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Internal Assessment Test – II

Sub:	**Engineering Mathematics - III**			Sub Code:	17MAT31	Branch:	CS,EEE, CV
Date:	15.10.2018	Duration:	90 min's	Max Marks: 50	Sem/ Sec:	III/ CS-A,EEE-A.CV-A	OBE

Question 1 is compulsory. Answer any SIX questions from Question 2 to 9.

No.	Question	MARKS	CO	RBT
1.	The following are the marks of 11 students in two papers (out of 100) Paper-I: 80, 45, 55, 56, 58, 60, 65, 68, 70, 75, 85 Paper-II: 82, 56, 50, 48, 60, 62, 64, 65, 70, 74, 90 Find the coefficient of correlation and the lines of regression.	[08]	CO4	L3
2.	Define Karl Pearson's co-efficient of correlation. If θ is the acute angle between the lines of regression, then show that $\tan\theta = \frac{\sigma x \sigma y}{\sigma_x^2+\sigma_y^2}\frac{1-r^2}{r}$. Explain the significance when $r = 0, \pm 1$.	[07]	CO4	L1
3.	The following table gives the production in thousand units of a certain commodity in different years. Year (x): 1975, 1985, 1995, 2005, 2015 Production (y): 8, 10, 12, 10, 16 Fit a straight line to the data and estimate the production in the year 2020.	[07]	CO6	L3
4.	Find the real root of the equation $xe^x = 2$ lying between 0 and 1 using Regula- Falsi method correct to 3 decimal places. Perform 4 iterations.	[07]	CO6	L3
5.	Find Z transforms of $\cos n\theta$ and $\sin n\theta$.	[07]	CO2	L3
6.	Find the inverse Z transform of $\frac{18z^2}{(2z-1)(4z+1)}$.	[07]	CO2	L3
7.	The area of a circle (A) corresponding to diameter (D) is given below. D: 80, 85, 90, 95, 100 A: 5026, 5674, 6362, 7088, 7854 Find the area corresponding to diameter 105 using an appropriate interpolation formula.	[07]	CO6	L3
8.	Apply Lagrange's method to find the value of x corresponding to f(x) = 15 from the following data: x: 5, 6, 9, 11 f(x): 12, 13, 14, 16	[07]	CO6	L3
9.	Evaluate $\int_2^8 \frac{dx}{\log_{10} x}$, using Simpson's (1/3)rd rule by taking 7 equidistance ordinates.	[07]	CO6	L3

35 + 4

59

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InternalAssessment Test – II

Sub:	**Engineering Mathematics - III**	Sub Code:	17MAT31	Branch:	CS
Date: 15.10.2018	Duration: 90 min's	Max Marks: 50	Sem/ Sec:	III/ CS-B,C.	OBE

Question 1 is compulsory. Answer any SIX questions from Question 2 to 9. — MARKS | CO | RBT

1. The following are the marks of 11 students in two papers (out of 100) — [08] CO4 L1

Paper- I	80	45	55	56	58	60	65	68	70	75	85
Paper -II	82	56	50	48	60	62	64	65	70	74	90

Find the coefficient of correlation and the lines of regression.

2. Define Karl Pearson's co-efficient of correlation. If θ is the acute angle between the lines of regression, then show that $\tan\theta = \frac{\sigma x \sigma y}{\sigma_x^2+\sigma_y^2}\frac{1-r^2}{r}$. Explain the significance when $r = 0,\pm1$. — [07] CO4 L1

3. The following table gives the production in thousand units of a certain commodity in different years. — [07] CO4 L1

Year (x)	1975	1985	1995	2005	2015
Production(y)	8	10	12	10	16

Fit a straight line to the data and estimate the production in the year 2020.

4. Find the real root of the equation $xe^x = 2$ lying between 0 and 1 using Regula- Falsi method correct to 3 decimal places. Perform 4 iterations. — [07] CO4 L1

5. Find Z transforms of $\cos n\theta$ and $\sin n\theta$. — [07] CO2 L1

6. Find the inverse Z transform of $\frac{18z^2}{(2z-1)(4z+1)}$. — [07] CO2 L1

7. The area of a circle (A) corresponding to diameter (D) is given below. — [07] CO6 L1

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

8. Find the interpolating polynomial for the given function given by the following table. hence find $f(12.5)$ and $f'(12.5)$ — [07] CO6 L3

x	10	11	12	13
y	21	23	27	33

9. Fit an exponential curve of the form $y = ae^{bx}$ by the method of least squares for the following data — [07] CO6 L3

No.of petals	5	6	7	8	9	10
No. of flowers	133	55	23	7	2	2

IAT-2

CS- A, B, C
CY- A, EEE-A.

Paper Solution - 17MAT31 - Maths-III

1 Consider x_i = Marks in Paper I, y_i = Marks in Paper II

x_i	y_i	$X_i = x_i - \bar{x}$	$Y_i = y_i - \bar{y}$	$X_i Y_i$	X_i^2	Y_i^2
80	82	15	16	240	225	256
45	56	-20	-10	200	400	100
55	50	-10	-16	160	100	256
56	48	-9	-18	162	81	324
58	60	-7	-6	42	49	36
60	62	-5	-4	20	25	16
65	64	0	-2	00	00	4
68	65	3	-1	-3	9	1
70	70	5	4	20	25	16
75	74	10	8	80	100	64
85	90	20	24	480	400	576.

Regression lines:-

1 Regression line of x on y.

$$(x-\bar{x}) = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$(x-65) = (0.92)\ \frac{11.33}{12.24}\ (y-66)$$

$$x-65 = (0.8516)(y-66)$$

$$\therefore \boxed{x = 0.8516\,y + 8.7982} \quad \text{— } 1$$

2 Regression line of y on x.

$$(y-\bar{y}) = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$$

$$(y-66) = (0.92)\left(\frac{12.24}{11.33}\right)(x-65)$$

$$(y-66) = (0.9938)(x-65)$$

$$\therefore \boxed{y = 0.9938x + 1.403} \quad \text{— } 1$$

Q.2 To find co-relation between two parameter x and y, Karl Pearson's co-efficient r is defined as.

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n \, \sigma_{\bar{x}} \, \sigma_y}$$

where $\bar{x}, \bar{y}$ are the mean of x and y respectively.

$\sigma_{\bar{x}} . \sigma_y$ are standard devition of x and y respectively.

n: number of parameter.

0'

→ value of r lies between -1 to 1.

→ If $r = 0$, there is no-corelation b/w x and y.

→ If $r = \pm 1$, there is perfect co-relation b/w x and y.

→ Regression line of x on y

$$(x-\bar{x}) = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$\therefore (y-\bar{y}) = \frac{1}{r}\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \quad \text{—— (1) —— (1)}$$

Slope of the regression line

$$m_1 = \frac{1}{r}\frac{\sigma_y}{\sigma_x} \quad \text{—— 0.5}$$

→ Regression line of y on x,

$$(y-\bar{y}) = r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \quad \text{—— (2) —— (1)}$$

$$\therefore \text{ slope } m_2 = r\frac{\sigma_y}{\sigma_x} \quad \text{—— 0.5}$$

If θ is the angle b/w two regression lines,

then $\tan\theta = \dfrac{m_1 - m_2}{1+m_1 m_2}$ —— (1)

(2)

$$= \frac{\frac{1}{r}\frac{\sigma_y}{\sigma_x} - r\frac{\sigma_y}{\sigma_x}}{1+\frac{\sigma_y}{\sigma_x}\frac{\sigma_y}{\sigma_x}} = \frac{\left(\frac{1-r^2}{r}\right)\frac{\sigma_y}{\sigma_x}}{\left(\frac{\sigma_x^2+\sigma_y^2}{\sigma_x^2}\right)}$$

$$= \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

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Q.3 ~~Let~~ take $X = x - 1995$

$X = x-1995$	y	X_i^2	$X_i y_i$
-20	8	400	-160
-10	10	100	-100
0	12	0	0
10	10	100	100
20	16	400	320
0	56	1000	160

2

Normal eq^n of the straight line

$$y = aX + b$$

$$\Sigma y = a\Sigma X + nb \quad \text{———} 1$$

$$\Sigma Xy = a\Sigma X^2 + b\Sigma X \quad \text{———} 1$$

$$\therefore \; 56 = a(0) + 5b \Rightarrow b = 11.2$$

$$160 = a(1000) + b(0) \Rightarrow a = 0.16$$

1

$$\therefore \; y = aX + b$$

$$y = (0.16)(x - 1995) + 11.2$$

$$y = 0.16x - 319.2 + 11.2$$

$$\therefore \boxed{y = 0.16x - 308}$$ 1

$\therefore$ production in $x = 2020$, $\boxed{y = 15.2}$ 1

4. $xe^{x} - 2 = 0$

$f(x) = xe^{x} - 2$

$f(0) = -2 < 0 \quad f(1) = 0.7183 > 0$

I iteration $\quad x_1 = \dfrac{a f(b) - b f(a)}{f(b) - f(a)}$ (1M)

$[0, 1] \quad a = 0 \quad b = 1$

$$x_1 = \frac{0 f(1) - 1 f(0)}{f(1) - f(0)}$$

$$= \frac{-1(-2)}{0.7183 - (-2)} = 0.73575$$

$$f(x_1) = f(0.73575) = 0.73575e^{0.73575} - 2$$

$$= -0.46445 < 0$$

II iteration $\quad f(0.73575) < 0$

$f(1) > 0 \quad [\overset{a}{0.73575}, \overset{b}{1}]$

$x_2 = ?$

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$$x_2 = \frac{0.73575\, f(1) - 1\, f(0.73575)}{f(1) - f(0.73575)}$$

$$= 0.8395$$

$$f(0.8395) = 0.8395\, e^{0.8395} - 2 = -0.05638 < 0$$

III iteration $f(0.8395) < 0$ $f(1) > 0$

$$[0.8395, 1]$$

$x_3 = ?$

$$x_3 = \frac{0.8395\, f(1) - 1 \cdot f(0.8395)}{f(1) - f(0.8395)}$$

$$x_3 = 0.8512$$

$$f(0.8512) = 0.8512\, e^{0.8512} - 2 = -0.0061 < 0$$

IV iteration $f(0.8512) < 0$ $f(1) > 0$

$$[0.8512, 1]$$

$$x_4 = \frac{0.8512\, f(1) - 1\, f(0.8512)}{f(1) - f(0.8512)}$$ 4M

$$= 0.8545$$

Ans 0.8545 1M

5 Find Z transforms of $\cos n\theta$ and $\sin n\theta$

$$\cos n\theta = \frac{e^{in\theta} + e^{-in\theta}}{2}$$

$$Z(\cos n\theta) = \frac{1}{2} Z\left(e^{in\theta} + e^{-in\theta}\right)$$

$$= \frac{1}{2} Z\left(e^{i\theta}\right)^n + \frac{1}{2} Z\left(e^{-i\theta}\right)^n$$

$$= \frac{1}{2}\left[\frac{z}{z-e^{i\theta}} + \frac{z}{z-e^{-i\theta}}\right]$$

$$= \frac{1}{2}\left[\frac{z(z-e^{-i\theta}) + z(z-e^{i\theta})}{z^2 - z(e^{i\theta}+e^{-i\theta}) + e^{i\theta}e^{-i\theta}}\right]$$

$$= \frac{1}{2}\left[\frac{2z^2 - z(e^{i\theta}+e^{-i\theta})}{z^2 - 2z\cos\theta + 1}\right]$$

$$= \frac{1}{2}\left(\frac{2z^2 - 2z\cos\theta}{z^2 - 2z\cos\theta + 1}\right) = \frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}$$

3M

$$\sin(n\theta) = \frac{e^{in\theta} - e^{-in\theta}}{2i}$$

$$Z(\sin n\theta) = \frac{1}{2i}\left\{Z\left(e^{in\theta}\right) - Z\left(e^{-in\theta}\right)\right\}$$

$$Z(\sin n\theta) = \frac{1}{2i}\left[\frac{z}{z-e^{i\theta}} - \frac{z}{z-e^{-i\theta}}\right]$$

$$= \frac{z}{2i}\left[\frac{1}{z-e^{i\theta}} - \frac{1}{z-e^{-i\theta}}\right]$$

$$= \frac{z}{2i}\left[\frac{(z-e^{-i\theta})-(z-e^{i\theta})}{z^2 - z(e^{i\theta}+e^{-i\theta}) + e^{i\theta}e^{-i\theta}}\right]$$

$$= \frac{z}{2i}\,\frac{(e^{i\theta}-e^{-i\theta})}{z^2-2z\cos\theta+1}$$

$$= \frac{z}{2i}\,\frac{2i\sin\theta}{z^2-2z\cos\theta+1} = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$$

3M

6. $$\bar{u}(z) = \frac{18z^2}{(2z-1)(4z+1)}$$

1M

$$\frac{\bar{u}(z)}{z} = \frac{18z}{(2z-1)(4z+1)}$$

$$= 18\left\{\frac{z}{(2z-1)(4z+1)}\right\}$$

$$= 18\left\{\frac{A}{2z-1} + \frac{B}{4z+1}\right\}$$

$$= 18\left\{\frac{A(4z+1)+B(2z-1)}{(2z-1)(4z+1)}\right\}$$

(10)

$$18z = 18\{A(4z+1) + B(2z-1)\}$$

$4A + 2B = 1$ ①

$A - B = 0 \Rightarrow A = B$ ②

① is $6B = 1$ $\boxed{B = 1/6} \Rightarrow \boxed{A = -1/6}$ (3M)

$$\frac{\bar{u}(z)}{z} = -\frac{1}{6}\frac{1}{2z-1} + \frac{1}{6}\frac{1}{4z+1}$$

$$\bar{u}(z) = -\frac{1}{6}\left(\frac{z}{2z-1}\right) + \frac{1}{6}\frac{z}{4z+1}$$ (1M)

$$-\frac{1}{6}\frac{1}{2}\left(\frac{z}{z-1/2}\right) + \frac{1}{6}\cdot\frac{1}{4}\left(\frac{z}{z+1/4}\right)$$

Taking inverse (1M)

$$u_n = -\frac{1}{12}(1/2)^n + \frac{1}{24}(-1/4)^n,$$

7. NGBF

$n = 5$ 4 backward differences

$$y_p = y_4 + \frac{p}{1!}\nabla y_4 + \frac{p(p+1)}{2!}\nabla^2 y_4$$

$$+ \frac{p(p+1)(p+2)}{3!}\nabla^3 y_4 + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_4$$

at $x_p = x_4 + ph$ $\quad 105 = 100 + p(5)$

$5 = 5p \Rightarrow p = 1$ (1M)

$x = D$	$y = A$	∇	∇^2	∇^3	∇^4
x_0 80	y_0 5026				
		∇y_1 648			
x_1 85	y_1 5674		$\nabla^2 y_2$ 40		
		∇y_2 688		$\nabla^3 y_3$ −2	
x_2 90	y_2 6362		$\nabla^2 y_3$ 38		
		∇y_3 726			4 $\nabla^4 y_4$
x_3 95	y_3 7088		40 $\nabla^2 y_4$	2 $\nabla^3 y_4$	
		766 ∇y_4			
x_4 100	y_4 7854				

$$f(105) = 7854 + 1(766) + \frac{(1)(1+1)}{2!}(40) \quad \text{4M}$$

$$+ \frac{1(1+1)(1+2)}{3!}(2) + \frac{1(1+1)(1+2)(1+3)}{4!}(4)$$

$$= 7854 + 766 + 40 + 2 + 4 = 8666$$

Area corresponding to $D = 105$ is 8666 1M

9. x – No. of petals
y – No. of flowers

x	y	Y $\log_e y$	xY	x^2
5	133	4.8903	24.4515	25
6	55	4.0073	24.0438	36
7	23	3.1355	21.9485	49
8	7	1.9459	15.5672	64
9	2	0.6931	6.2379	81
10	2	0.6931	6.9310	100
45		15.3652	99.1799	355

3M

NE's $\Sigma Y = nA + b\Sigma x$

$\Sigma xY = A\Sigma x + b\Sigma x^2$

$15.3652 = 6A + 45b$

$99.1799 = 45A + 355b$

$A = 9.4433 \quad b = -0.9177$ 2M

Solving $a = e^{9.4433} \sim 12623.3$

Curve $y = ae^{bx} = 12623.3\, e^{-0.9177x}$ 1M

Q8

x	y	Δ	Δ^2	Δ^3
10 x_0	21 y_0			
		Δy_0 2	$\Delta^2 y_0$ 2	$\Delta^3 y_0$ 0
11 x_1	23 y_1			
		1 Δy_1	$\Delta^2 y_1$ 2	
12 x_2	24 y_2			
		6 Δy_2		
13 x_3	33 y_3			

3M

$$y = f(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$x = 10 + p(1)$$

at $x = x_0 + ph$

$$p = x - 10$$

$$f(x) = 21 + \frac{x-10}{1!}(2) + \frac{(x-10)(x-11)}{2}(2) + \frac{(x-10)(x-11)(x-12)}{6}(0)$$

2M

$$= 21 + 2x - 20 + x^2 - \cancel{19x + 190}\ 21x + 110$$

$$\cancel{= x^2 - 17x + 91}$$

1M

$$= x^2 - 19x + 111$$

Ans $x^2 - 19x + 111$

$$f(12.5) = 29.75$$

1M

$$f'(x) = 2x - 19$$

$$f'(12.5) = 6$$

Q.8

x	x_0 5	x_1 6	x_2 9	x_3 11
$y=f(x)$	12 y_0	13 y_1	14 y_2	16 y_3

here, $y=f(x)=15$ is given. $x=?$

Applying Lagrange's inverse interpolation formula,

$$x=\left\{\frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)}\right\}(x_0)$$

$$+\left\{\frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}\right\}(x_1)$$

$$+\left\{\frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)}\right\}(x_2)$$

$$+\left\{\frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}\right\}(x_3)$$

3

$$\therefore x=\frac{(15-13)(15-14)(15-16)}{(12-13)(12-14)(12-16)}(5)+\frac{(15-12)(15-14)(15-16)}{(13-12)(13-14)(13-16)}(16)$$

$$+\frac{(15-12)(15-13)(15-16)}{(14-12)(14-13)(14-16)}(9)+\frac{(15-12)(15-13)(15-14)}{(16-12)(16-13)(16-14)}\times 11$$

$$=\frac{10}{8}-\frac{18}{3}+\frac{54}{4}+\frac{66}{24}$$ 3

$\therefore \boxed{x=11.5}$ 1

Q.9 $I = \int_{2}^{8} \frac{dx}{\log_{10} x}$,

here 7 ordinates are given $\therefore n = 6$.

$\therefore h = \frac{b-a}{n} = \frac{8-2}{6} = 1$ (1)

x	2	3	4	5	6	7	8
$y = \frac{1}{\log_{10} x}$	3.3219	2.0959	1.6609	1.4306	1.2850	1.1832	1.1073
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(2)

by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule,

$$I = \frac{h}{3}\left[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\right] \quad \text{— (01)}$$

$$= \frac{1}{3}\left[(3.3219 + 1.1073) + 2(1.6609 + 1.2850) + 4(2.0959 + 1.4306 + 1.1832)\right]$$

$$= \frac{1}{3}\left[29.1598\right]$$

(02)

$$\boxed{I = 9.7199} \quad \text{— (1)}$$

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