CMR
INSTITUTE OF
TECHNOLOGY

USN | | | | | | | | | | |

CMRIT

Internal Assesment Test - III

| Sub: | Engineering Maths-III | | | | | | | Code: | 17MAT31 |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 19/ 11 /2018 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 3 | Branch: | ISE-A & B, ECE-C |

**First question is compulsory. Answer any six questions from rest**

| | Marks | OBE CO | OBE RBT |
|---|---|---|---|
| 1. Compute the coefficient of correlation and the equation of lines of regression for the given data. | [8] | CO4 | L3 |

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

| | Marks | CO | RBT |
|---|---|---|---|
| 2. Fit a second degree parabola of the form $y = ax^2 + bx + c$ for the given data. | [7] | CO6 | L3 |

| x | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|
| y | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

| | Marks | CO | RBT |
|---|---|---|---|
| 3. A heavy cable hangs freely under gravity between two fixed points.Show that the shape of the cable is Catenary. | [7] | CO3 | L3 |


CMR
INSTITUTE OF
TECHNOLOGY

USN | | | | | | | | | | |

CMRIT

Internal Assesment Test - III

| Sub: | Engineering Maths-III | | | | | | | Code: | 17MAT31 |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 19/ 11 /2018 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 3 | Branch: | ISE-A & B, ECE-C |

**First question is compulsory. Answer any six questions from rest.**

| | Marks | OBE CO | OBE RBT |
|---|---|---|---|
| 1. Compute the coefficient of correlation and the equation of lines of regression for the given data. | [8] | CO4 | L3 |

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

| | Marks | CO | RBT |
|---|---|---|---|
| 2. Fit a second degree parabola of the form $y = ax^2 + bx + c$ for the given data. | [7] | CO6 | L3 |

| x | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|
| y | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

| | Marks | CO | RBT |
|---|---|---|---|
| 3. A heavy cable hangs freely under gravity between two fixed points.Show that the shape of the cable is Catenary. | [7] | CO3 | L3 |

4. Fit a curve of the form $y = ae^{bx}$ for the given data. [7] CO6 L3

| x | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| y | 133 | 55 | 23 | 7 | 2 | 2 |

5. Derive Euler's equation in standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. [7] CO3 L3

6. Find the extremal of the functional $\int_a^b \left(x^2 (y')^2 + 2y^2 + 2xy\right) dx$ [7] CO3 L3

7. Find the total work done by the force $\vec{F} = (3xy)\hat{i} - y\hat{j} + 2xz\hat{k}$ in moving a particle round the circle $x^2 + y^2 = 4$. [7] CO5 L3

8. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x \text{ and } y = x^2$. [7] CO5 L3

9. Verify Stoke's theorem for the vector $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by $x = 0, x = a, y = 0, y = b$. [7] CO5 L3

4. Fit a curve of the form $y = ae^{bx}$ for the given data. [7] CO6 L3

| x | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| y | 133 | 55 | 23 | 7 | 2 | 2 |

5. Derive Euler's equation in standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. [7] CO3 L3

6. Find the extremal of the functional $\int_a^b \left(x^2 (y')^2 + 2y^2 + 2xy\right) dx$ [7] CO3 L3

7. Find the total work done by the force $\vec{F} = (3xy)\hat{i} - y\hat{j} + 2xz\hat{k}$ in moving a particle round the circle $x^2 + y^2 = 4$. [7] CO5 L3

8. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x \text{ and } y = x^2$. [7] CO5 L3

9. Verify Stoke's theorem for the vector $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by $x = 0, x = a, y = 0, y = b$. [7] CO5 L3

# Internal Assessment - III

19/11/2018 ENGINEERING MATHS-3 IS A, B ECE - C

Code: 17MAT31

## Solution Manual.

1) Coefficient of correlation & Regression.

| $x$ | $y$ | $x-y=z$ | $x^2$ | $y^2$ | $z^2$ |
|---|---|---|---|---|---|
| 1 | 9 | -8 | 1 | 81 | 64 |
| 2 | 8 | -6 | 4 | 64 | 36 |
| 3 | 10 | -7 | 9 | 100 | 49 |
| 4 | 12 | -8 | 16 | 144 | 64 |
| 5 | 11 | -6 | 25 | 121 | 36 |
| 6 | 13 | -7 | 36 | 169 | 49 |
| 7 | 14 | -7 | 49 | 196 | 49 |
| $\Sigma x=28$ | $\Sigma y=77$ | $\Sigma z=-49$ | $\Sigma x^2=140$ | $\Sigma y^2=875$ | $\Sigma z^2=347$ |

$\bar{x}=\frac{\Sigma x}{n}=\frac{28}{7}=4$ ; $\bar{y}=\frac{\Sigma y}{n}=\frac{77}{7}=11$ ; $\bar{z}=\frac{\Sigma z}{n}=-7$

3m

$\sigma_x^2=\frac{\Sigma x^2}{n}-(\bar{x})^2=\frac{140}{7}-(4)^2=4$

$\sigma_y^2=\frac{\Sigma y^2}{n}-(\bar{y})^2=\frac{875}{7}-(11)^2=4$

$\sigma_z^2=\frac{\Sigma z^2}{n}-(\bar{z})^2=\frac{347}{7}-(-7)^2=0.57$. 2m

$$r = \frac{\sigma_x^2+\sigma_y^2-\sigma_{x-y}^2}{2\sigma_x\sigma_y} = \frac{4+4-0.57}{2\cdot\sqrt{4}\sqrt{4}} \sim 0.93$$

$\therefore \boxed{r = 0.93}$

1m

Lines of regression is

$y-\bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$

$y-11 = \frac{(0.93)(2)}{2}(x-4)$

$\Rightarrow y = 0.93x+7.28$

1m

$x-\bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

$x-4 = \frac{(0.93)2}{2}(y-11)$

$\Rightarrow x = 0.93y-6.23$

1m

2) Fit $y = ax^2+bx+c$ for given data

| $x$ | $y$ | $x^2$ | $x^3$ | $x^4$ | $xy$ | $x^2y$ |
|---|---|---|---|---|---|---|
| 1.0 | 1.1 | 1 | 1 | 1 | 1.9 | 1.1 |
| 1.5 | 1.3 | 2.25 | 3.375 | 5.0625 | 1.95 | 2.925 |
| 2.0 | 1.6 | 4.0 | 8.0 | 16.00 | 3.2 | 6.4 |
| 2.5 | 2.0 | 6.25 | 15.625 | 39.0625 | 5.0 | 12.4 |
| 3.0 | 2.7 | 9.00 | 27. | 81.00 | 8.1 | 24.3 |
| 3.5 | 3.4 | 12.25 | 42.875 | 150.0625 | 11.9 | 41.65 |
| 4.0 | 4.1 | 16.00 | 64 | 256.00 | 16.4 | 65.6 |
| | | $\Sigma x^2 = 50.75$ | $161.875 = \Sigma x^3$ | $548.1875 = \Sigma x^4$ | $47.65 = \Sigma xy$ | $154.475 = \Sigma x^2y$ |

3m

Normal equations are

$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2y$

$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy$

$a\Sigma x^2 + b\Sigma x + nc = \Sigma y$

$16.2 = 7c+17.5b+50.75a$

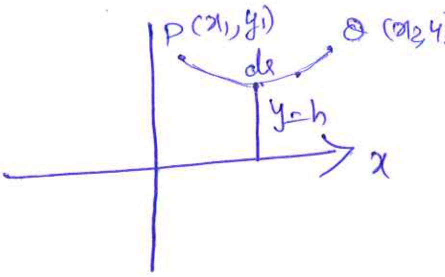
$47.65 = 17.5c+50.75b+161.875a$

$154.475 = 50.75c+161.875b+548.1875a$

Solving $a \sim 0.244$ $b \sim -0.198$ $c \sim 1.04$ 3m

$\therefore$ required curve is $y = 0.244x^2-0.198x+1.04$ // 1m

3) A heavy cable hangs freely under gravity between two fixed points. S.T the shape of the cable is a catenary.




* Let $P(x_1, y_1)$ & $Q(x_2, y_2)$ be the two fixed points of the cable.
* $ds \rightarrow$ be the elementary arc length of the cable.
* $\rho \rightarrow$ be the density (ie) Mass / unit length.

$\therefore$ $\rho\, ds$ is the mass of the element. 1m

* $g \rightarrow$ acceleration due to gravity.

$\therefore$ Potential energy of the element $= mgh = \rho ds \cdot g \cdot y$

(x-axis is taken as line of reference) 1m

$\therefore$ Total Potential energy $= T = \int_P^Q (\rho ds) \cdot g\, y\, dx$

$$= \int_{x_1}^{x_2} \rho g y \cdot \frac{ds}{dx} \cdot dx = \int_{x_1}^{x_2} \rho g y \cdot \sqrt{1+y'^2}\, dx.$$

$\rho g$ is a constant. $\therefore f(x, y, y') = y\sqrt{1+y'^2}$.

$f$ is independent of $x$.

$\therefore$ Let us use the Euler's equation of the form

$$f - y' \cdot \frac{\partial f}{\partial y'} = \text{Constant.}$$ 1m

$$y\sqrt{1+y'^2} - y' \cdot \frac{y}{2\sqrt{1+y'^2}} \cdot 2y' = c.$$

$$\frac{y(1+y'^2) - yy'^2}{\sqrt{1+y'^2}} = c$$

$$\Rightarrow y'^2 = \frac{y^2 - c^2}{c^2} \quad \text{(ie)} \quad \frac{dy}{\sqrt{y^2 - c^2}} = \frac{1}{c}\, dx.$$

Integrating

$$\int \frac{dy}{\sqrt{y^2 - c^2}} = \frac{1}{c} \int dx + c.$$

$$\cosh^{-1}\left(y/c\right) = x/c + K$$

$$\therefore y = c \cosh\left(\frac{x+a}{c}\right) \quad \text{where } a = kc$$ 4m

This is equation of a catenary.

It can be proved that this corresponds to the Minimum Value of $S$.

5

4) Fit curve of the form $y = ae^{bx}$

given $y = ae^{bx}$

$\log_e y = \log_e a + bx \log_e e$

$Y = A + bx$ ; $\boxed{Y = \log_e y}$ & $\boxed{A = \log_e a}$

1m

| $x$ | $Y = \log_e y$ | $xY$ | $x^2$ |
|---|---|---|---|
| 5 | 4.8903 | 24.4515 | 25 |
| 6 | 4.0073 | 24.0438 | 36 |
| 7 | 3.1355 | 21.9485 | 49 |
| 8 | 1.9459 | 15.5672 | 64 |
| 9 | 0.6931 | 6.2379 | 81 |
| 10 | 0.6931 | 6.9310 | 100 |
| $\Sigma x = 45$ | $\Sigma Y = 15.3652$ | $\Sigma xY = 99.1799$ | $\Sigma x^2 = 355$ |

2m

$\therefore$ Normal equations are $\Sigma Y = nA + b\Sigma x$ ;
$\Sigma xY = A\Sigma x + b\Sigma x^2$ $(n = 6)$

$$\left.\begin{array}{l} 6A + 45b = 15.3652 \\ 45A + 355b = 99.1799 \end{array}\right\}$$ solving $A = 9.4433$ ; $b = -0.9177$

$\Rightarrow a = e^{A} = 12623.3$ 3m

$\therefore$ Required curve of fit is $y = (12623.3)e^{-0.9177x}$

1m

5) Euler's equation in standard form:

Statement:

A necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y')\,dx$ where $y(x_1) = y_1$ & $y(x_2) = y_2$ to be extremum is

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0.$$ 1m

Proof:

* Let $I$ be an extremum along some curve $y = y(x)$ passing through $P(x_1, y_1)$ and $Q(x_2, y_2)$.
* Let $y = y(x) + h\alpha(x)$ — (1) be the neighbouring curve, where $h$ is small. Since at $P$ & $Q$ these two curves meet we have

$$\alpha(x_1) = 0 \text{ at } P \quad \& \quad \alpha(x_2) = 0 \text{ at } Q \qquad (2)$$

When $h = 0$ these two curves coincide making $I$ an extremum.

This requires $\frac{dI}{dh} = 0$, when $h = 0$, ($I \to$ function of $h$)

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \frac{\partial}{\partial h} f\left(x, y(x) + h\alpha(x), y'(x) + h\alpha'(x)\right) dx.$$

Applying chain rule for the partial derivative R.H.S

We have
$$\frac{dI}{dh} = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial h} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial h} + \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial h}\right) dx \qquad (3)$$

but $h$ is independent of $x$ & hence $\frac{\partial x}{\partial h} = 0$.

Let us consider (1) $\frac{\partial y}{\partial h} = \alpha(x)$ & from (4) $\frac{\partial y'}{\partial h} = \alpha'(x)$

Using these results in (3) We have

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \left( \frac{\partial f}{\partial y} \alpha(x) + \frac{\partial f}{\partial y'} \alpha'(x) \right) dx. \quad \text{——(5)}$$

3m

Keeping the first term in R.H.S of (5) as it is and integrating the second term by parts we have,

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \frac{\partial f}{\partial y} \alpha(x) dx + \left\{ \left( \frac{\partial f}{\partial y'} \alpha(x) \right)_{x_1}^{x_2} - \int_{x_1}^{x_2} \alpha(x) \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) dx \right\}$$

$$= \int_{x_1}^{x_2} \frac{\partial f}{\partial y} \alpha(x) dx + \left\{ \frac{\partial f}{\partial y'} \alpha(x_2) - \frac{\partial f}{\partial y'} \alpha(x_1) \right\} - \int_{x_1}^{x_2} \alpha(x) \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) dx.$$

But $\alpha(x_1) = 0 = \alpha(x_2)$

$$\therefore \frac{dI}{dh} = \int_{x_1}^{x_2} \left( \frac{\partial f}{\partial y} - \frac{d}{dx}\left( \frac{\partial f}{\partial y'} \right) \right) \alpha(x)\, dx.$$

But we have already stated that $\frac{dI}{dh}$ must be zero when $h=0$ for $I$ to be extremum.

$\therefore$ Integrand in the RHS must be zero.

Since $\alpha(x)$ is arbitrary we must have $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

This is the required Euler's equation being the necessary condition for the extremum of the functional

$$I = \int_{x_1}^{x_2} f(x, y, y')\, dx.$$

3m

(8)

(6)

$$\int_a^b \left(x^2 y'^2 + 2y^2 + 2xy\right) dx.$$

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \quad \text{becomes}$$

$$(4y + 2x) - \frac{d}{dx}\left(2x^2 y'\right) = 0$$

$$(4y+2x) - \left(2x^2 y'' + 4xy'\right) = 0$$

Simplifying $x^2 y'' + 2xy' - 2y = x$. 2m

put $x = e^t$ $(t = \log x)$

Then $xy' = Dy$ ; $x^2 y'' = D(D-1)y$ , $D = \frac{d}{dt}$

$\therefore$ We get $\left(D(D-1) + 2D - 2\right) y = e^t$

$$\left(D^2 + D - 2\right) y = e^t$$

A. Eqn is $m^2 + m - 2 = 0 \Rightarrow m = 1, -2$

$\therefore y_c = c_1 e^t + c_2 e^{-2t} = c_1 x + \frac{c_2}{x^2}$ 2m

$y_p = \frac{e^t}{D^2 + D - 2} = \frac{e^t}{0}$, replacing D by 1

$= t \frac{e^t}{2D+1} = t \frac{e^t}{2(1)+1} = \frac{te^t}{3} = \frac{x \log x}{3}$ 2m

$\therefore y = y_c + y_p$ (ie) $y = c_1 x + \frac{c_2}{x^2} + \frac{x \log x}{3}$ 1m

7) $\vec{F} = (3xy)\,i - y\,j + 2xz\,k$

Total Work done $W = \int_C \vec{F}\cdot d\vec{r}$ 1m

$\vec{r}$ $x^2+y^2=4$ can be represented in Parametric $x = 2\cos\theta$ $y = 2\sin\theta$ ; $z=0$

$0 \le \theta \le 2\pi$ 2m

$$W = \int_C \vec{F}\cdot d\vec{r} = \int 3xy\,dx - y\,dy + 2zx\,dz \quad 1m$$

$$W = \int_0^{2\pi} 3(4\cos\theta\sin\theta)(-2\sin\theta)d\theta - \int_0^{2\pi} 4\sin\theta\cos\theta\,d\theta$$

$$= -24\int_0^{2\pi} \sin^2\theta\cos\theta\,d\theta - 2\int_0^{2\pi} \sin 2\theta\,d\theta$$

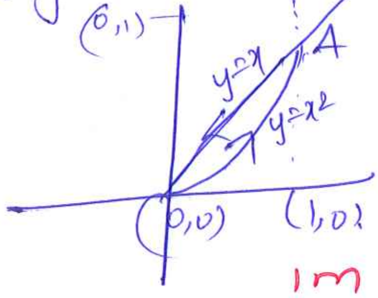
$$= -24\left(\frac{\sin^3\theta}{3}\right)_0^{2\pi} - 2\left(\frac{-\cos 2\theta}{2}\right)_0^{2\pi}$$

$$= 0 \quad 3m$$

$\therefore$ Total Work done is 0.

8) Verify Green theorem: $\int_C (xy+y^2)dx + x^2dy$.

C is bounded by $y=x$ & $y=x^2$




Point of intersection put $y=x$ in $y=x^2$

1m

$\therefore \; x^2-x=0 \Rightarrow x=0,1 \Rightarrow y=0,1$

$\therefore$ points are $(0,0)$ & $(1,1)$

T.P $\int_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy$

L.H.S $\int_C Mdx + Ndy = I_1 + I_2$ where

$I_1 = \int_{OA} (xy+y^2)dx + x^2dy = \int_0^1 (x\cdot x^2+x^4)dx + x^2\cdot 2x\,dx$

$I_1 = \int_0^1 (3x^3+x^4)\,dx = 3\left(\frac{x^4}{4}\right)_0^1 + \left(\frac{x^5}{5}\right)_0^1 = \frac{19}{20}$

$I_2 = \int_{AO} (xy+y^2)\,dx + x^2dy = \int_{x=1}^{0} (x\cdot x + x^2)dx = \int_0^1 3x^2dx = -1$

$LHS = I_1 + I_2 = \frac{19}{20} - 1 = -1/20.$ — (1) 3m

R.H.S. $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 2x - (x+2y) = x - 2y$.

$$\text{R.S.S} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$= \int_0^1 \int_{y=x^2}^{x} (x - 2y)\, dy\, dx.$$

$$= \int_0^1 \left(xy - y^2\right)_{y=x^2}^{x} dx$$

$$= \int_0^1 \left(x^4 - x^3\right) dx = \left(\frac{x^5}{5} - \frac{x^4}{4}\right)_0^1$$

$$\text{R.H.S} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20} \quad \text{—} \quad (2)$$ 3m

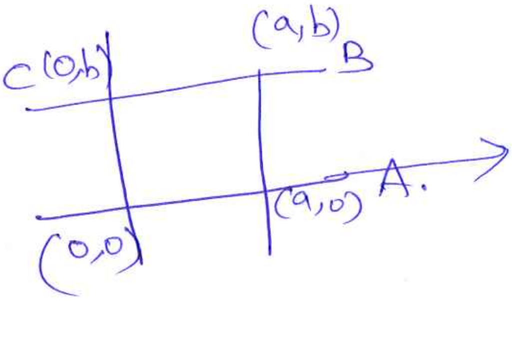
from (1) & (2) proved.

———— x ————

9) Verify Stokes theorem

$\vec{F} = (x^2+y^2)\, i - 2xy\, j$

T.P $\int_C F \cdot dr = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds.$ 1m



$$\int_C \vec{F}\cdot\vec{dr} = \int_{OA} \vec{F}\cdot\vec{dr} + \int_{AB} \vec{F}\cdot\vec{dr} + \int_{BC} \vec{F}\cdot\vec{dr} + \int_{CO} \vec{F}\cdot\vec{dr}$$
$$= I_1 + I_2 + I_3 + I_4$$

Along OA $y=0 \Rightarrow dy=0$ & $x$ varies from 0 to a

$$I_1 = \int_{x=0}^{a} x^2\,dx = \left(\frac{x^3}{3}\right)_0^a = \frac{a^3}{3}$$

$$I_2 = \int_0^b -2ay\,dy = -(ay^2)_0^b = -ab^2 \quad \left(\begin{array}{l} x=a \\ dx=0 \\ 0\le y\le b\end{array}\right)$$

$$I_3 = \int_0^a (x^2+b^2)\,dx = \left(\frac{x^3}{3}+b^2x\right)_a^0 = \frac{-a^3}{3} - ab^2$$

(Along BC $y=b \Rightarrow dy=0$ & $a \le x \le 0$)

$$I_4 = \int_{y=b}^{0} 0\,dy = 0$$

3m

$$\therefore \int_C F\cdot dr = \frac{a^3}{3} - ab^2 - \frac{a^3}{3} - ab^2 = -2ab^2 \quad \text{— (1)}$$

$$\nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+y^2 & -2xy & 0 \end{vmatrix} = -4yk$$

$$(\nabla\times F)\cdot\hat{n}\,ds = (-4yk)(dy\,dz\,i + dz\,dx\,j + dx\,dy\,k)$$
$$= -4y\,dx\,dy$$

$$\therefore \iint_S (\nabla\times F)\cdot\hat{n}\,ds = \int_{x=0}^{a}\int_0^b -4y\,dx\,dy = -4\int_{x=0}^{a}\left(\frac{y^2}{2}\right)_0^b dx$$

$$R.H.S = -2ab^2 \quad \text{— (2)}$$

3m

From (1) & (2) Proved.