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CMRIT

Internal Assesment Test - I

Sub:	Engineering Maths-III							Code:	17MAT31
Date:	07/ 09 /2018	Duration:	90 mins	Max Marks:	50	Sem:	3	Branch:	MECH –A,B

Answer all questions

		Marks	OBE CO	OBE RBT
1.	Express y as a Fourier series upto 2nd harmonics given the following values.	[8]	CO1	L3

x	0	1	2	3	4	5
y	4	8	15	7	6	2

		Marks	OBE CO	OBE RBT
2.	Using Newton's divided difference formula find f(3) and f(5) from the following data.	[7]	CO6	L3

x	2	4	9	10
y	4	56	711	980

		Marks	OBE CO	OBE RBT
3.	Find a root of the equation $3x = \cos x + 1$ correct to 4 decimal places.	[7]	CO6	L3

4. Fit an interpolating polynomial using Lagrange's [illegible] interpolation formula and hence find y at x=3 given [7] CO6 L3

x	0	1	2	5
y	2	3	12	147

5. From the following table find the number of students who obtained i)less than 45 marks ii)less than 75 marks. [7] CO6 L3

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

6. Obtain the half range Fourier sine series of $f(x)=\begin{Bmatrix} \frac{1}{4}-x \ in \ 0 \le x \le \frac{1}{2} \\ x-\frac{3}{4} in \ \frac{1}{2} \le x \le 1 \end{Bmatrix}$. [7] CO1 L3

7. Obtain the Fourier series expansion of f(x) $= x(2\pi - x) \ in \ 0 \le x \le 2\pi$. Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots$ [7] CO1 L3

Internal Test - I

Branch - TCE/CIV/ISE ECE-"C"

1. Expand y as a Fourier series upto 2nd harmonics given the following Table values

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Soln.

$0 \le x \le 6$ $\quad 2l = 6 \Rightarrow l = 3$ $\quad N = 6$

$$y = \frac{a_0}{2} + \left(a_1 \cos\frac{\pi x}{3} + b_1 \sin\frac{\pi x}{3}\right) + \left(a_2 \cos\frac{2\pi x}{3} + b_2 \sin\frac{2\pi x}{3}\right)$$ (1m)

$\boxed{\theta = \frac{\pi x}{3}}$

x	y	$\theta = \pi x/3$	$y\cos\theta$	$y\cos 2\theta$	$y\sin\theta$	$y\sin 2\theta$
0	4	0	4	4	0	0
1	8	60°	4	-4	6.928	6.928
2	15	120°	-7.5	-7.5	12.99	-12.99
3	7	180°	-7	7	0	0
4	6	240°	-3	-3	-5.196	5.196
5	2	300°	1	-1	-1.732	-1.732
	42		-8.5	-4.5	12.99	-2.598

(3m)

$a_0 = \frac{2}{N}\Sigma y = \frac{1}{3}(42) = 14$ (1m)

$a_1 = \frac{2}{N}\Sigma y\cos\theta = \frac{1}{3}(-8.5) = -2.833$

$a_2 = \frac{2}{N}\Sigma y\cos 2\theta = \frac{1}{3}(-4.5) = -1.5$ (1m)

$b_1 = \frac{2}{N}\Sigma y\sin\theta = 4.33$

$b_2 = \frac{2}{N}\Sigma y\sin 2\theta = \frac{1}{3}(-2.598) = -0.866$ (1m)

$$\boxed{y = 7 - 2.833\cos\frac{\pi x}{3} + 4.33\sin\frac{\pi x}{3} - 1.5\cos\frac{2\pi x}{3} - 0.88\sin\frac{2\pi x}{3}}$$ (1m)

2. Using Newton's divided difference formula find $f(3)$ and $f(5)$ from the following data

x	2	4	9	10
y	4	56	711	980

Sol: we have to find $f(x)$ where we have by data

$f(2)=4,\ f(4)=56,\ f(9)=711\ \ f(10)=980$.

x	$f(x)$	I DD	II DD	III DD
$x_0=2$	$y_0=4$			
		$f(x_0,x_1)=\frac{56-4}{4-2}=26$		
$x_1=4$	$y_1=56$		$f(x_0,x_1,x_2)=\frac{131-26}{9-2}=15$	
		$f(x_1,x_2)=\frac{711-56}{9-4}=131$		$f(x_0,x_1,x_2,x_3)$ $=\frac{23-15}{10-2}=1$
$x_2=9$	$y_2=711$		$f(x_1,x_2,x_3)=\frac{269-131}{10-4}=23$	
		$f(x_2,x_3)=\frac{980-711}{10-9}=269$		
$x_3=10$	$y_3=980$			

$f(x)=f(x_0)+(x-x_0)[x_0\,x_1]+(x-x_0)(x-x_1)[x_0,x_1,x_2]$ - - (3m)

(1m)

$f(x)=4+(x-2)26+(x-2)(x-4)15+(x-2)(x-4)(x-9)$

$=4+(x-2)[(x^2+2x+2)]$

$\boxed{f(x)=x^3-2x}$ (2m)

$\boxed{f(3)=21\quad f(5)=115}$ (1m)

(P2) Find a real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places

Sol Given equation is $\cos x = 3x - 1$

Let $f(x) = \cos x - 3x + 1$

observe that $f(0.5) = \cos 0.5 - 3 \times 0.5 + 1$

$= 0.37756 > 0$ and

$f(1) = \cos 1 - 3 \times 1 + 1 = -1.4597 < 0$.

Since $f(0.5)$ and $f(1)$ are opposite signs

1M a real root of given equation lies between 0.5 and 1

Let $a = 0.5$, $b = 1$

Regula falsi method is described by

3M $$x^* = \frac{af(b) - bf(a)}{f(b) - f(a)} \quad \text{or} \quad x^* = f(a) - \left(\frac{b-a}{f(b)-f(a)}\right) f(a)$$

update a, b If $f(a)$, $f(x^*)$ of opposite signs then $b = x^*$ else $a = x^*$

Iteration 1

Now: $$x^* = \frac{af(b) - bf(a)}{f(b) - f(a)} = 0.60275$$

and $f(x^*) = \cos 0.60275 - 3 \times 0.60275 + 1$

$= 0.0155 > 0$

update a, b $\rightarrow$: $a = 0.60275$, $b = 1$

Iteration 2

$$x^* = \frac{a f(b) - b f(a)}{f(b) - f(a)} = 0.60692$$

and $f(x^*) = 0.0006 > 0$

update $a, b \rightarrow a = 0.60692, b = 1$

Iteration 3

3M

$$x^* = \frac{a f(b) - b f(a)}{f(b) - f(a)} = 0.6071$$

$\therefore$ An approximate root for given

equation is 0.607.

We can also apply Newton Raphson method to solve this problem.

Newton Raphson method is described

by

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad i = 0, 1, 2, 3 \dots$$

Qn Find an interpolating polynomial using Lagranges interpolation formula and hence find y at $x=3$.

x	0	1	2	5
y	2	3	12	147

Sol Given

x	$x_0=0$	$x_1=1$	$x_2=2$	$x_3=5$
y	$y_0=2$	$y_1=3$	$y_2=12$	$y_4=147$

Lagranges interpolating polynomial is given by

4M

$$y = y(x) \approx \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

i.e. $$y \approx \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} \times 2 + \frac{x(x-2)(x-5)}{(1-0)(1-2)(1-5)} \times 3$$

$$+ \frac{x(x-1)(x-5)}{(2-0)(2-1)(2-5)} \times 12 + \frac{x(x-1)(x-2)}{(5-0)(5-1)(5-2)} \times 147$$

2M i.e. $$y \approx -\frac{1}{5}(x-1)(x-2)(x-5) + \frac{3}{4}x(x-2)(x-5) - 2x(x-1)(x-5) + \frac{49}{20}x(x-1)(x-2)$$

1M $\therefore$ y at $x=3$ is given by

$$y \approx -\frac{1}{5}(3-1)(3-2)(3-5) + \frac{3}{4}3(3-2)(3-5) - 2 \times 3(3-1)(3-5) + \frac{49}{20}3(3-1)(3-2) = 35$$

5. From the following find the No of Students who obtained (i) less than 45 marks (ii) less than 75 marks.

marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

Sol: We shall reconstitute the given table with $f(x)$ representing the number of students less than x marks. That is

less than 40 marks 31 students

less than 50 marks $31+42=73$ students

" 60 marks $73+51=124$ " (1m)

" less 70 marks $124+35=159$ "

" less 80 marks $159+31=190$ "

(i) We need to find $f(45)$ being the no of students scoring less than 45 marks

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0=40$	31	42	9	-25	37
50	73	51	-16	12	
60	124	35	-4		
70	159	31			
80	190				

(2m)

(a) We shall find $f(45)$

$$y_r = y_0 + r\,\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 - \cdots$$

$$r = \frac{x-x_0}{h} \;;\; r = \frac{45-40}{10} = 0.5$$

$$f(45) = 31 + (0.5)42 + \frac{(0.5)(0.5-1)}{2}(9) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(-25)$$

$$+ \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)(37)}{24}$$

(2m)

$f(45) = 47.86 \simeq 48$ Then the no of students obtained less than 45 marks 48.

$$y(75) = y_n + q\,\nabla y_n + \frac{q(q+1)}{2!}\nabla^2 y_n + \cdots$$

$$= 190 + (-1/2)31 + \frac{(-1/2)(-1/2+1)}{2!}(-4) + (-1/2)\frac{(-1/2+1)(-1/2+2)}{3!} \times (-1) + \cdots$$

$$= 190 - 15.5 + 0.25 - 0.75$$

$$1.4452$$

$$\boxed{y(75) = 172.5 \approx 173}$$ (2m)

6. obtain the half Range fourier Series of $f(x) = \begin{cases} 1/4 - x & \text{in } 0 \le x \le 1/2 \\ x - 3/4 & \text{in } 1/2 \le x \le 1 \end{cases}$

Soln: $f(x)$ is defined in $(0,1)$ comparing with half the range $(0,1)$ we have $l = 1$.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x \quad \text{(1m)} \quad \text{where} \quad b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi}{l}\right)x\,dx$$

(1m)

$$b_n = 2\left[\int_0^{1/2}(1/4 - x)\sin n\pi x\,dx + \int_{1/2}^{1}(x - 3/4)\sin n\pi x\,dx\right]$$

Apply Bernoulli's Rule.

$$b_n = 2\left[(1/4 - x)\frac{-\cos n\pi x}{n\pi} - (-1)\frac{-\sin n\pi x}{n^2\pi^2}\Big|_0^{1/2}\right.$$

$$\left. + \left[(x - 3/4)\frac{-\cos n\pi x}{n\pi} - 1 - \frac{\sin n\pi x}{n^2\pi^2}\Big|_{1/2}^{1}\right]\right]$$

$$b_n = 2\left\{\left[-\frac{1}{n\pi}(1/4 - x)\cos n\pi x\Big|_0^{1/2} - \frac{1}{n^2\pi^2}[\sin n\pi x]\Big|_0^{1/2}\right.\right.$$

$$\left. -\frac{1}{n\pi}\left[(x - 3/4)\cos n\pi x\Big|_{1/2}^{1} + \frac{1}{n^2\pi^2}[\sin n\pi x]\Big|_{1/2}^{1}\right]\right\}$$

$$b_n = 2\left\{+\frac{1}{4n\pi}\left(\cos n\pi/2 + 1 - \cos n\pi - \cos n\pi/2\right) - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}\right\}$$

$$= 2\left\{\frac{1}{4n\pi}(1 - \cos n\pi) - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}\right\} = \frac{1}{2n\pi}\left\{1 - (-1)^n\right\} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$

(4m)

$$\boxed{f(x) = \sum_{n=1}^{\infty}\left[\frac{1}{2n\pi}\left\{1 - (-1)^n\right\} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}\right]\sin n\pi x}$$ (1m)

7. Obtain the Fourier Series Expansion of $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$. Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$

Soln $f(x) = x(2\pi - x)$, $f(2\pi - x) = (2\pi - x)(2\pi - (2\pi - x)) = x(2\pi - x)$

$f(x)$ is even function. $\therefore \boxed{b_n = 0}$ (1m)

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x(2\pi - x)\,dx = \frac{2}{\pi}\int_0^{\pi}(2\pi x - x^2)\,dx = \frac{2}{\pi}\left[2\pi \frac{x^2}{2} - \frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi^3 - \frac{\pi^3}{3}\right] = \frac{2}{\pi}\left[\frac{2\pi^3}{3}\right]$$

$$\boxed{a_0 = \frac{4\pi^2}{3}}$$ (1m)

$$a_n = \frac{2}{\pi}\int_0^{\pi}(2\pi x - x^2)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(2\pi x - x^2)\frac{\sin nx}{n} - (2\pi - 2x)\left(-\frac{\cos nx}{n^2}\right) + (-2)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[0 + 2\pi \frac{\cos 0}{n^2}\Big|_0^{\pi}\right] = \frac{-4}{\pi}\left[\pi\frac{\cos 0}{n^2}\right] = -\frac{4}{n^2}$$

$$\boxed{a_n = -\frac{4}{n^2}} = -\frac{4}{n^2}$$ (3m)

$$f(x) = x(2\pi - x) = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} -\frac{4}{n^2}\cos nx \quad \text{--- (1)}$$ (1m)

$x = \pi$

$$\pi^2 = \frac{2\pi^2}{3} - 4\left[\frac{-1}{1^2}\cos\pi + \frac{1}{2^2}\cos 2\pi - \frac{1}{3^2}\cos 3\pi - \dots\right]$$

$$\pi^2 - \frac{2\pi^2}{3} = -4\left(-\frac{1}{1^2}\cos\pi + \frac{1}{2^2}\cos 2\pi - \frac{1}{3^2}\cos 3\pi\right)$$

$$\boxed{\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots}$$ (1m)