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Internal Assesment Test - II

Sub:	Engineering Maths-III					Code:	17MAT31
Date:	15/ 10 /2018	Duration:	90 mins	Max Marks: 50	Sem: 3	Branch:	ISE-A & B, TCE EC-B, C

First question is compulsory. Answer any six questions from rest

		Marks	OBE CO	OBE RBT
1.	Solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 2^n$. Given that $u_0 = 2,\ u_1 = 1$.	[8]	CO2	L3
2.	Find the Z-transform of $\cosh\left(\frac{n\pi}{2} + \theta\right)$	[7]	CO2	L3
3.	If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find the values of u_0, u_1, u_2, u_3.	[7]	CO2	L3
4.	Find the Z-transform of n^3 and hence find Z-transform of $e^{an} n^3$ by damping rule.	[7]	CO2	L3

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5. Find the complex Fourier transform of $f(x)=\begin{cases}1 & for\ |x|\le a\\ 0 & for |x|>a\end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$. [7] CO2 L3

6. Find the infinite Fourier cosine transform of e^{-x^2}. [7] CO2 L3

7. Find the complex Fourier transform of $e^{-a^2x^2}$, a>0. [7] CO2 L3

8. Solve the integral equation $\int_0^\infty f(x)\sin\alpha x\,dx=\begin{cases}10\,, & 0\le\alpha<1\\ 20\,, & 1\le\alpha<2\\ 0\,, & \alpha>2\end{cases}$ [7] CO2 L3

9. Find the real root of the equation $x^3-3x+4=0$ by Regula-Falsi method. [7] CO6 L3

10. Evaluate $\int_0^1 \frac{x\ dx}{1+x^2}$ using Weddle's rule by taking seven ordinates and hence find $\log_e 2$. [7] CO6 L3

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IAT-2 Solution M3

1) $u_{n+2} - 2u_{n+1} + u_n = 2^n$; Given $u_0 = 2$; $u_1 = 1$

Taking z-transforms on both sides.

$Z_T(u_{n+2}) - 2Z_T(u_{n+1}) + Z_T(u_n) = Z_T(2^n)$ 1m

(by linearity property)

$z^2\left(\bar{U}(z) - u_0 - \frac{u_1}{z}\right) - 2z(\bar{U}(z) - u_0) + Z_T(u_n) = \frac{z}{z-2}$

Substituting $u_0 = 2$ & $u_1 = 1$ we get [by left shifting rule and $Z(k^n) = \frac{z}{z-k}$]

$\bar{U}(z)(z^2 - 2z + 1) - z(2z - 3) = \frac{z}{z-2}$

$\therefore \bar{U}(z) = \frac{z(2z-3)}{(z-1)^2} + \frac{z}{(z-2)(z-1)^2}$ 1m

Let $\bar{U}(z) = P(z) + Q(z)$ —— (1)

Now $P(z) = \frac{z(2z-3)}{(z-1)^2} = \frac{Az}{(z-1)} + \frac{Bz}{(z-1)^2}$

$z(2z-3) = Az(z-1) + Bz$

Put $z = 1 \Rightarrow \boxed{B = -1}$

Comparing coefficient of z we get $\boxed{A = 2}$

$\therefore P(z) = \frac{2z}{(z-1)} - \frac{z}{(z-1)^2}$ —— (2) 2m

$$q(z) = \frac{z}{(z-2)(z-1)^2} = \frac{Az}{(z-2)} + \frac{Bz}{(z-1)} + \frac{Cz}{(z-1)^2}$$

$$1 = A(z-1)^2 + B(z-1)(z-2) + C(z-2)$$

Put $z=2$ We get $\boxed{A=1}$

Put $z=1$ We get $\boxed{C=-1}$

Put $z=0$ $1 = A + B(-1)(-2) + C(-2)$ $\therefore \boxed{B=-1}$

$$\therefore q(z) = \frac{z}{(z-2)} - \frac{z}{(z-1)} - \frac{z}{(z-1)^2} \quad \text{—③}$$ 2m

SubitITute ② & ③ in eq ① We get.

$$\bar{U}(z) = \frac{2z}{(z-1)} - \frac{z}{(z-1)^2} + \frac{z}{z-2} - \frac{z}{(z-1)} - \frac{z}{(z-1)^2}$$

$$\bar{U}(z) = \frac{z}{z-2} + \frac{z}{(z-1)} - \frac{2z}{(z-1)^2}.$$ 1m

Applying inverse z-T on both sides. We get

$$\boxed{U_n = 2^n - 1^n - 2 \times n.}$$ 1m

by using the formulae $z_T^{-1}\left(\frac{z}{z-k}\right) = k^n$

$$z_T^{-1}\left(\frac{z}{(z-1)^2}\right) = n.$$

2) $Z.T$ of $\cosh\left(\frac{n\pi}{2}+\theta\right)$

Let $u_n = \cosh\left(\frac{n\pi}{2}+\theta\right) = \frac{1}{2}\left[e^{\left(\frac{n\pi}{2}+\theta\right)} + e^{-\left(\frac{n\pi}{2}+\theta\right)}\right]$ 1m

$u_n = \frac{1}{2}\left[e^{\theta}\, e^{n\pi/2} + e^{-\theta}\cdot e^{-n\pi/2}\right]$

$Z_T(u_n) = \frac{1}{2}\left[e^{\theta}.\ Z_T(e^{n\pi/2}) + e^{-\theta} Z_T(e^{-n\pi/2})\right]$ 1m

(by linearity property)

We have $Z_T(k^n) = \frac{z}{z-k}$. here $k_1 = e^{\pi/2}$ & $k_2 = e^{-\pi/2}$

$\therefore Z_T(e^{n\pi/2}) = \frac{z}{z-e^{\pi/2}}$ & $Z_T(e^{-n\pi/2}) = \frac{z}{z-e^{-\pi/2}}$.

$\therefore Z_T(u_n) = \frac{1}{2}\left[e^{\theta}.\frac{z}{z-e^{\pi/2}} + e^{-\theta}.\frac{z}{z-e^{-\pi/2}}\right]$ 2m

$= \frac{z}{2}\left[\frac{e^{\theta}(z-e^{-\pi/2}) + e^{-\theta}(z-e^{\pi/2})}{(z-e^{\pi/2})(z-e^{-\pi/2})}\right]$

$= \frac{z}{2}\left[\frac{z(e^{\theta}+e^{-\theta}) - \left\{e^{(\pi/2-\theta)} + e^{-(\pi/2-\theta)}\right\}}{z^2 - z(e^{\pi/2}+e^{-\pi/2})+1}\right]$

$= \frac{z}{2}\left[\frac{2z\cosh\theta - 2\cosh(\pi/2-\theta)}{z^2 - 2z\cosh(\pi/2)+1}\right]$ 2m

$Z_T(u_n) = \frac{z^2\cosh\theta - z\cosh(\pi/2-\theta)}{z^2-2z\cosh(\pi/2)+1}.$ 1m

3

Given $\bar{U}(z) = \dfrac{2z^2+3z+12}{(z-1)^4}$

We have $u_0 = \lim_{z\to\infty} \bar{U}(z)$

$u_1 = \lim_{z\to\infty} z(\bar{U}(z) - u_0)$; $u_2 = \lim_{z\to\infty} z^2\left(\bar{U}(z) - u_0 - \frac{u_1}{z}\right)$

& $u_3 = \lim_{z\to\infty} z^3\left(\bar{U}(z) - u_0 - \frac{u_1}{z} - \frac{u_2}{z^2}\right)$ by I. V. T 2m

$\therefore u_0 = \lim_{z\to\infty} \dfrac{2z^2+3z+12}{(z-1)^4} = \lim_{z\to\infty} \dfrac{z^2}{z^4}\left(\dfrac{2+3/z+12/z^2}{(1-1/z)^4}\right) = 0$ 1m

$u_1 = \lim_{z\to\infty} \cancel{z}\cdot \dfrac{\cancel{z^2}\left(2+3/z+\frac{12}{z^2}\right)}{z^4(1-1/z)^4} = 0$ 1m

$u_2 = \lim_{z\to\infty} z^2\left(\dfrac{2z^2+3z+12}{(z-1)^4}\right) = \lim_{z\to\infty} \dfrac{\cancel{z^4}(2+3/z+12/z^2)}{\cancel{z^4}(1-1/z)^4} = 2$ 1m

$u_3 = \lim_{z\to\infty} z^3\left(\dfrac{2z^2+3z+12}{(z-1)^4} - \dfrac{2}{z^2}\right)$

$= \lim_{z\to\infty} \dfrac{z(11z^3+8z-2)}{(z-1)^4} = \lim_{z\to\infty} \dfrac{\cancel{z^4}(11+8/z^2-2/z^3)}{\cancel{z^4}(1-1/z)^4}$

$\boxed{u_3 = 11}$ 2m

Hence $\boxed{u_0=0}$ $\boxed{u_1=0}$ $\boxed{u_2=2}$ & $\boxed{u_3=11}$

4) Find Z.T of n^3 & hence find Z.T of $e^{an} \cdot n^3$

Z.T of $n^3 = Z_T(n^3) = -z \cdot \frac{d}{dz}\left(Z_T(n^2)\right)$ 1m

$$= -z \cdot \frac{d}{dz}\left(\frac{z^2+z}{(z-1)^3}\right)$$

$$= -z\left\{\frac{(z-1)^3(2z+1) - (z^2+z) \cdot 3(z-1)^2}{(z-1)^6}\right\}$$

$$= -z(z-1)^2\left\{\frac{(z-1)(2z+1) - 3(z^2+z)}{(z-1)^6}\right\}$$

$$= \frac{-z(-z^2-4z-1)}{(z-1)^4}$$

$$Z_T(n^3) = \frac{z^3+4z^2+z}{(z-1)^4}$$ 2m

$\therefore Z_T(e^{an} n^3) = Z_T((e^a)^n n^3)$ of the form $Z_T(k^n n^3)$

$\therefore$ By applying Damping Rule. 1m

$$Z_T(k^n \cdot n^3) = \left\{Z_T(n^3)\right\}_{z \to z/k} = \frac{(z/k)^3 + 4(z/k)^2 + (z/k)}{(z/k - 1)^4}$$

$$Z_T(k^n \cdot n^3) = \frac{kz^3 + 4k^2z^2 + k^3z}{(z-k)^4}$$ 2m

$$\therefore Z_T(e^{an} n^3) = \frac{e^a \cdot z^3 + 4(e^a)^2 \cdot z^2 + (e^a)^3 z}{(z-e^a)^4}$$ 1m

5) $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$.

$$F(u) = \int_{-\infty}^{\infty} f(x)\, e^{iux}\, dx$$ 1m

$$F(u) = \int_{-a}^{a} 1 \cdot e^{iux}\, dx = \left(\frac{e^{iux}}{iu}\right)_{-a}^{a} = \frac{1}{iu}\left(e^{iua} - e^{-iua}\right)$$

$$F(u) = \frac{2\sin au}{u}$$ Since $\begin{cases} e^{iua} = \cos ua + i\sin ua \\ \& \; e^{-iua} = \cos ua - i\sin ua \end{cases}$

To evaluate $\int_{0}^{\infty} \frac{\sin x}{x}\, dx$

$F(u) = \frac{2\sin au}{u}$ 2m applying Inverse Fourier Sine transform

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2\sin au}{u} \cdot e^{-iux}\, du$$ 1m

$$f(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin au}{u} \cdot e^{-iux}\, du.$$

Lets put $x = 0$. $\therefore f(0) = 1$

$$\therefore 1 = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin au}{u}\, du.$$ $(\because e^0 = 1)$ 2m

$$\frac{2}{\pi} \int_{0}^{\infty} \frac{\sin au}{u}\, du = 1$$ $\left(\because \frac{\sin au}{u}\right.$ is an even function of u

Put $a = 1$ $$\boxed{\int_{0}^{\infty} \frac{\sin u}{u}\, du = \pi/2}$$ 1m

6) Find infinite Fourier cosine tranform of e^{-x^2}

$$F_c(u) = \int_0^\infty f(x) \cos ux \, dx$$

$$F_c(u) = \int_0^\infty e^{-x^2} \cos ux \, dx.$$ 1m

Applying Leibnitz rule for differentiation under the integral sign.

$$F_c'(u) = \int_0^\infty e^{-x^2} (-\sin ux . x) \, dx = \int_0^\infty (-xe^{-x^2}) . \sin ux \, dx.$$

$$\int u \, dv = uv - \int v \, du \quad \text{here } dv = -xe^{-x^2} \Rightarrow v = \int -xe^{-x^2} dx$$

put $x^2 = t$; $2x\,dx = dt$

$$v = \int -e^{-t} \frac{dt}{2} = +\frac{e^{-t}}{2} = +\frac{e^{-x^2}}{2}$$

$$\therefore F_c'(u) = \left(\sin ux . \frac{+e^{-x^2}}{2} \right)_0^\infty - \int_0^\infty \frac{e^{-x^2}}{2} (\cos ux) . u \, dx.$$

$$2F_c'(u) = 0 - u \int_0^\infty e^{-x^2} \cos ux \, dx \Rightarrow 2F_c'(u) = F_c(u) \times u$$

$$2 . \frac{F_c'(u)}{F_c(u)} = -u.$$ Integrating both sides 4m

$$\log F_c(u) = -\frac{u^2}{4} + \log k \Rightarrow F_c(u) = ke^{-u^2/4}$$ 1m

To find k; put $u = 0 \Rightarrow F_c(0) = \int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ (gamma function)

$$\therefore F_c(0) = ke^0 = k \Rightarrow \boxed{k = \frac{\sqrt{\pi}}{2}}$$

$$\boxed{F_c(u) = \sqrt{\pi}/2 \; e^{-u^2/4}}$$

1m

7) $f(x) = e^{-a^2x^2}$; $a > 0$

$$F(u) = \int_{-\infty}^{\infty} e^{-a^2x^2} \cdot e^{iux}\, dx = \int_{-\infty}^{\infty} e^{-a^2\left(x^2 - \frac{iux}{a^2}\right)} dx.$$

1m 1m

Completing the square we get

$$F(u) = \int_{-\infty}^{\infty} e^{-a^2\left(x - \frac{iu}{2a^2}\right)^2} \cdot e^{-u^2/4a^2}\, dx.$$

Put $a\left(x - \frac{iu}{2a^2}\right) = t \Rightarrow dx = \frac{dt}{a}$ & t varies from $-\infty$ to ∞

$$F(u) = e^{-u^2/4a^2} \int_{-\infty}^{\infty} e^{-t^2} \cdot \frac{dt}{a} \quad 3m$$

& $\int_0^{\infty} e^{-t^2}\, dt = \sqrt{\pi}/2$

$$\therefore F(u) = \frac{\sqrt{\pi}}{a} \cdot e^{-u^2/4a^2}. \quad 2m$$

8) $\int_0^{\infty} f(x) \sin \alpha x\, dx$ is given (ie) fourier Sine transform

To solve the above integral let us use inverse fourier Sine transform.

$$\therefore f(x) = \frac{2}{\pi} \int_0^{\infty} F_s(\alpha) \cdot \sin \alpha x \cdot d\alpha. \quad 1m$$

$$f(x) = \frac{2}{\pi}\left[\int_0^1 10\sin\alpha x\, d\alpha + \int_1^2 20\sin\alpha x\, d\alpha + \int_2^\infty 0.\sin\alpha x\, d\alpha\right] \quad 1m$$

$$= \frac{2}{\pi}\left[\left[\frac{-10\cos\alpha x}{x}\right]_0^1 + \left[\frac{-20\cos\alpha x}{x}\right]_1^2\right]$$

$$= \frac{-20}{\pi x}\left(\cos x - 1 + 2\cos 2x - 2\cos x\right)$$

$$f(x) = \frac{20}{\pi x}\left(1 + \cos x - 2\cos 2x\right) \quad 5m$$

10) $\int_0^1 \frac{x\,dx}{1+x^2}$ by Weddle rule

here $h = \frac{1-0}{6} = 1/6$; $n = 6$ 1m

$x = 0$	$1/6$	$1/3$	$1/2$	$2/3$	$5/6$	1	
$y = 0$	$6/37$	$3/10$	$2/5$	$6/13$	$\frac{30}{61}$	$1/2$	2m

$$\therefore \int_a^b y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right] \quad 1m$$

$$\therefore \int_0^1 \frac{x\,dx}{1+x^2} = 0.3466. \quad 1m$$

To find $\log_e 2$ Integrating theoretically.

$$\int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_0^1 \frac{2x\,dx}{1+x^2}$$

$$= \frac{1}{2}\left(\log_e(1+x^2)\right)_0^1$$

$$= \frac{1}{2}\log_e 2 - \frac{1}{2}\log_e 1.$$

$$\int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\log_e 2 \qquad \text{Since } \log_e 1 = 0$$

$$\Rightarrow \quad 0.3466 = \frac{1}{2}\cdot\log_e 2$$

$$\Rightarrow \boxed{\log_e 2 = 0.6932}$$

2m

a) $f(x) = x^3 - 3x + 4$

$f(0) = 4 > 0$; $f(-2) = 2 > 0$

$f(-3) = -14 < 0$.

$\therefore$ Root lies between -2 & -3. 1m

<u>I iteration</u> ~~a = -2.1 ; b = -2.2~~

$f(-2.1) = 1.039$ & $f(-2.2) = -0.048$

$\therefore$ $a = -2.2$; $b = -2.1$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = -2.196$$ 2m

$f(x_1) = f(-2.196) = -0.002 < 0$

$\therefore$ Root lies between $(-2.196 ; -2.1)$

<u>II iteration :</u>

$a = -2.196$ $b = -2.1$

$f(a) = -0.002$ $f(b) = 1.039$

$$x_2 = \frac{(-2.196)(1.039) - (2.1)(0.002)}{1.039 + 0.002} = -2.1958$$

$\therefore$ $\boxed{x_2 = -2.196}$ 2m

$x_3 = -2.196$ 2m.

$\therefore$ The real root is -2.196.