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Internal Assesment Test - I

Sub:	Engineering Maths-III							Code:	17MAT31
Date:	07/ 09 /2018	Duration:	90 mins	Max Marks:	50	Sem:	3	Branch:	ISE-A & B

Answer all the questions

	Marks	OBE CO	OBE RBT
1. Express y as a Fourier series upto 2^{nd} harmonics given the following values.	[8]	CO1	L3
2. Using Newton's divided difference formula find f(3) and f(5) from the following data.	[7]	CO6	L3
3. Evaluate $\int_0^1 \frac{x\ dx}{1+x^2}$ using Simpson's one third rule by taking seven ordinates and hence find $\log_e 2$.	[7]	CO6	L3

Data for Question 1:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Data for Question 2:

x	2	4	9	10
y	4	56	711	980

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4. Fit an interpolating polynomial of the form x=f(y) by using Lagrange's inverse interpolation formula and hence find x at y=5 given [7] CO6 L3

x	2	10	17
y	1	3	4

5. From the following table find the number of students who obtained i)less than 45 marks ii)less than 75 marks. [7] CO6 L3

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

6. Obtain the half range Fourier sine series of $f(x)=\begin{cases} \frac{1}{4}-x \text{ in } 0\le x\le \frac{1}{2} \\ x-\frac{3}{4} \text{ in } \frac{1}{2}\le x\le 1 \end{cases}$. [7] CO1 L3

7. Obtain the Fourier series expansion of f(x) $= x(2\pi - x)$ *in* $0\le x\le 2\pi$. Hence deduce that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\dots$. [7] CO1 L3

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Internal Test - I Branch - TCE/CIV/RSE ECE-"C"

1. Expand y as a Fourier series upto 2nd harmonics given the following Table values

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Soln: $0 \le x \le 6$ $2l = 6 \Rightarrow l = 3$ $N = 6$

$$y = \frac{a_0}{2} + \left(a_1 \cos\frac{\pi x}{3} + b_1 \sin\frac{\pi x}{3}\right) + \left(a_2 \cos\frac{2\pi x}{3} + b_2 \sin\frac{2\pi x}{3}\right)$$ (1m)

$\boxed{\theta = \frac{\pi x}{3}}$

x	y	$\theta = \pi x/3$	$y\cos\theta$	$y\cos 2\theta$	$y\sin\theta$	$y\sin 2\theta$
0	4	0	4	4	0	0
1	8	60°	4	-4	6.928	6.928
2	15	120°	-7.5	-7.5	12.99	-12.99
3	7	180°	-7	7	0	0
4	6	240°	-3	-3	-5.196	5.196
5	2	300°	1	-1	-1.732	-1.732
	42		-8.5	-4.5	12.99	-2.598

(3m)

$a_0 = \frac{2}{N}\Sigma y = \frac{1}{3}(42) = 14$ (1m)

$a_1 = \frac{2}{N}\Sigma y\cos\theta = \frac{1}{3}(-8.5) = -2.833$

$a_2 = \frac{2}{N}\Sigma y\cos 2\theta = \frac{1}{3}(-4.5) = -1.5$ (1m)

$b_1 = \frac{2}{N}\Sigma y\sin\theta = 4.33$

$b_2 = \frac{2}{N}\Sigma y\sin 2\theta = \frac{1}{3}(-2.598) = -0.866$ (1m)

$$\boxed{y = 7 - 2.833\cos\frac{\pi x}{3} + 4.33\sin\frac{\pi x}{3} - 1.5\cos\frac{2\pi x}{3} - 0.88\sin\frac{2\pi x}{3}}$$ (1m)

2. Using Newton's divided difference formula find $f(3)$ and $f(5)$ from the following data

x	2	4	9	10
y	4	56	711	980

Sol: we have to find $f(x)$ where we have by data

$f(2)=4,\ f(4)=56,\ f(9)=711\ \ f(10)=980.$

x	$f(x)$	I DD	II DD	III DD
$x_0=2$	$y_0=4$			
		$f(x_0,x_1)=\frac{56-4}{4-2}=26$		
$x_1=4$	$y_1=56$		$f(x_0,x_1,x_2)=\frac{131-26}{9-2}=15$	
		$f(x_1,x_2)=\frac{711-56}{9-4}=131$		$f(x_0,x_1,x_2,x_3)$ $=\frac{23-15}{10-2}=1$
$x_2=9$	$y_2=711$		$f(x_1,x_2,x_3)=\frac{269-131}{10-4}=23$	
		$f(x_2,x_3)=\frac{980-711}{10-9}=269$		
$x_3=10$	$y_3=980$			

(3m)

$f(x)=f(x_0)+(x-x_0)[x_0x_1]+(x-x_0)(x-x_1)[x_0,x_1,x_2]-\ -$

(1m)

$f(x)=4+(x-2)26+(x-2)(x-4)15+(x-2)(x-4)(x-9)$

$=4+(x-2)[(x^2+2x+2)]$

$\boxed{f(x)=x^3-2x}$ (2m)

$\boxed{f(3)=21\quad f(5)=115}$ (1m)

3. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ using Simpson's one third Rule By taking seven ordinates and hence find $\log_e 2$

Sol: $h=\frac{1-0}{6}$ $n=6.$

x	$x_0=0$	$x_1=1/6$	$x_2=1/3$	$x_3=1/2$	$x_4=2/3$	$x_5=5/6$	$x_6=1$
$y=\frac{x}{1+x^2}$	0	6/37	3/10	2/5	6/13	30/61	1/2

(2m)

Simpson's $1/3^{rd}$ Rule $I = \int_a^b y\,dx = \frac{h}{3}[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)]$

$= \frac{1}{18}[(0+1/2)+4(6/37+2/5+30/61)+2(3/10+6/13)]$

$= 0.3466$. —— (a) (3m)

Now we shall deduce value of $\log_e 2$.

Integrating theoretically

$$\int_0^1 \frac{x}{1+x^2}dx = \frac{1}{2}\int_0^1 \frac{2x\,dx}{1+x^2} = \frac{1}{2}\log(1+x^2)\Big|_0^1$$

$$= \frac{1}{2}[\log 2 - \log 1] = \frac{1}{2}\log 2 \quad \text{—(b)}$$

compare (a) & (b) $\boxed{\log_e 2 = 0.6932}$ (2m)

4. Fit an interpolating polynomial of the form $x = f(y)$ by using lagrange's inverse interpolation formula and hence find x at $y=5$ given

x	2	10	17
y	1	3	4

Soln: $x = f(y) = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)}x_0 + \frac{(y-y_0)(y-y_2)x_1}{(y_1-y_0)(y_1-y_2)} + \frac{(y-y_0)(y-y_1)x_2}{(y-y_0)(y_2-y_1)}$ (2m)

$$= \frac{(y-3)(y-4)}{(-2)(-3)}2 + \frac{(y-1)(y-4)}{2(-1)}10 + \frac{(y-1)(y-3)17}{3}$$

$$x = \frac{1}{3}(y-3)(y-4) - 5(y-1)(y-4) + \frac{17}{3}(y-1)(y-3)$$

$$x = \frac{1}{3}(y^2-7y+12) - 5(y^2-5y+4) + \frac{17}{3}(y^2-4y+3)$$

Thus $x = f(y) = y^2+1$. is the required polynomial (3m)

at $y=5$ $x = 5^2+1 = 26$ (2m)

5. From the following find the No of Students who obtained i) less than 45 marks (ii) less than 75 marks.

marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

Sol: We shall reconstitute the given table with $f(x)$ representing the number of students less than x marks. That is

less than 40 marks 31 Students

less than 50 marks $31+42=73$ Students

" 60 marks $73+51=124$ "

(1m)

" less 70 marks $124+35=159$ "

" less 80 marks $159+31=190$ "

(i) We need to find $f(45)$ being the no of students scoring less than 45 marks

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0=40$	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

(2m)

(a) We shall find $f(45)$

$$y_x = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 - -$$

$$s = \frac{x-x_0}{h} \; ; \; s = \frac{45-40}{10} = 0.5$$

$$f(45) = 31 + (0.5)42 + \frac{(0.5)(0.5-1)}{2}(9) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(-25)$$

$$+ \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)(37)}{24}$$

(2m)

$f(45) = 47.86 \Rightarrow 48$ Thus the no of students obtained less than 45 marks 48.

$$y(75) = y_n + q\nabla y_n + \frac{q(q+1)}{2!}\nabla^2 y_n + \cdots$$

$$= 190 + (-1/2)31 + \frac{(-1/2)(-1/2+1)}{2!}(-4) + \frac{(-1/2)(-1/2+1)(-1/2+2)}{3!}(-4) + \cdots$$

$$\boxed{y(75) = 172.8 \approx 173}$$ (2m)

6. obtain the half range fourier Series of $f(x) = \begin{cases} 1/4 - x & \text{in } 0 \le x \le 1/2 \\ x - 3/4 & \text{in } 1/2 \le x \le 1 \end{cases}$

Soln: $f(x)$ is defined in $(0,1)$ comparing with half the range $(0,1)$ we have $l = 1$.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x \quad \text{(1m)} \quad \text{where } b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi}{l}\right)x\,dx$$ (1m)

$$b_n = 2\left[\int_0^{1/2}(1/4 - x)\sin n\pi x\,dx + \int_{1/2}^{1}(x - 3/4)\sin n\pi x\,dx\right]$$

Apply Bernoulli's Rule.

$$b_n = 2\left[\left((1/4 - x)\frac{-\cos n\pi x}{n\pi} - (-1)\frac{-\sin n\pi x}{n^2\pi^2}\right)\Big|_0^{1/2} + \left((x - 3/4)\frac{-\cos n\pi x}{n\pi} - 1\cdot\frac{-\sin n\pi x}{n^2\pi^2}\right)\Big|_{1/2}^{1}\right]$$

$$b_n = 2\left\{\left[-\frac{1}{n\pi}(1/4 - x)\cos n\pi x\Big|_0^{1/2} - \frac{1}{n^2\pi^2}[\sin n\pi x]\Big|_0^{1/2}\right.\right. \left. - \frac{1}{n\pi}\left[(x - 3/4)\cos n\pi x\right]\Big|_{1/2}^{1} + \frac{1}{n^2\pi^2}[\sin n\pi x]\Big|_{1/2}^{1}\right\}$$

$$b_n = 2\left\{+\frac{1}{4n\pi}\left(\cos n\pi/2 + 1 - \cos n\pi - \cos n\pi/2\right) - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}\right\}$$

$$= 2\left\{\frac{1}{4n\pi}(1 - \cos n\pi) - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2}\right\} = \frac{1}{2n\pi}\left\{1 - (-1)^n\right\} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$ (4m)

$$\boxed{f(x) = \sum_{n=1}^{\infty}\left[\frac{1}{2n\pi}\left\{1 - (-1)^n\right\} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}\right]\sin n\pi x}$$ (1m)

7. obtain the Fourier Series Expansion of $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$ Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} -$

Soln $f(x) = x(2\pi - x)$ if $f(2\pi - x) = (2\pi - x)(2\pi - (2\pi - x)) = x(2\pi - x)$

$f(x)$ is even function $\therefore$ $\boxed{b_n = 0}$ (1m)

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x(2\pi - x)\,dx = \frac{2}{\pi}\int_0^{\pi}(2\pi x - x^2)\,dx = \frac{2}{\pi}\left[2\pi x^2/2 - x^3/3\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi^3 - \pi^3/3\right] = \frac{2}{\pi}\left[2\pi^3/3\right]$$

$$\boxed{a_0 = \frac{4\pi^2}{3}}$$ (1m)

$$a_n = \frac{2}{\pi}\int_0^{\pi}(2\pi - x^2)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(2\pi - n^2)\frac{\sin nx}{n} - (-2x)\left(-\frac{\cos nx}{n^2}\right) + (-2)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[-2x\frac{\cos nx}{n^2}\Big|_0^{\pi}\right] = \frac{-4}{\pi}\left[\pi\frac{\cos n\pi}{n^2}\right] = -4\frac{(-1)^n}{n^2}$$

$$\boxed{a_n = -\frac{4(-1)^n}{n^2}}$$ (3m)

$$f(x) = x(2\pi - x) = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} -\frac{4}{n^2}(-1)^n \cos nx \quad \text{—①}$$ (1m)

$x = \pi$

$$\pi^2 = \frac{2\pi^2}{3} - 4\left[-\frac{1}{1^2}\cos\pi + \frac{1}{2^2}\cos 2\pi - \frac{1}{3^2}\cos 3\pi - - -\right)$$

$$\pi^2 - \frac{2\pi^2}{3} = -4\left(-1/1^2 \cos\pi + \frac{1}{2^2}\cos 2\pi - \frac{1}{3^2}\cos 3\pi\right.$$

$$\boxed{\frac{\pi^2}{12} = 1/1^2 - 1/2^2 + 1/3^2 - - - -}$$ (1m)