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Internal Assessment Test – III

Sub:	Engineering Mathematics - III				Sub Code:	17MAT31	Branch :	CS
Date:	19.11.2018	Duration:	90 min's	Max Marks:	50	Sem/Sec:	III CS-B,C	OBE

	Question 1 is compulsory. Answer any SIX questions from Question 2 to 9	MARKS	CO	RBT
1.	State and prove Euler's theorem.	[08]	CO3	L3
2.	Test for an extremum of the functional $I=\int_0^1 (xy+y^2-2y^2y')dx, y(0)=1, y(1)=2$	[07]	CO3	L3
3.	Use Simpson's rule to evaluate $\int_1^7 \frac{dx}{x}$ and hence find $\log_e 7$.	[07]	CO6	L3
4.	The velocity **v** of a particle at distance **s** from a point on its path is given by the table below. Estimate the time taken to travel 60 metres using Weddle's rule.	[07]	CO6	L3

S metres	0	10	20	30	40	50	60
V m/sec	47	58	64	65	61	52	38

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①

Eulers eqn

A nec condn for $I = \int_{x_1}^{x_2} f(x\, y\, y')\,dx$

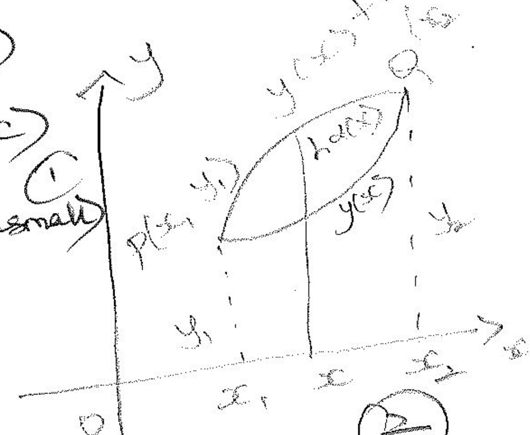
where $y(x_1) = y_1$ and $y(x_2) = y_2$

to be an extremum is that Euler's eqn

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

<u>Proof</u> Let I be an extremum along some curve $y = y(x)$ passing thro' $P(x_1\, y_1)$ and $Q(x_2\, y_2)$

Let $y = y(x) + h\alpha(x)$ ① be the nghg curve (h small) joining these pts



②

so that $\alpha(x_1) = 0$ at P
$\alpha(x_2) = 0$ at Q ②

when $h = 0$ the 2 curves coincide making I an extremum.

(i.e) $$I = \int_{x_1}^{x_2} f\left(x, y(x) + h\alpha(x), y' + h\alpha'(x)\right) dx$$

is an extremum when $h = 0$. ③

This req $\frac{dI}{dh} = 0$, when $h = 0$, treating I as a fn of h

3

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \frac{\partial f}{\partial y}\alpha(x)\,dx + \left\{ \left[\frac{\partial f}{\partial y'}\alpha(x)\right]_{x_1}^{x_2} - \int_{x_1}^{x_2} \alpha(x)\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)dx \right\}$$

$$= \int_{x_1}^{x_2} \frac{\partial f}{\partial y}\alpha(x)\,dx + \left\{ \frac{\partial f}{\partial y'}\alpha(x_2) - \frac{\partial f}{\partial y'}\alpha(x_1) \right\}$$

$$- \int_{x_1}^{x_2} \alpha(x)\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)dx$$

But $\alpha(x_1) = 0 = \alpha(x_2)$

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)\right]\alpha(x)\,dx$$

We have already stated that 2 $\frac{dI}{dh} = 0$ when $h=0$ for I to be an extremum. Integrand in RHS must be zero

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \qquad 1$$

is the nec condn for I to be an extremum.

$\delta I = 0$ rep the nec condn for I to be an extremum.

Geodesics

Given 2 arb pts P and Q on a surface S there exist infinite no. of curves on S having P and Q as their extremities. Of these the curve whose length is the least is called as the geodesic bet the pts P and Q on the given surface. A geodesic on a surface is a curve which the distance bet the 2 pts of the surface is min.

P.T the S.D bet 2 pts on a plane is along a st line joining them PT geodesics on a plane are st lines

Geodesics on a plane are st lines

We can also S.T this is the st line joining the pts $P(x_1, y_1)$ $Q(x_2, y_2)$

$$y(x_1) = y_1 \quad y(x_2) = y_2$$

$$y_1 = c_1 x_1 + c_2 \quad y_2 = c_1 x_2 + c_2$$

$$\frac{y_2 - y_1}{x_2 - x_1} = c_1$$

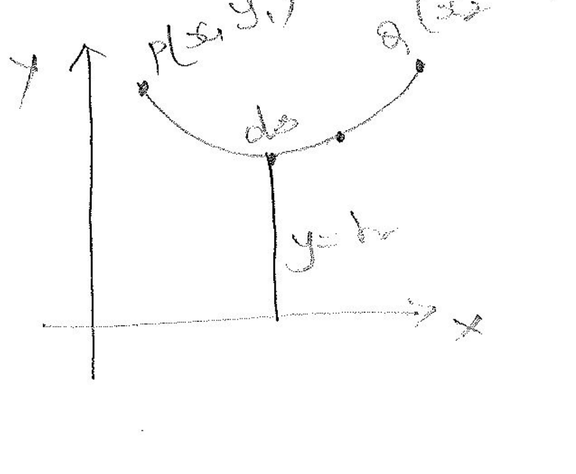
$$y - y_1 = c_1(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$ eqn of line PQ (1)

Hanging cable (chain) problem

A heavy cable hangs freely under gravity bet 2 fixed pts. S.T the shape of the cable is a catenary.



2. $I = \int_0^1 (xy + y^2 - 2y^2y')\,dx$

$y(0) = 1 \quad y(1) = 2$

$f = xy + y^2 - 2y^2y'$

$f_x = y \quad f_y = x + 2y - 4yy' \quad f_{y'} = -2y^2$ 1

Eulers eqn particular form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ 2

$x + 2y - 4yy' - \frac{d}{dx}(-2y^2) = 0$

$x + 2y - 4yy' + 4yy' = 0$ 3

$x + 2y = 0 \quad y = -x/2$ doesn't satisfy the conditions $y(0) = 1$ & $y(1) = 2$

$y = -\frac{x}{2}$ not the extremum of I 1

3. $h = \frac{7-1}{6} = 1$ 1

$n = 6$ 7 ordinates

$x_0 = 1, \; x_1 = 1 \quad x_2 = 2, \; x_3 = 3 \quad x_4 = 4$

$x_5 = 5 \quad x_6 = 6$

$y = ~~\log~~ \frac{1}{x}$

Exact value of $\int_1^7 \frac{dx}{x} = (\log x)_1^7$

$= \log_e 7$ — (2)

(1) & (2) $\int_1^7 \frac{dx}{7} = \log_e 7 \sim 1.9587$ 1

4. $h = 10 \quad v = \frac{ds}{dt}$

$I = \int_0^{60} dt = \int_0^{60} \frac{1}{v} \cdot ds$ 2

$y = \frac{1}{v}$

s	0	10	20	30	40	50	60
$y=\frac{1}{v}$	0.0213	0.0172	0.0156	0.0154	0.0164	0.0192	0.0263

$I = \frac{10}{3}\left[0.0213 + 0.0263 + 2(0.0156 + 0.0164) + 4(0.0172 + 0.0154 + 0.0192)\right]$ 4

$= 1.0627$

Time taken to travel 60 meters

$= 1.0627$ sec 1

13

$$y = \frac{1}{6}\left\{(129-81)x^2 - 7x + 405x - 512x + 12 - 324 + 384\right\}$$

3

519
405

$$y = \frac{1}{6}\left\{48x^2 - 114x + 72\right\}$$

1

396
324

$$y = 8x^2 - \cancel{19}x + 12$$

$$p(x) = 8x^2 - 19x + 12$$

Verify $p(1) = 1$; $p(3) = 27$; $p(4) = 64$

6. $x_0 = 30$ $y_0 = -30$

$x_1 = 34$ $y_1 = -13$ 1

$x_2 = 38$ $y_2 = 3$

$x_3 = 42$ $y_3 = 18$

$$x = \frac{(y+13)(y-3)(y-18)}{(-30+13)(-30-3)(-30-18)} 30$$

$$+ \frac{(y+30)(y-3)(y-18)}{(-13+30)(-13-3)(-13-18)} 34$$

$$+ \frac{(y+30)(y+13)(y-18)}{(3+30)(3+13)(3-18)} (38)$$

3

$$+ \frac{(y+30)(y+13)(y-3)}{(18+30)(18+13)(18-3)} (42)$$

RHS $N_x - M_y = 1-(-1) = 2$

$$\iint_R 2\,dx\,dy = 2\int_{x=0}^{1}\int_{y=x^2}^{\sqrt{x}} dy\,dx$$

$$= 2\int_{x=0}^{1} (y)_{x^2}^{\sqrt{x}}\,dx$$

$$= 2\int_{x=0}^{1}\left(\sqrt{x} - x^2\right)dx$$

$$= 2\left\{\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right\}_0^1 = 2\left(\frac{2}{3} - \frac{1}{3}\right)$$

$$= 2/3$$

(2)

LHS $\displaystyle\int_{OA} (x-y)\,dx + (x+y)\,dy$

$y = x^2 \quad dy = 2x\,dx$

$$\int_{x=0}^{1} (x - x^2)\,dx + (x + x^2)\,2x\,dx$$

$$= \int_{x=0}^{1}\left(x - x^2 + 2x^2 + 2x^3\right)dx$$

$$= \int_{x=0}^{1}\left(x + x^2 + 2x^3\right)dx$$

$$= \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{2x^4}{4}\right)_0^1$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{2} = \frac{4}{3}$$

x	y	I	II	III
x_0 15	36 y_0			
		0.4		
			0.04	
x_1 25	40 y_1			0
		1		
			0.04	
x_2 30	45 y_2			
		0.6		
x_3 35	48 y_3			

3

NGIF

$$f(x) = 36 + (x-15)\,0.4 + (x-15)(x-25)(0.04) + 0$$

$x = 26$

$$f(x) = 36 + (26-15)(0.4) + (26-15)(26-25)(0.04)$$

$$= 36 + 11(0.4) + (11)(1)(0.04)$$ 2

$$\simeq 36 + 4.4 + 0.44$$

$$\simeq 40.84$$

$$\simeq 41$$ 1