

Internal Assessment Test 1 – Sept. 2018

Sub: Fluid Mechanics

Sub Code: 17CV33

Branch: Civil

Date: 10/09/2018

Duration: 90 min

Max Marks: 50

Sem / Sec:

III – A & B

Answer all questions

- 1 (a) Define the terms – a) Surface tension b) Specific volume. [04]
- (b) A cube of 0.25m sides and mass 28kg slides down a plane inclined at 2V:3H covered by a thin film of oil of viscosity 2.2×10^{-3} Pa-s. If the thickness of the film is 0.02mm, determine the steady state velocity of the block. [06]
- 2 (a) Derive an expression for capillary rise in a glass tube. *22.162 m/c* [04]
- (b) State and prove Pascal's Law. [06]
- 3 (a) State Hydrostatic Law with figure. [02]
- (b) Given 2 pipes x and y, carrying liquids of sp. gravity 0.9 and 0.8 respectively, the pipe x being 400mm above pipe y. A differential U tube Hg manometer is used to find the difference in their pressures. The Hg level in the limb of the manometer connected to pipe y is 50cm below the pipe's center, while the level in the other limb is 0.2m below that pipe's center. Find the difference in the pressure in the pipes in terms of meters of water. [08]
- 4 (a) Difference between (i) Specific gravity and Specific weight, (ii) Absolute pressure and gauge pressure, (iii) Simple manometer and differential manometer, (iv) Centre of pressure and centre of gravity. (each 2.5) [10]
- 5 (a) Define total pressure and centre of pressure. Derive an expression for the total pressure and depth of centre of pressure from the free surface of liquid of an inclined plane surface submerged in the liquid. [10]

All the Best

Pradeep
6/9/2018
Sign of CI

selected
Sign of CCI

Sign of HOD

1a) Surface tension, σ

It is defined as a tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquid such that the contact surface behave like a membrane under tension.

$$= \frac{\text{Force}}{\text{unit length of free surface}}$$

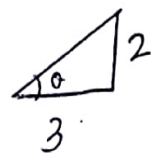
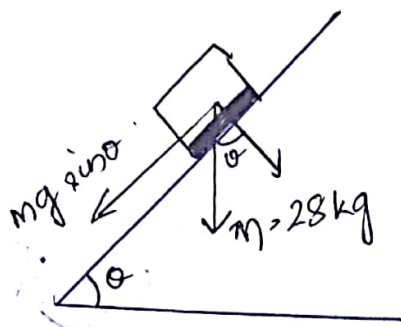
$$= \frac{\text{surface energy}}{\text{unit area}} \quad \begin{array}{l} \text{S.I unit} \\ - \text{N/m} \end{array}$$

b) Specific volume - It is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

$$\text{S.I unit, } m^3/kg$$

$$\text{Specific volume of water} = \frac{1}{1} = \frac{1}{1000} = 10^{-3} m^3/kg$$

b)



$$\tan \theta = \frac{2}{3}$$

$$\theta = \tan^{-1}(2/3)$$

$$\theta = 33.69^\circ$$

side of cube, a , 0.25m

Mass of cube = 28 kg

$$\text{Wt of cube} = 28 \times 9.81 = 274.68\text{ N}$$

$$\mu = 2.2 \times 10^{-3} \text{ Pa s}$$

$$t = 0.02\text{ mm} = 0.02 \times 10^{-3} \text{ m}$$

By Newton's Law of Viscosity

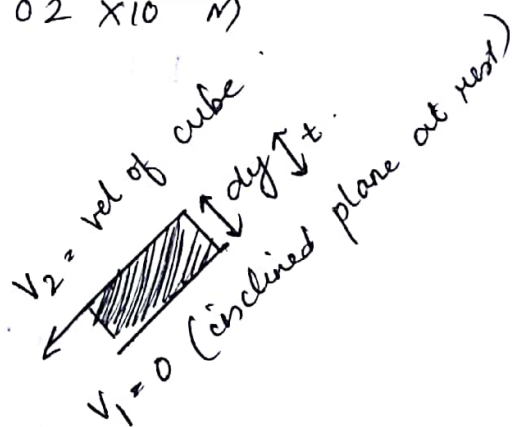
$$\tau = \mu \frac{du}{dy}$$

L.H.S

$$\tau = \frac{F}{A} = \frac{mg \sin \theta}{a \times a}$$

$$= \frac{28 \times 9.81 \times \sin 33.69}{0.25 \times 0.25}$$

$$= 2437.83 \text{ N/m}^2 \quad \text{--- 2}$$



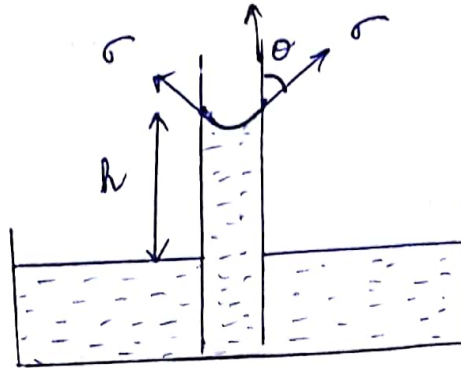
R.H.S

$$2. \mu \frac{du}{dy} = \frac{2.2 \times 10^{-3} (v_2 - v_1)}{t}$$

$$2437.83 = \frac{2.2 \times 10^{-3} v_2}{0.02 \times 10^{-3}}$$

$$v_2 = 22.162 \text{ m/s} \quad \text{--- 2}$$

2 a) Capillary rise in glass tube



Consider a glass tube of small diameter 'd' open at both ends and inserted in a liquid.

Let

θ - contact angle of liquid surface (b/w liquid & glass)

σ - surface tension force for unit length

ρ - density of liquid

h - height of the liquid to the adjacent general level of liquid.

Under a state of F_1 equilibrium, the weight of liquid of height 'h' is balanced by the surface tension force at the surface of the liquid in the tube, F_2 .

wt of liquid in tube, $F_1 = mg \downarrow$

$$= \rho \times \text{vol} \times g$$

$$= \rho \times \frac{\pi}{4} d^2 h \cdot g \quad \text{--- (1) ---}$$

Force due to surface tension, $F_2 \uparrow$

$$= \sigma \times \text{circumference} \times \cos \theta$$

$$F_2 = \sigma \times \pi d \cos \theta \quad \text{--- (2) ---}$$

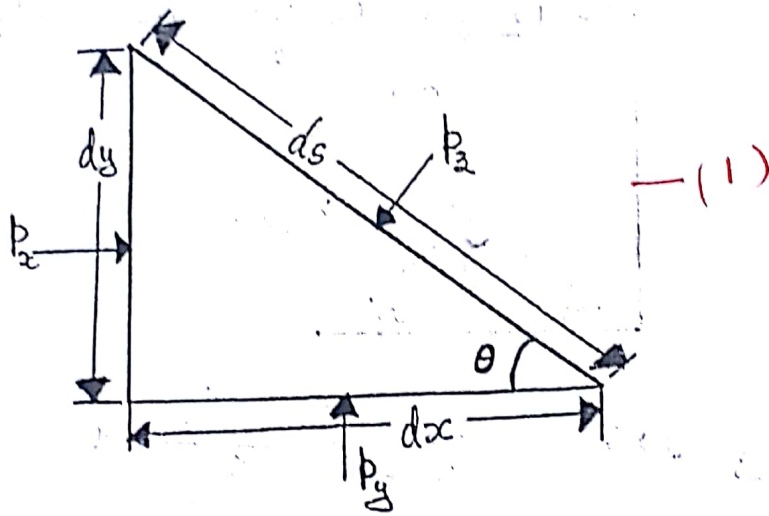
Equating (1) & (2)

$$F_1 = F_2$$

$$\rho \times \frac{\pi}{4} d^2 h g = \sigma \pi d \cos \theta$$

$$h = \frac{4 \sigma \cos \theta}{\rho g d} \quad \text{--- 2 ---}$$

Pascal's law :- It states that the pressure or intensity of pressure in a static fluid is equal in all direction is called Pascal's law. —(1)



Consider an arbitrary fluid Element of wedge shape of fluid mass at rest. The fluid Element of very small dimension dx, dy, dz and the width of the Element \perp to plane of the paper be unity. Let P_x, P_y and P_z be the intensity of pressure acting on the faces AB, BC, CA respectively. Let angle be θ at C.

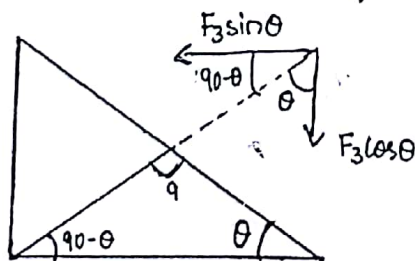
The force acting on the wedge are forces due to pressure P_x, P_y, P_z acting normal to the surface. The second force is weight of the fluid Element.

The forces are calculated below :-

Force due to pressure P_x on the face AB $F_1 = P_x dy \times 1$

Force due to pressure P_y on the face BC $F_2 = P_y dy \times 1$

Force due to pressure P_z on the face AC $F_3 = P_z ds \times 1$



$$F_4 = mg = \rho \times \text{volume} \times g = \rho \times \frac{dx \times dy \times x}{2} \times g$$

for fluid at rest $\Sigma F_x = 0$
 $F_1 - F_3 \sin \theta = 0$
 $P_x dy - P_3 ds \sin \theta = 0$

Substitute $dy = ds \sin \theta$

$$P_x ds \sin \theta - P_3 ds \sin \theta = 0$$

$$\boxed{P_x = P_3} \quad \text{--- 1}$$

III^{ly} resolving forces along y $\Sigma F_y = 0$

$$F_2 - F_4 - F_3 \cos \theta = 0$$

$$P_y dy - \frac{\rho dx dy g}{2} - P_3 ds \cos \theta = 0$$

$$\boxed{P_y = P_3} \quad \text{--- 2}$$

$[F_4 = 0, \text{ since } dx \text{ \& } dy \text{ are very small and their product can be neglected, } dy = ds \cos \theta]$

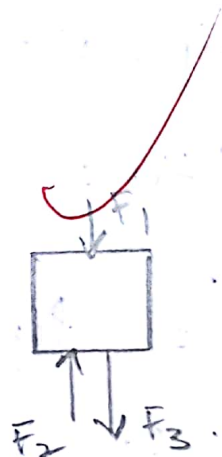
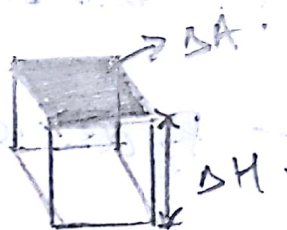
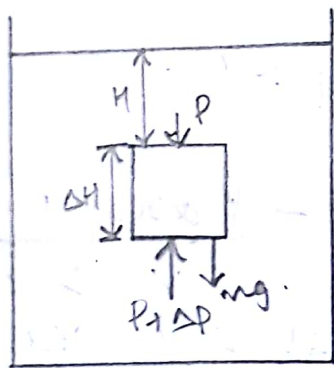
Equating ① and ②

$$P_3 = P_y = P_x$$

Since θ is an arbitrary constant, the Equation proves that pressure is same in all direction at a point in a static liquid.

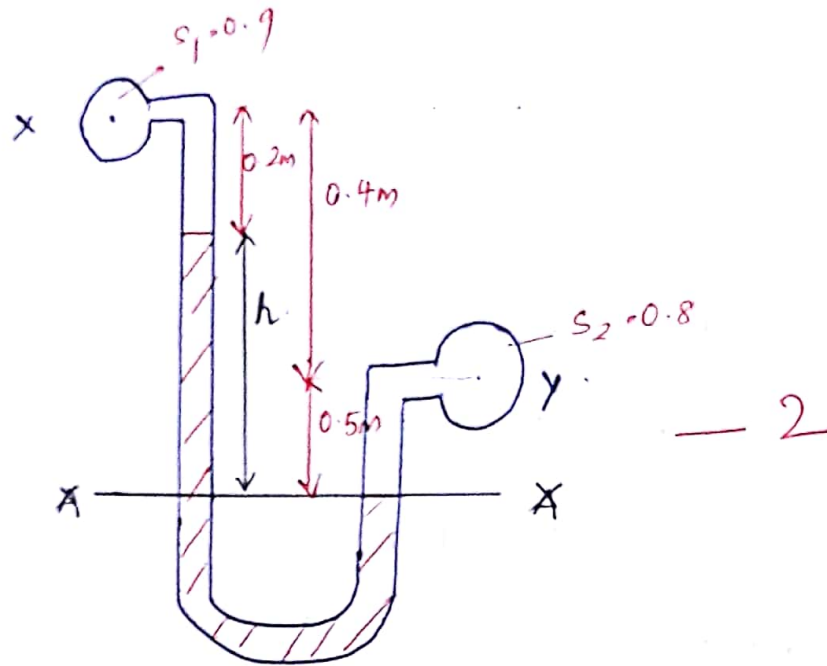
3A.) (a.) Hydrostatic law: It states that, in a fluid at rest, the pressure P inside increases vertically down-ward, & is equal to the specific weight of the fluid.

$$\Rightarrow P = \rho g. \quad [\text{units } \text{N/m}^2] \quad -2$$



From the figure above, by applying equilibrium conditions, hydrostatic law can be proved as $P = \rho g$.

3b)



Taking datum as ~~X-X~~ A-A,

L.H.S = R.H.S

$$P_x + s_1 \times 10^3 \times g \times 0.2 + 13.6 \times 10^3 \times g \times 0.7$$

$$= P_y + s_2 \times 10^3 \times g \times 0.5 \quad - 2$$

$$P_x + 0.9 \times 10^3 g \times 0.2 + 13.6 \times 10^3 g \times 0.7 =$$

$$P_y + 0.8 \times 10^3 g \times 0.5$$

$$P_y - P_x = 10^3 \times g (0.9 \times 0.2 + 13.6 \times 0.7$$

$$- 0.8 \times 0.5) = 10^3 \times g \times 9.3 \quad - 2$$

In terms of w water

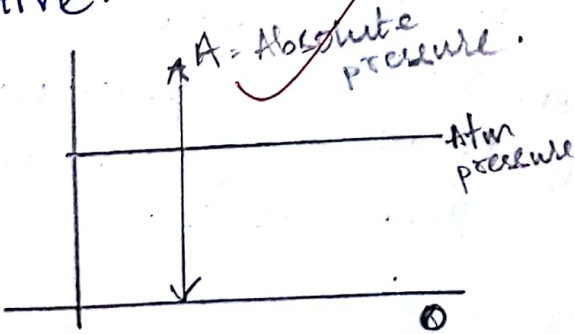
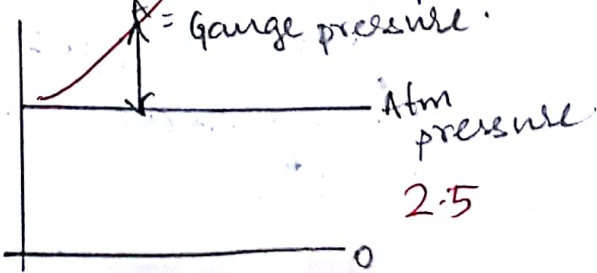
$$\frac{P_y - P_x}{\rho_w g} = \frac{P_y - P_x}{10^3 \times g} = 9.3 \text{ m of water} \quad - 2$$

Q.A.) (a.)

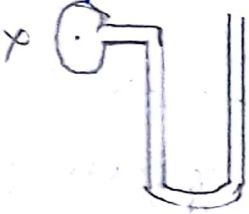
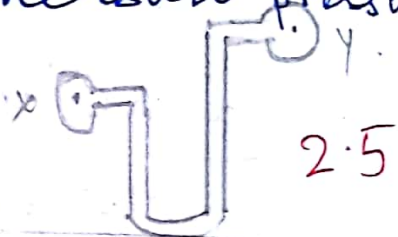
(i) Specific gravity & specific weight.

Specific gravity	Specific weight.
<p>1) It is defined as ratio of density of fluid to the density of water at reference temperature.</p> <p>2) It is denoted by 'r'.</p> <p>2) It has no units.</p> $r = \frac{\text{density of fluid}}{\text{density of water}}$	<p>1) It is defined as the ratio of mass^{weight} of the fluid, to the volume.</p> <p>2) It is denoted by 's'.</p> <p>2) Its units are N/m^3.</p> $s = \frac{\text{weight of the fluid}}{\text{Volume}}$ <p style="text-align: right;">- 2.5</p>

(ii.) Absolute pressure & Gauge pressure.

Absolute pressure.	Gauge pressure.
<p>1) The pressure is measured from absolute '0'.</p> <p>2) The pressure is always positive.</p> 	<p>1) The pressure is measured from atmospheric temperature.</p> <p>2) The pressure is always to positive, if negative it becomes vacuum pressure.</p>  <p style="text-align: right;">2.5</p>

(iii) Simple manometer & Differential manometer.

Simple manometer	Differential manometer
<p>1) It measures the value of the pressure of the fluid required.</p> <p>2) It consists of only one pt to measure pressure, other pt is left open to atmosphere.</p>  <p>3) It can be inclined.</p>	<p>1) It only measures the difference in the values of pressures of the fluids.</p> <p>2) It consists of 2 points consisting of different fluids with manometric fluid to measure pressure difference.</p>  <p>3) It can't be inclined.</p>

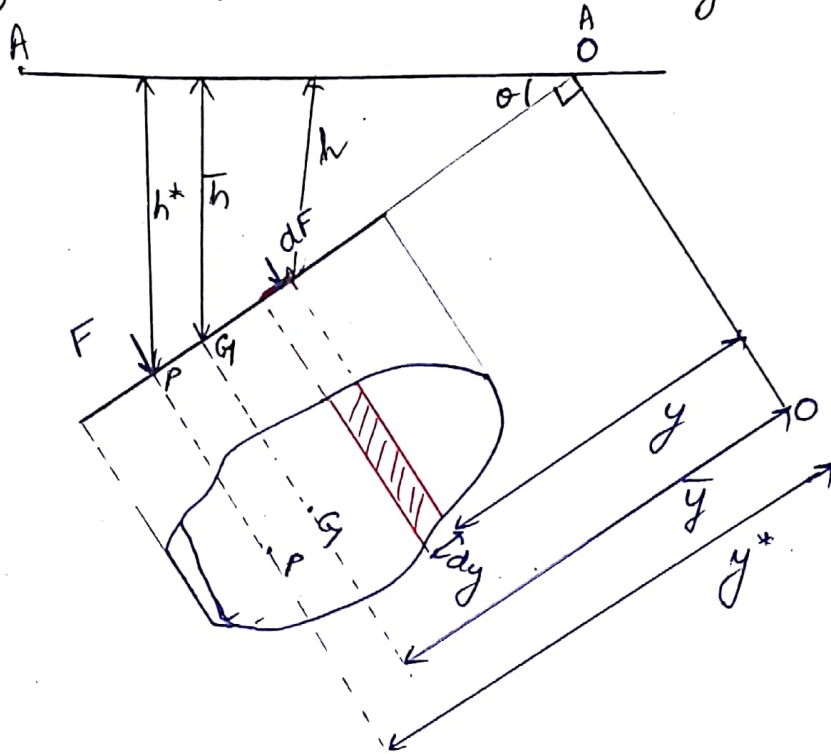
(iv) Centre of pressure & centre of gravity.

Centre of pressure	Centre of gravity.
<p>1) It is the pt where moments due to pressure is equal to the resultant moment.</p> <p>2) The distance of centre of pressure from free surface is given as h^*.</p>	<p>1) It is, the point where entire ^{wt} mass of the body is said to be concentrated.</p> <p>2) The distance of ^{2.5} centre of gravity from free surface is given as \bar{h}.</p>

Inclined surface Immersed in liquid

Definition - 2

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with



the free surface^(AA') of the liquid as shown in figure.

Let

$A \rightarrow$ Total area of surface.

$\bar{h} \rightarrow$ depth of C.G. (G) from free surface.

$h^* \rightarrow$ depth of centre of pressure from free surface

Let the plane of surface if produced meet the free liquid at O. Then O-O is the axis I^* to the plane of surface.

$\bar{y} -$ distance of C.G. from O-O-axis

$y^* \rightarrow$ distance of C.P. from O-O-axis

From figure.

$$\sin \theta = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{h y^*} = \frac{h}{y} \quad \text{--- (1)}$$

1) Total Pressure, F

Consider a small strip of area dA at a distance of h from free surface and y from $O-O$ axis.

Pressure on the strip, $p = \rho g h$

Pressure force, dF on the strip

$$dF = \rho g h dA \quad \text{--- (2)} \quad \left[h \text{ and } dA \text{ are not } \perp \right]$$

$$\text{sub } h = y \sin \theta$$

$$dF = \rho g y dA \sin \theta \quad \text{--- (2)}$$

$$F = \int dF = \rho g \sin \theta \int y dA \quad \rightarrow \quad \begin{array}{l} \text{1}^{\text{st}} \text{ Moment of} \\ \text{area} = \\ \int y dA = A \bar{y} \end{array}$$

$$= \rho g \sin \theta A \bar{y}$$

$$= \rho g A \bar{h} \quad \text{--- (3)} \quad \left[\bar{y} \sin \theta = \bar{h} \right] \quad \text{--- 3}$$

2) Centre of pressure, h^*

a) Sum of moments

Moment of force dF about $O-O$ axis

$$dM = dF \times y$$

sub dF from (2)

$$dM = \rho g y dA \sin \theta \cdot y$$

$$dM = \rho g y^2 \sin \theta dA$$

$$M_1 = \int dM = \rho g \sin \theta \int y^2 dA \quad \left[\begin{array}{l} \text{2}^{\text{nd}} \text{ Moment of} \\ \text{area} \end{array} \right]$$

$$= \rho g \sin \theta I_0 \quad \text{--- (4)} \quad \int y^2 dA = I_0$$

where I_0 is the M.I about O-O axis

(b) Moment of Resultant Force

$$M_2 = F x y^* \quad \text{--- (5)}$$

Equating (4) & (5)

$$\rho g \sin \theta I_0 = F x y^*$$

$$\rho g \sin \theta I_0 = \rho g A \bar{h} x y^*$$

$$\text{sub } y^* = \frac{h^*}{\sin \theta}$$

$$\sin \theta \cdot I_0 = A \bar{h} \cdot \frac{h^*}{\sin \theta}$$

$$\frac{I_0 \sin^2 \theta}{A \bar{h}} = h^*$$

$$h^* = \left[I_G + \frac{A \bar{h}^2}{\sin^2 \theta} \right] \frac{\sin^2 \theta}{A \bar{h}}$$

$$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h}$$

$$h^* = \bar{h} + \frac{I_G \sin^2 \theta}{A \bar{h}}$$

$$I_0 = I_G + \frac{A \bar{y}^2}{\sin^2 \theta}$$

$$= I_G + \frac{A \bar{h}^2}{\sin^2 \theta}$$

--- 3c