$= \frac{3.5 f(4) - 4 f(3.5)}{f(4) - f(3.5)}$$

$$= \frac{3.5(0.3979) - 4(-0.5440)}{(0.3979) - (-0.5440)}$$

$$\boxed{x_1 = 3.7887}, \quad \boxed{f(x_1) = \cancel{-0.03013}\ -0.00109} < 0$$

(1)

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$\underline{\text{II}^{nd}}$ Iteration: take $a = 3.7887\cancel{68}$, $b = 4$

$$x_2 = \frac{(3.7887\cancel{68})\, f(4) - 4\, f(\cancel{3.78768}\ 3.7887)}{f(4) - f(3.78768)}$$

$$= \frac{(3.78768)(0.3979) - 4(\cancel{-0.00403}\ -0.00109)}{(0.3979) - (\cancel{-0.03013}\ -0.00109)}$$

2

$$\boxed{x_2 = 3.7892}\ ,\quad f(x_2) = 0.000147 \quad < 0$$

$\underline{\text{III}^{rd}}$ Iteration: take $a = 3.7892$, $b = 4$

$f(a) = -0.000147$

$f(b) = 0.3979$

2

$$x_3 = \frac{(3.7892)(0.3979) + 4(0.000147)}{0.3979 + 0.000147}$$

$$\boxed{x_3 = 3.7892}$$

$\therefore$ the root of the eq^n is $\underline{3.7892}$.

1

4. $f(x) = \begin{cases} -\pi, & -\pi \leq x < 0 \\ x, & 0 < x \leq \pi. \end{cases}$

Fourier series Expansion of $f(x)$ is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$\rightarrow$ ①.

↳ here $f(x)$ is neither even nor odd.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} (-\pi)\, dx + \int_{0}^{\pi} x\, dx \right]$$

$$= \frac{1}{\pi} \left[(-\pi) \left[x \right]_{-\pi}^{0} + \left[\frac{x^2}{2} \right]_0^{\pi} \right]$$

$$= \frac{1}{\pi} \left[(-\pi) \left\{ 0 + \pi \right\} + \frac{\pi^2}{2} \right]$$

$$\boxed{a_0 = -\pi + \pi/2} \quad -①$$

$$\boxed{a_0 = -\pi/2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\cos(nx)\,dx + \int_{0}^{\pi} x\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left\{(-\pi)\frac{\sin nx}{n}\right\}_{-\pi}^{0} + \left\{(x)\frac{\sin(nx)}{n} - (1)\left\{-\frac{\cos nx}{n^2}\right\}\right\}_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\{0\} + \left\{\pi\frac{\sin(n\pi)}{n} + \frac{\cos(n\pi)}{n^2}\right\} - \left\{0 + \frac{1}{n^2}\right\}\right]$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$= \frac{1}{n^2\pi}\left[(-1)^n - 1\right]$$

$$a_n = \begin{cases} 0 & ; n\text{-even} \\ \dfrac{-2}{n^2\pi} & ; n\text{-odd} \end{cases} \quad -(2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\sin(nx)\,dx + \int_{0}^{\pi} x\sin(nx)\,dx\right]$$

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$$b_n = \frac{1}{\pi}\left[(-\pi)\left(-\frac{\cos nx}{n}\right)_{-\pi}^{0} + \left\{ x \cdot \left\{\frac{-\cos(nx)}{n}\right\} - (1)\left\{\frac{-\sin(nx)}{n^2}\right\}\right\}_{0} \right]$$

$$= \frac{1}{\pi}\left[(-\pi)\left\{-\frac{1}{n} + \frac{(-1)^n}{n}\right\} + \left\{-\frac{\pi}{n}(-1)^n + 0\right\} - \left\{0\right\} \right]$$

$$= \frac{1}{n} - \frac{(-1)^n}{n} - \frac{(-1)^n}{n}$$

$$\boxed{b_n = \frac{1}{n}\left\{1 - 2(-1)^n\right\}}$$

2

$\therefore$

$$f(x) = -\pi/4 + \sum_{\substack{n=1 \\ n\text{-odd}}}^{\infty} \left(\frac{-2}{n^2\pi}\right)\cos(nx) + \sum_{n=1}^{\infty} \frac{1}{n}\left\{1 - 2(-1)^n\right\} \sin(nx)$$

2

To deduce the required series,

Put $x = 0$,

at $x = 0$, the series converges to,

$$\frac{1}{2}\left[f(0^+) - f(0^-)\right] = \frac{1}{2}\left[0 + (-\pi)\right] = -\pi/2.$$

$$f(0) = -\frac{\pi}{4} + \sum_{\substack{n=1 \\ n\text{-odd}}}^{\infty} \frac{-2}{n^2\pi} + 0.$$

$$-\pi/2 = -\pi/4 - \frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$-\frac{\pi}{4} = -\frac{2}{\pi}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]$$

$$\therefore \boxed{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}} \quad — \ 2$$

Q.5

$$f(x) = x^3 - 3x + 1$$

$$f'(x) = 3x^2 - 3 \quad — \ \tfrac{1}{2}$$

by Newton's Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, 3, \ldots \quad \tfrac{1}{2}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

take $x_0 = 0.25$

$$x_1 = 0.25 - \frac{f(0.25)}{f'(0.25)}$$

$$= 0.25 - \left[\frac{0.2656}{-2.8125}\right] \quad 2$$

$$\boxed{x_1 = 0.34443}$$

$$x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$$

$$= 0.344\cancel{43} - \left[\frac{f(0.34443)}{f'(0.34443)}\right]$$

$$= 0.34443 - \left[\frac{0.00757}{-2.6441}\right]$$

$$\boxed{x_2 = 0.34729} \quad — ①$$

$$x_3 = 0.34729 - \left[\frac{f(0.34729)}{f'(0.34729)}\right]$$

$$= 0.34729 - \left[\frac{0.000016}{-2.63816}\right]$$

$$= \cancel{0.34772}$$

$$\boxed{x_3 = 0.34729} \quad — ①$$

$$\boxed{x_4 = 0.34729.} \quad — ①$$

$\therefore$ The root of the eq^n $\boxed{x = 0.3472}$.

①

$f(x) = e^x$ in $0 < x < 1$

half-range Fourier sine series expansion of $f(x)$ is,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{— (1).} \quad \text{— (1)}$$

here, $l = 1$. — (1)

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= 2\int_0^1 e^x \sin(n\pi x)\,dx$$

$$= 2\left[\frac{e^x}{1+n^2\pi^2}\left\{\sin(n\pi x) - (n\pi)\cos(n\pi x)\right\}\right]_0^1 \quad \left[\because \int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left\{a\sin bx - b\cos bx\right\}\right]$$

— (3)

$$= 2\left[\frac{e}{1+n^2\pi^2}\left\{\sin(n\pi) - (n\pi)\cos(n\pi)\right\}\right]$$

$$- 2\left[\frac{1}{1+n^2\pi^2}\left\{0 - (n\pi)\right\}\right]$$

— (1)

$$= \frac{-2e(n\pi)(-1)^n}{1+n^2\pi^2} + \frac{2n\pi}{1+n^2\pi^2} \quad \boxed{= \frac{2n\pi}{1+n^2\pi^2}\left\{-e(-1)^n + 1\right\}.}$$

$$\therefore \boxed{e^x = \frac{2n\pi}{1+n^2\pi^2}\left(1 - e(-1)^n\right)\sin(n\pi x)} \quad \text{— (1)}$$

Q.7

Let $f(x) = \tan x - x$

$$f'(x) = \sec^2 x - 1 = \tan^2 x \quad - (1)$$

Initial approximation $x_0 = 4.5$.

1st Iteration: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= 4.5 - \frac{f(4.5)}{f'(4.5)}$$

$$= 4.5 - \frac{[\tan(4.5) - (4.5)]}{\tan^2(4.5)}$$

$$\boxed{x_1 = 4.4936}$$

2.5

2nd Iteration: $x_2 = x_1 - \left[\frac{f(x_1)}{f'(x_1)}\right]$

$$= 4.4936 - \frac{[\tan(4.4936) - 4.4936]}{\tan^2(4.4936)}$$

$$\boxed{x_2 = 4.4934}$$

2.5

$\therefore$ The root of the eqn is $\boxed{x = 4.4934}$

(1)

$$f(x) = \frac{\pi - x}{2} \quad \text{in } 0 < x < 2\pi$$

The Fourier series of $f(x)$ is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

0.5 1

$$f(2\pi - x) = \frac{\pi - (2\pi - x)}{2}$$

$$= \frac{-\pi + x}{2}$$

$$= -\frac{(\pi - x)}{2}$$

$$= -f(x)$$

0.5

$\therefore$ given fu^n is an odd fu^n in $(0, 2\pi)$

$\therefore \; a_0 = 0, \quad a_n = 0.$ 1

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cancel{\cos(nx)} \sin(nx)\, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \left(\frac{\pi - x}{2}\right) \sin(nx)\, dx$$

$$= \frac{2}{\pi} \left[\left(\frac{\pi - x}{2}\right)\left(-\frac{\cos(nx)}{n}\right) - \left(-\frac{1}{2}\right)\left(-\frac{\sin(nx)}{n^2}\right) \right]_0^{\pi}$$