

17CV32

Strength of
materials

I IAT

Solutions.

2018

HOOKE'S LAW

"Under Elastic Zone. Stress is directly proportional to Applied Strain"

(Normal)

(Linear)

Stress \propto Strain with elastic limit.

$$\sigma \propto t$$

$$\sigma = E t$$

E = young's modulus or ~~Young's~~ modulus of elasticity, expressed as N/mm^2 (UOM).

$$\sigma = \frac{\text{Load}}{\text{Area/s}} = \frac{P}{A}$$

$$t = \frac{\Delta L}{L}$$

$$\therefore \frac{P}{A} = E * \frac{\Delta L}{L}$$

$$\therefore \Delta L = \Delta = \frac{PL}{AE}$$

Within the limit of proportionality
Stress \propto Strain.
(Applied)

Hooke's Law applied to shear.

Shear Stress \propto Shear Strain. Within elastic limit.

$$\tau \propto \phi^{0.5}$$

$$\tau = (\text{constant}) * (\phi) = G * \phi = G r$$

G = Modulus of rigidity.

$$\tau = G * \frac{\Delta L}{L}$$

(Numericals.)

Stress-Strain diagram for ferrous and Non-ferrous Materials.

Understanding of Mechanical behaviours of Material is very much essential for using such material in construction and Machine parts. Normally behaviour of material under Tension (axial), compression (axial), shear, bending, Torsion loads are studied in Laboratory Environment. Apart from this other properties like Impact, Hardness are studied.

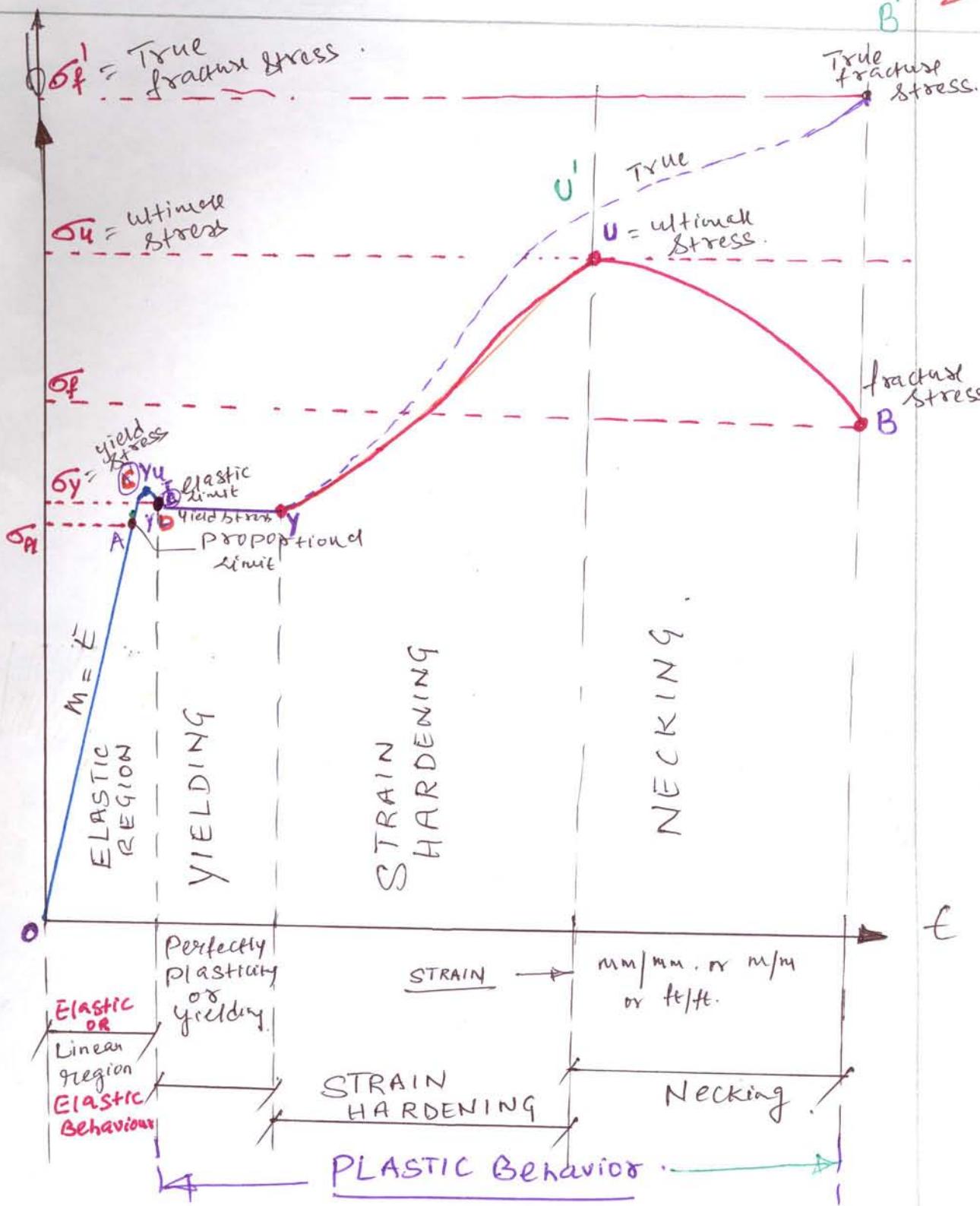
Various apparatus used are as Universal testing machine - UTM - Tension, compression, bending, shear machine - CTM - Compression Testing machine, Torsion Testing machine, Impact Testing machine, Hardness Tester, etc.

The Strength of a material depends on its ability to sustain a load without undue deformation or failure. This property is inherent in the material itself and must be determined by Experiment.

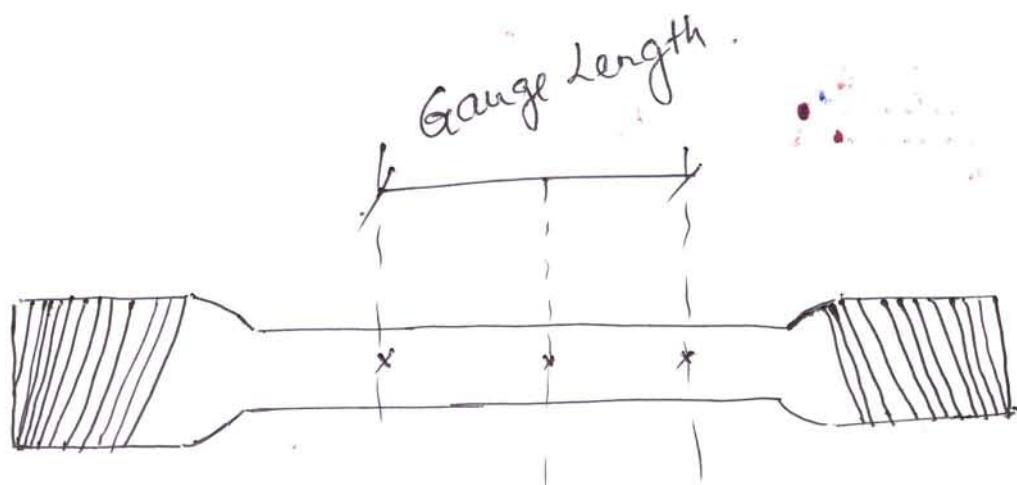
One of the most important tests to perform in this regard is Tension or compression Test.

Although many important mechanical properties of material can be determined from this test, it is used primarily to determine the relationship between Average normal stress and average normal strain in many engineering materials such as Metals, Ceramics, Polymers and Composites (RCC).





Conventional or Engineering And True Stress-Strain diagram for Ductile Material - Steel (Ferrous) under Tensile Load application.



Mild Steel Specimen.

of Standard Shape - Circular, Rectangular, Square.

- Test on Standard Specimen, conducted using Universal testing machine upto destruction.
- The ends of the specimen are gripped in the UTM and one of the grips is moved apart, thus exerting tensile load on the Specimen.
- The Load is applied and recorded through Dial gauges.
- The Elongation or Extension is recorded for applied loads @ each stages / increments in applied load.
- During initial stages → use Extensometer / Strain gauges - electronic (This is fixed along the specimen between gauge length) - electric resistance strain gauge.
- Later stages - use Dial gauge / scale fixed on UTM.
- From Loads applied and deformation measured from start to failure calculate S.F.L & Plot it.

Engineering Stress and Strain (Nominal) And True Stress and Strain.

$$\text{Engineering Stress (Nominal Stress)} = \frac{\text{APPLIED Load}}{\text{Original or initial area of } \sigma_s} = \frac{P}{A_0}$$

$$\text{Engineering Strain (or Nominal Strain)} = \frac{\Delta \text{ or } \delta \text{ or } \delta l}{L_0}$$

= $\frac{\text{change in length}}{\text{original length}}$

$$\text{True Stress} = \frac{\text{Applied Load}}{\text{actual } \sigma_s \text{ @ the time of load measurement}} = \frac{P}{A_{xx}}$$

$$\text{True Strain} = \frac{\text{Change in length } \Delta \text{ or } \delta \text{ or } \delta l}{\text{Specimen gauge length } @ \text{the time of load measured.}} = \frac{\delta l}{L_{xx}}$$

Stress - strain behaviour of Ductile ferrous material under normal temperature is shown in the graph. The behaviour of Mild Steel under Tensile Load can be summarised under:

(A) ELASTIC Behaviour

From O to A

- Initially the curve is a straight line between O to A. It is linear and also proportional. i.e. Stress is proportional to strain between point O to A.
- Beyond point A, the proportionality between stress and strain no longer exist.
- Hence Point 'A' is called Proportionality limit. & Stress @ A σ_A = Stress @ Elastic Limit or Proportionality limit

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For Low Carbon Steel - Range is 210 MPa to 350 MPa
For High Strength Steel (prestressing wire) > 550 MPa

- This Zone is referred as Elastic Zone. because.
If we remove or release External Load applied on the material, Material will regain its original shape (ie It comes back to its original length (for $P=0$))
- The Slope of this Linear Curve gives a Material constant known as Modulus of Elasticity OR Elastic Modulus.
- $\sigma \propto t$, $E = \sigma/t$

→ Between A-C-D-Y - yielding zone.

- [Point A-C] → Proportionality Limit to upper yield point
- At point 'C' there is an increase in strain without appreciable increase in load and curve drops suddenly to point 'D'. Then again with slight increase in load, strain increases slightly and practically constant upto Y with increase in load.
This Phenomenon of increase in load is called yielding.

point C = upper yield point] absent in brittle materials
point D = lower yield point

→ Stress @ point Y = σ_y = yield stress.

→ Stress occurring here is called

→ The deformation occurring here is called plastic deformation.

→ Plastic range $> \frac{10}{600}$ time Elastic range.

→ $\epsilon_{plastic range}$ is called perfectly plastic

→ Zone A-C-D-Y is called perfectly plastic zone. [Deforms without an increase in the applied load]

→ If External load is removed & now, material will not come back to its original shape. permanent set or deformation is created in material.

(C) STRAIN Hardening $\gamma - \nu$

- When yielding has ended, any increase in small external load, the curve rises continuously but becomes flatter until it reaches a maximum referred to as the Ultimate Stress on Stress. Preferred to as the Ultimate Stress on Stress.
- During strain hardening the material undergoes changes in its crystalline structure resulting in increased resistance of the material to further deformation. [Elongation of the test specimen in this region requires an increase in tensile load & therefore the stress-strain diagram has a positive slope]

$U - B$

D NECKING & Failure.

- yield stress \rightarrow yield strength.
- ultimate stress \rightarrow ultimate strength.
- Breaking stress \rightarrow Breaking strength.

D Necking & Failure.

$U + B$

At the ultimate stress, the cross-sectional area begins to decrease. in a localized region of the specimen, instead of over its entire length. This phenomenon is caused by slip planes formed within the material and the actual actual strain produced are caused by shear stress. As a result, Neck gradually tends to form in this region as the specimen elongates further and finally breaks at Fracture Stress. Of specimen breaks at Fracture Stress. Of the failure is cup & cone for elastic material like mild steel.

NOTE

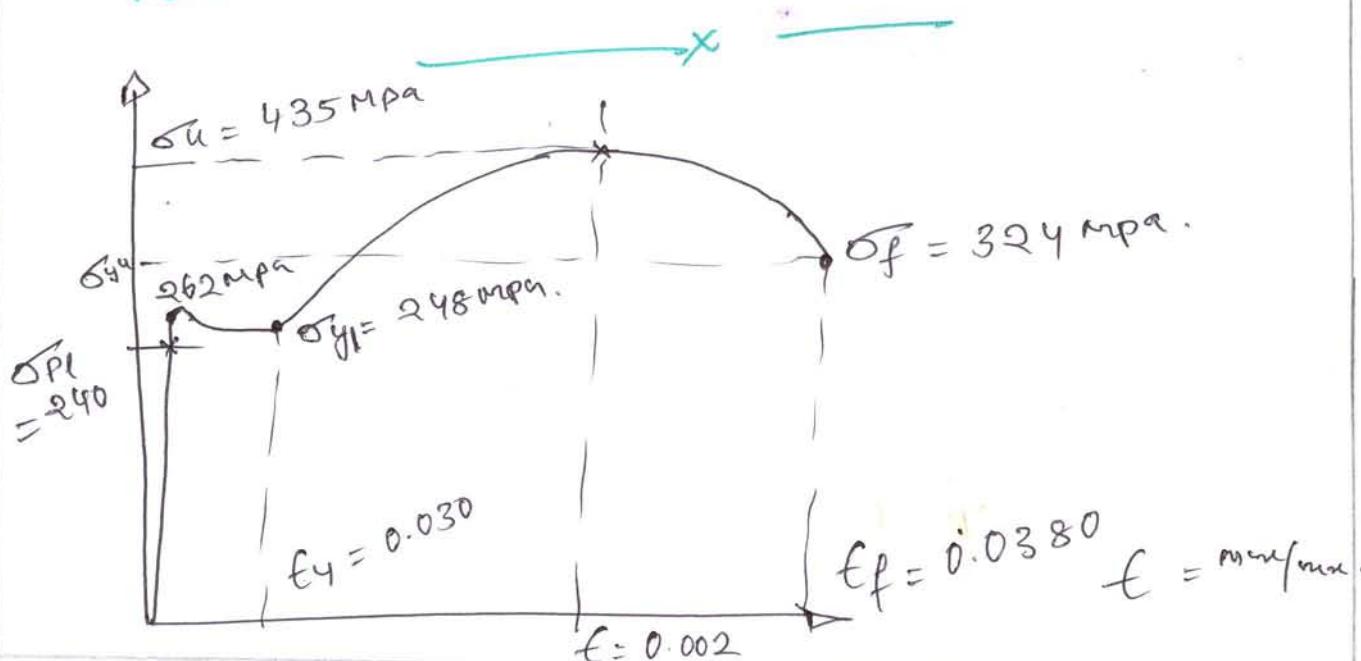
Stress-strain diagram O-A-C-Y-U'-B' is based on True Stress & true strain.

concept.

Steel \rightarrow Ductile-Ferrous Material
presence of clearly defined yield point followed by large plastic strains is an important characteristic

Metals such as structural steel that undergo large permanent strains before failure are classified as ductile.

- "Ductility is that property that enables a bar of steel to be bent into circular arc or drawn into a thin wire without breaking"
- \rightarrow A desirable feature of ductile material is that visible distortion's occur if the load becomes too large, thus providing an opportunity to take remedial action before an actual fracture occurs."
- \rightarrow Material exhibiting ductile behaviour are capable of absorbing large amounts of strain energy prior to fracture.



Ductile Material

Any material that can be subjected to large strains before it ruptures is called ductile material.

Engineers often choose ductile materials because they are capable of absorbing shock or energy and if they become over loaded, they will usually exhibit large deformation before failing.

Measure of Ductility

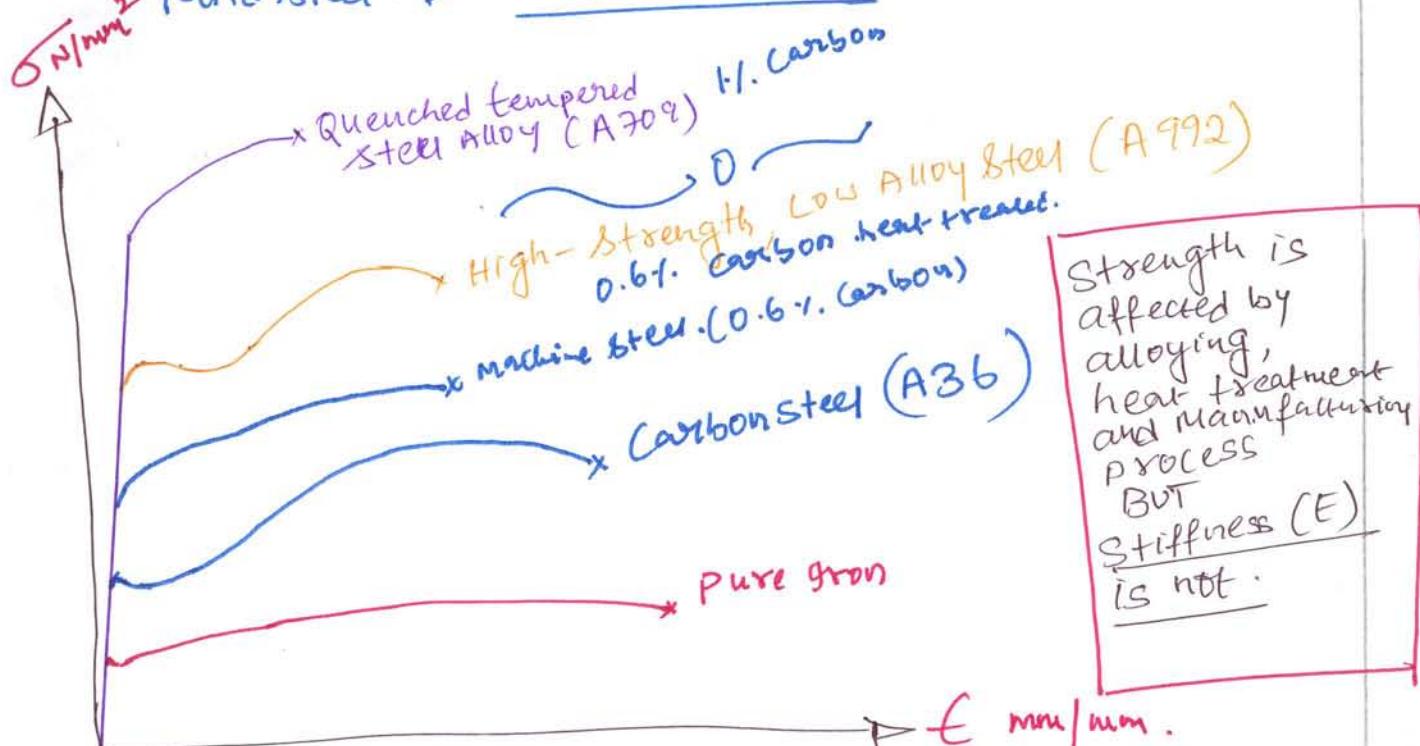
% Elongation \neq % Reduction in area.
@ the time of fracture.

$$\text{Percent elongation} = \frac{L_{\text{fracture}} - L_{\text{original}}}{L_{\text{original}}} \times 100$$

Mild Steel \rightarrow 35% to 40%.

$$\text{Percent reduction in Area} = \frac{A_{\text{original}} - A_{\text{final}}}{A_{\text{original}}} \times 100$$

$\sigma \text{ N/mm}^2$ Mild Steel \rightarrow 55% to 65%.

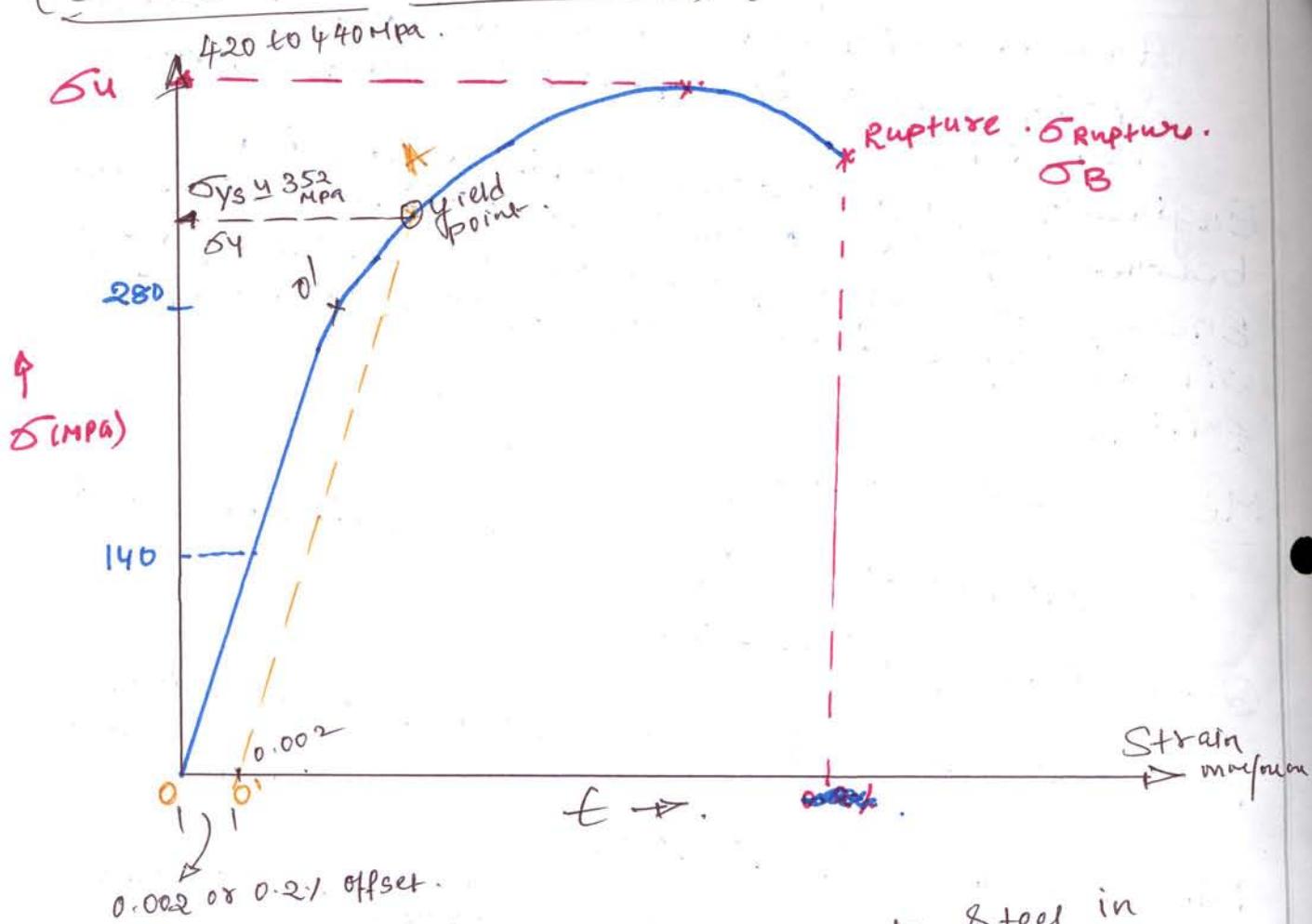


Strength is affected by alloying, heat treatment and manufacturing process
BUT Stiffness (E) is not.

Stress strain diagrams for iron & different grades of steel

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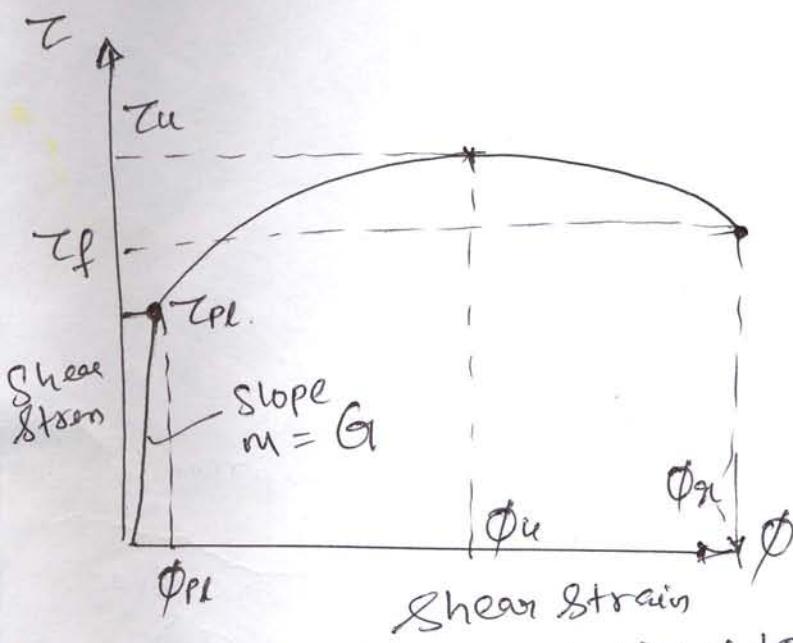
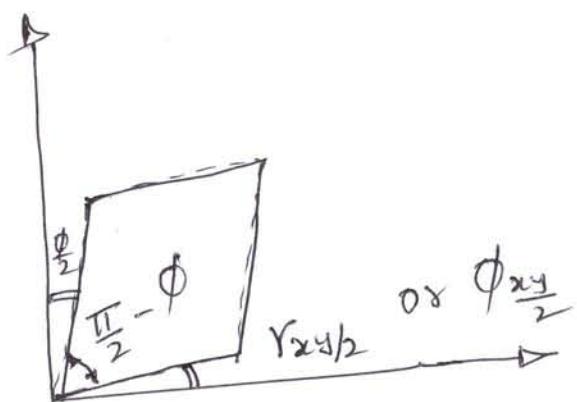
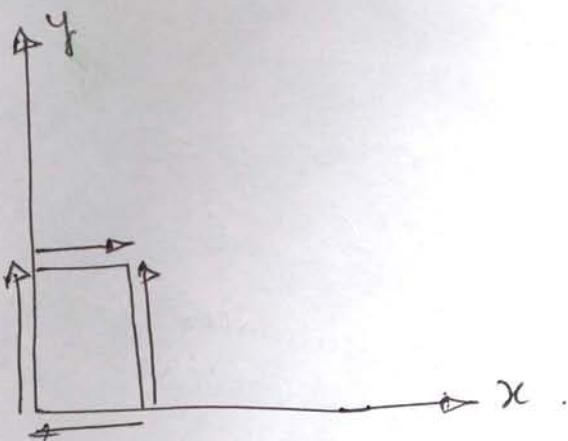
Stress-Strain Diagram for Aluminum Alloy
(Ductile material- Non ferrous) (Tensile Load)



- Behaviour of Aluminium is similar to Steel in elastic zone i.e Stress & Strain is linear.
- Aluminium, Brass, Molybdenum and Zinc will exhibit ductile stress-strain character similar to Steel. Whereby they undergo elastic stress-strain behaviour, yielding @ constant stress, strain hardening and finally necking until rupture or breaking. However constant yielding will not occur beyond its elastic range.
- They do not exhibit well defined yield point similar to Steel.
- Standard practice of defining yield point of such materials is through graphical procedure called "Offset Method".
- In Offset Method, 0.2% strain i.e 0.002 mm/mm is selected from X axis of the graph and a parallel line to ~~is~~ initial line OO' is constructed, to meet the stress-strain curve @ A. From this point, it is projected to Y axis to determine yield stress σ_y & δ_y .

Modulus of Rigidity OR Shear Modulus.

1x5



$$G = \frac{\tau_{PL}}{\phi_{PL}}$$

Behaviour of material subjected to torsion.
& Study of Shear stress vs Shear strain.

Hooke Law $\rightarrow \tau \propto \phi$ within elastic limit
 $\phi \rightarrow \text{radians.}$

$$\boxed{\tau = G\phi}$$

$G = \frac{\text{Shear modulus of elasticity or modulus of rigidity}}{N/m^2, N/mm^2, \text{ Pascal.}}$

G OR C or N

Bulk modulus.

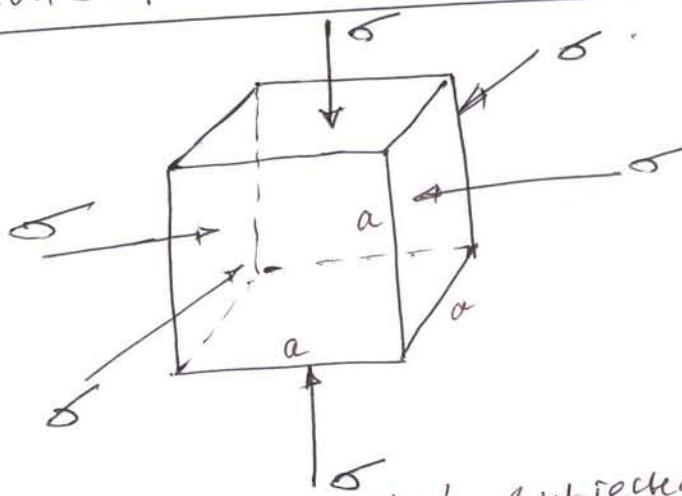
$$K = \frac{\text{Volumetric Stresses}}{\text{Tolumetric Strain}}$$

$$K = \frac{\sigma}{\left(\frac{dV}{V}\right)}$$

If $\sigma_x = \sigma_y = \sigma_z = \sigma$ (Tension or compression)

$$K = \sigma / \epsilon_V, \epsilon_V = \frac{dV}{V}$$

Relationship between volumetric strain & linear strain.



Consider a cube of side 'a' subjected to three mutually perpendicular, equal direct stresses.

Let δa or Δa be the shortening of each side. After straining, each side becomes $(a - \delta a)$.

$$\text{Linear Strain} = \frac{\delta a}{a}$$

$$\begin{aligned} \text{volumetric change} &= (a - \delta a)^3 - a^3 \\ &= a^3 - 3a^2 \delta a + 3a \delta a^2 - \delta a^3 \end{aligned}$$

~~Neglecting higher power of δa~~

$$\text{Volumetric change} = -3a^2 \delta a$$

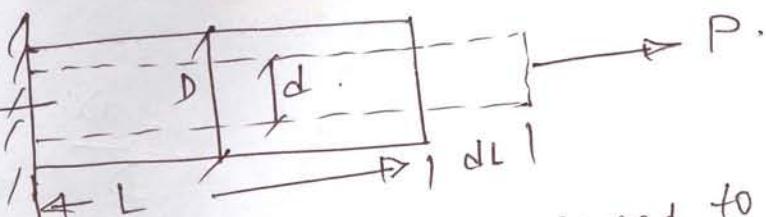
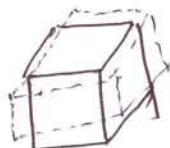
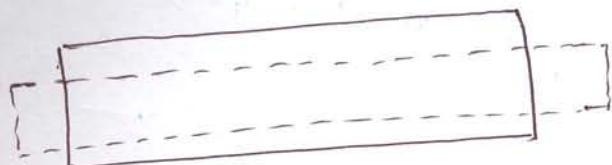
$$\text{Volumetric Strain} = \frac{-3a^2 \delta a}{a^3} = -3 \cdot \frac{\delta a}{a}$$

$$\boxed{\text{Volumetric Strain} = -3 * \text{linear strain.}}$$

$$\therefore \text{Volumetric Strain} = \frac{dV}{V} = \epsilon_{xy} + \epsilon_{yz} + \epsilon_{xz} = \epsilon_x + \epsilon_y + \epsilon_z$$

Relationship between CE, G & K.

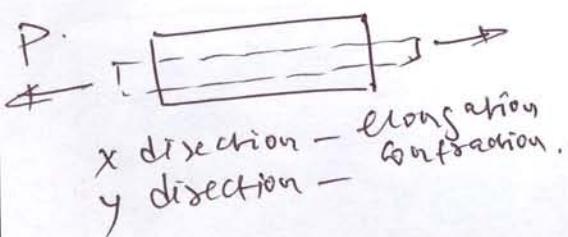
Poisson's Ratio.



When a body is subjected to direct stress, it undergoes deformation in the direction of load. If the observations are made @ rt angles to the direction of load, it can be seen that there is deformation of the opposite kind in the body.

Thus when a bar is subjected to a simple tensile loading. There is an elongation along the length of the bar, simultaneously the width or depth of the bar experiences shortening i.e. shortening in lateral direction.

$\mu = \frac{\text{lateral strain}}{\text{axial strain}}$



$\mu = \frac{\text{lateral strain}}{\text{axial strain}}$

"The Ratio of Strain in the Lateral direction to the Strain in axial direction is defined as Poisson's Ratio"

If is denoted by μ or $\nu = \frac{1}{E}$

$\mu \rightarrow 0.25 \text{ to } 0.33$ for most of the material

$\mu = \frac{\text{Strain in } Y \text{ direction.}}{\text{Strain in } X \text{ direction.}}$
 When Load is applied along X direction.

$$\epsilon_{xy} = \cancel{\epsilon_{yy}}$$

$$\mu_{xy} = \frac{\epsilon_y}{\epsilon_x}, \quad P \rightarrow X \text{ direction.}$$

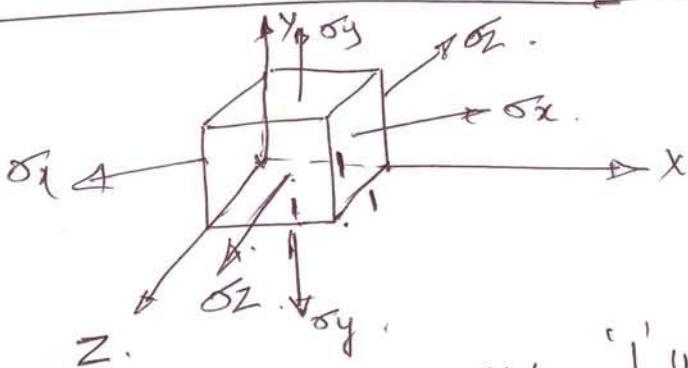
$$\text{Lateral Strain} = \epsilon_y = \frac{D-d}{D} \quad \left[\epsilon_y = \frac{d-D}{D} \text{ (we)} \right]$$

$$\text{Longitudinal Strain} = dL/L.$$

Note.

In case of a prismatic tension member, if the strain in X direction of the load is σ_x/E , then the strain in the other two directions will be $-\frac{1}{m}(\frac{\sigma_x}{E})$ & $-\frac{1}{m}\frac{\sigma_x}{E}$ respectively.

Relationship between $E, G + K$ & μ .



Consider a cube of side '1' unit.
 Subjected to Tensile stresses σ_x, σ_y & σ_z in 3 directions [1] as shown above.

Under the action of These stresses
The body deforms into a rectangular
parallelepiped and its volume is given by.

$$V' = \frac{(1+\epsilon_x)}{L} \cdot \frac{(1+\epsilon_y)}{B} \cdot \frac{(1+\epsilon_z)}{D}$$

$$L = B = D = 1$$

$$V = \text{original volume} = 1$$

$$\therefore V' = (1 + \epsilon_x + \epsilon_y + \epsilon_x \epsilon_y)(1 + \epsilon_z)$$

$$= 1 + \epsilon_x + \epsilon_y + \epsilon_x \epsilon_y + \epsilon_z + \epsilon_x \epsilon_z + \epsilon_y \epsilon_z + \epsilon_x \epsilon_y \epsilon_z$$

Since $\epsilon_x, \epsilon_y, \epsilon_z$ are very small QTY, we can ignore.
 $\epsilon_x \epsilon_y, \epsilon_x \epsilon_z, \epsilon_y \epsilon_z, \epsilon_x \epsilon_y \epsilon_z$.

$$\therefore V' = 1 + \epsilon_x + \epsilon_y + \epsilon_z$$

$$\text{Change in volume} = \Delta V \text{ or } \delta V = \epsilon_x + \epsilon_y + \epsilon_z + 1 - 1$$

$$\boxed{\delta V = \epsilon_x + \epsilon_y + \epsilon_z}$$

$$\text{volumetric strain} = \epsilon_v = \frac{\epsilon_x + \epsilon_y + \epsilon_z}{1}$$

NOW

$$\epsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \mu (\sigma_y + \sigma_z))$$

$$\text{Similarly } \epsilon_y = \frac{1}{E} (\sigma_y - \mu (\sigma_z + \sigma_x))$$

$$\epsilon_z = \frac{1}{E} (\sigma_z - \mu (\sigma_x + \sigma_y))$$

$$\therefore \epsilon_v = \frac{1}{E} (\sigma_x - \mu (\sigma_y + \sigma_z)) + \frac{1}{E} (\sigma_y - \mu (\sigma_z + \sigma_x))$$

$$+ \frac{1}{E} (\sigma_z - \mu (\sigma_x + \sigma_y))$$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\mu}{E} (\sigma_x + \sigma_y + \sigma_z)$$

$$\epsilon_v = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E} (1 - 2\mu)$$

Now If ~~$\sigma_x = \sigma_y = \sigma_z = \sigma$~~ $\sigma_x = \sigma_y = \sigma_z = \sigma$

Then

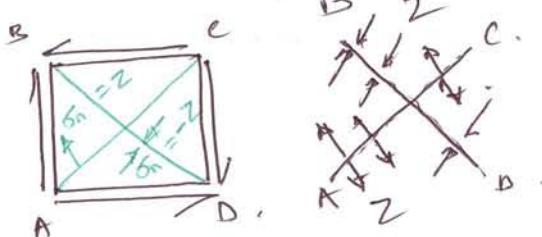
$$\epsilon_v = \frac{3\sigma}{E} (1 - 2\mu)$$

check

$$\text{OR } E = \frac{3\sigma}{\epsilon_v} (1 - 2\mu)$$

But by defn $\frac{3\sigma}{\epsilon_v}$ = products of Bulk modulus K.

$$\boxed{\therefore E = 3K \times (1 - 2\mu)}$$



Consider a Case of pure shear acting on an element.

The Normal compressive & Tensile stresses along the Diagonals BD & AC are indicated in figs.

The magnitude of Normal stress = $\pm \sigma$

Also the Diagonal strain are equal to half the Shear strain.

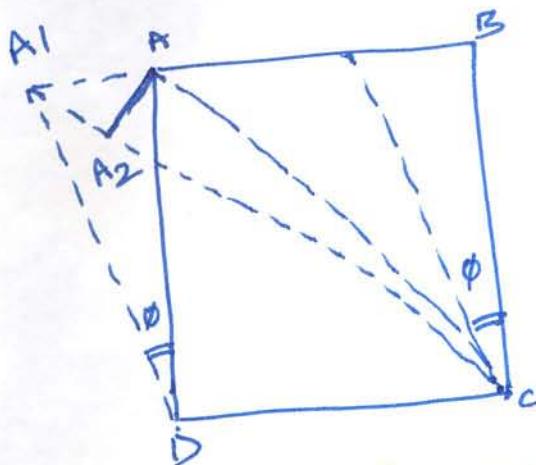
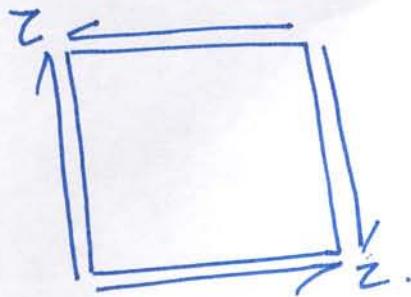
$$\text{ie } \epsilon_{AC} = \phi/2$$

$$\begin{aligned} \epsilon_{AC} &= \phi/2 \\ &= \frac{\tau}{2G} \end{aligned}$$

Shear Stress / Bulk Modulus

$\tau_G = 2\epsilon_{AC}$

Relation Between E & G .



Consider a square block ABCD of side 'a'
and thickness unity \perp to the plane of
paper.

Let the block be subjected to pure shear
stress of intensity Z .

Due to these stresses the block will be
subjected to a deformation such that
the diagonal AC is elongated
and diagonal BD is shortened.

Consider the diagonal AC.

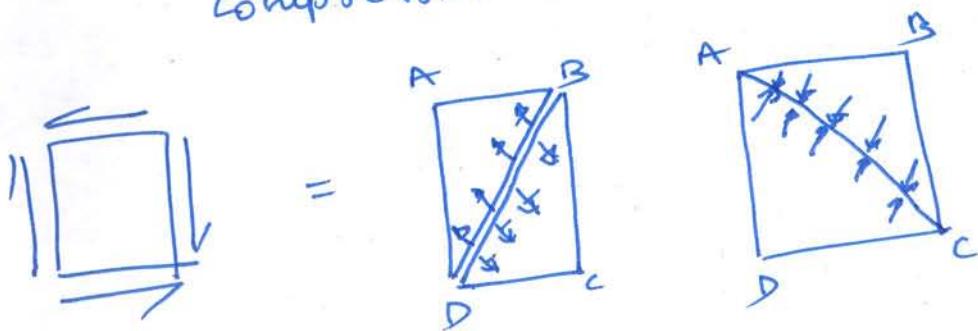
To Compute Increase in Length

of Diagonal AC.

→ Consider the effect of Diagonal tensile & compressive stresses developed along the diagonal due to pure shear stress.

$$\rightarrow \sigma_t = \sigma_c = \sigma_n = \tau.$$

→ Strain in the length of the Diagonal AC = Strain in the length of AC due to diagonal tensile stresses on the plane BD + Strain in the length of AC due to diagonal compressive stresses on the plane AC.



$$\text{Strain of AC} = \frac{\tau}{E} + \mu \frac{\tau}{E}$$

$$= \underline{\underline{\frac{\tau}{E}(1+\mu)}} \rightarrow ①$$

Strain of in diagonal AC from Geometry.

of distorted shape-

Let the block ABCD deforms to the position A₁B₁C₁D through the angle ϕ

Increase in length of the diagonal AC
 $\underline{\underline{= A_1C - AC}}$

Let AA_2 be 1° to A_1C .

Since the angle $\angle A_1CA_2$ is very small

$$\boxed{AC = A_2C}$$

∴ Increase in length of in diagonal $AC = CA_1 - CA_2$

$$= A_1A_2 \\ = AA_1 \cos \angle AA_1A_2$$

But $\angle AA_1A_2 \approx \angle BAC \approx 45^\circ$

∴ Increase in length of in diagonal $AC = CA_1 - CA_2$

∴ Increase in length of the diagonal $AC = AA_1 \cos 45^\circ \\ = \frac{AA_1}{\sqrt{2}}$

$$\text{But Shear strain } \phi = \frac{AA_1}{AD} \\ = \frac{AA_1}{a},$$

$$\therefore AA_1 = a \cdot \phi$$

∴ Increase in length of in diagonal $AC = \frac{a\phi}{\sqrt{2}}$

$$\text{But } AC = a\sqrt{2}$$

Increase in length
Original length

$$\therefore \text{Strain of diagonal} = \frac{AC}{\text{Original length}} = \frac{a\phi}{\sqrt{2}} \cdot \frac{1}{a\sqrt{2}} = \phi/2$$

NOW

$$\frac{\phi}{l_2} = \frac{Z}{E} (1+\mu)$$

$$E = 2 \cdot \frac{Z}{\phi} (1+\mu)$$

$$E = 2G(1+\mu).$$

Since the tensile & compressive Normal Stresses along 2 diagonal's are equal in magnitude = Z .

$$\epsilon_{AC} = \frac{\sigma}{E} + \mu \frac{\sigma}{E}$$

$$= \frac{\sigma}{E} (1+\mu) = \frac{Z}{E} (1+\mu) \checkmark$$

$$\therefore \frac{Z}{2G} = \frac{Z}{E} (1+\mu)$$

$$\boxed{E = 2G(1+\mu)}$$

$$G = \frac{E}{2(1+\mu)}$$

$$\therefore G = \frac{3K(1+2\mu)}{2(1+\mu)}$$

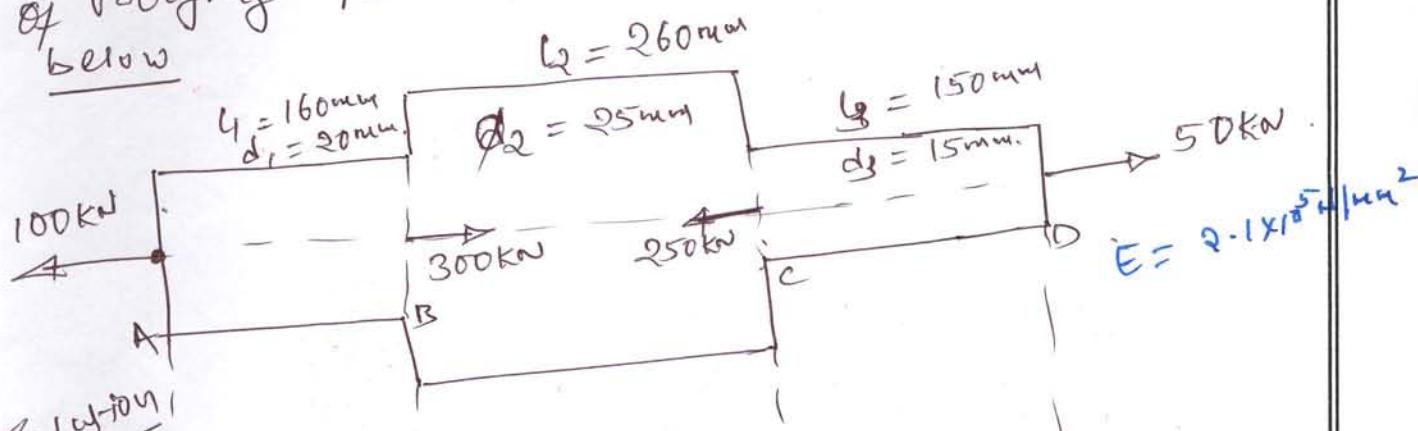
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$$\boxed{E = \frac{9KG}{(3K+G)}}$$

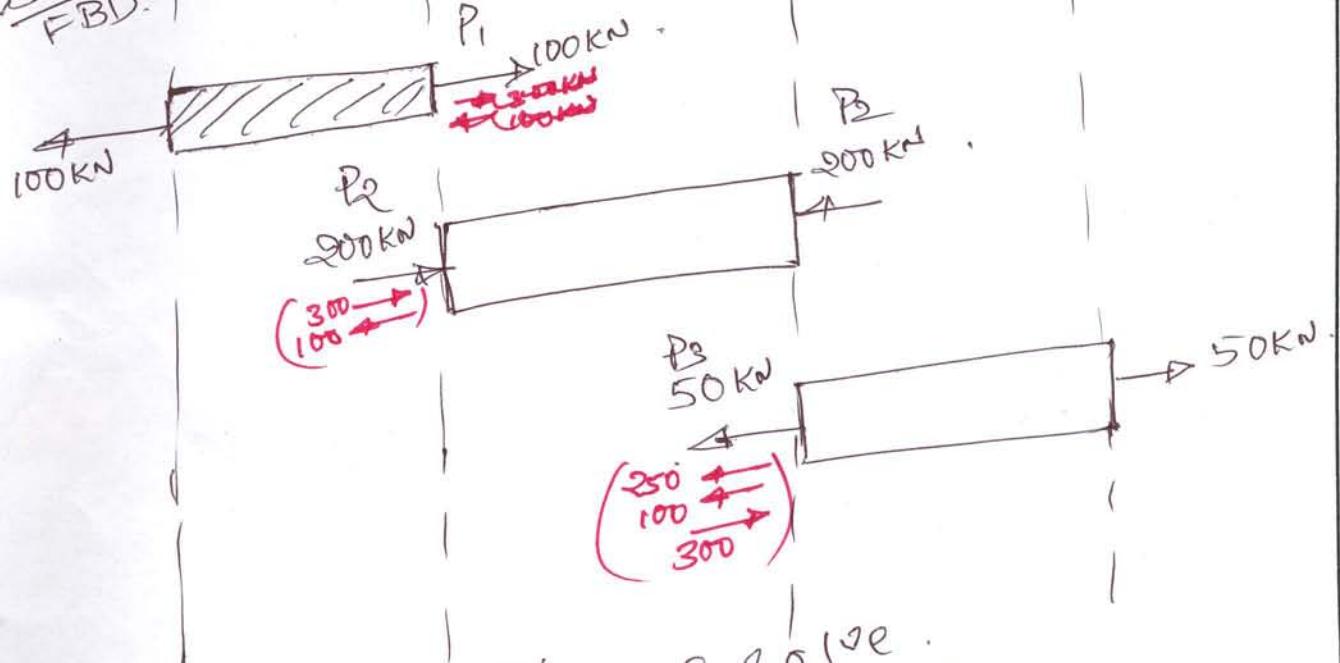
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10CV33
Dec 16 / Jan 17

Determine the Net elongation of a circular bar of varying Q/S subject to forces as shown below



Solution
FBD - I



$$\text{use: } \delta = \sum \frac{PL}{AE} \text{ & } \delta = 0.1\%.$$

$$A_1 = \frac{\pi}{4} \times 20^2 = 314.1593 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4} \times 25^2 = 490.88 \text{ mm}^2$$

$$A_3 = \frac{\pi}{4} \times 15^2 = 176.72 \text{ mm}^2$$

SOLVE Q.B ON SIMILAR LINES

Q3, Q8, Q12, Q27, Q30
Q33, Q38, Q40.
Q42, Q1

$$\delta = \frac{100 \times 10^3 \times 160}{314.1593 \times 2.1 \times 10^5} + \frac{-200 \times 10^3 \times 260}{490.88 \times 2.1 \times 10^5} + \frac{50 \times 10^3 \times 150}{176.72 \times 2.1 \times 10^5}$$

$$\delta = 0.243 \text{ mm} + (-0.505 \text{ mm}) + 0.202 \text{ mm}$$

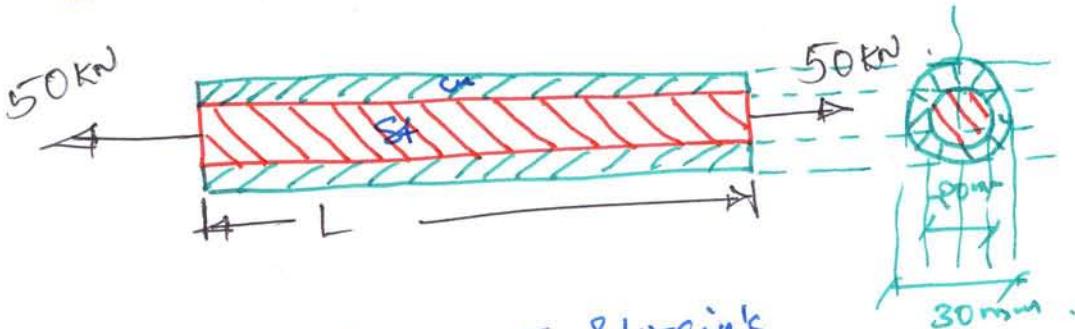
$$\delta = -0.0598 \text{ mm} \quad \underline{\underline{-0.06 \text{ mm}}}$$

QB

Q4

~~Q42~~~~Q42~~

A Compound bar consists of a Steel rod of 20 mm diameter rigidly fitted into a Copper tube of 20 mm outer dia & 5 mm thick. Determine the stresses induced in the different materials when a compound bar is subjected to an axial load (Tensile) of ~~100 KN~~ 50 KN. If $L = 3m$ $E_{st} = 200 GPa$ & $E_{cu} = 120 GPa$. Find $\Delta = ?$



a) Let ϵ_{st} & ϵ_{cu} be the strains in Steel & Copper.

$$\text{Then } \epsilon_{st} = \epsilon_{cu} \rightarrow (1)$$

$$\frac{\sigma_{st}}{E_{st}} = \frac{\sigma_{cu}}{E_{cu}}$$

$$\therefore \sigma_{st} = \frac{E_{st}}{E_{cu}} \sigma_{cu} = \frac{200 \times 10^9 N/m^2}{120 \times 10^9 N/m^2} \sigma_{cu}$$

$$\boxed{\sigma_{st} = 1.67 \sigma_{cu}}$$

(b) Let P_1 & P_2 be the load taken by Steel & Copper respectively.

$$\text{Then } P = P_1 + P_2 = 50 \times 10^3 N.$$

Steel &

$$A_{st} = \frac{\pi}{4} \times 20^2 = 314.2 \text{ mm}^2$$

$$A_{cu} = \frac{\pi}{4} (30^2 - 20^2) \\ = 392.7 \text{ mm}^2$$

$$50 \times 10^3 = \sigma_{st} A_{st} + \sigma_{cu} A_{cu}$$

$$50 \times 10^3 = 1.67 \sigma_{cu} A_{st} + \sigma_{cu} A_{cu} \\ = \sigma_{cu} (1.67 A_{st} + A_{cu})$$

$$\therefore \sigma_{cu} = \frac{50 \times 10^3}{1.67 A_{st} + A_{cu}} = \frac{50 \times 10^3}{1.67 \times 314.2 + 392.7} \\ = 54.51 \text{ N/mm}^2$$

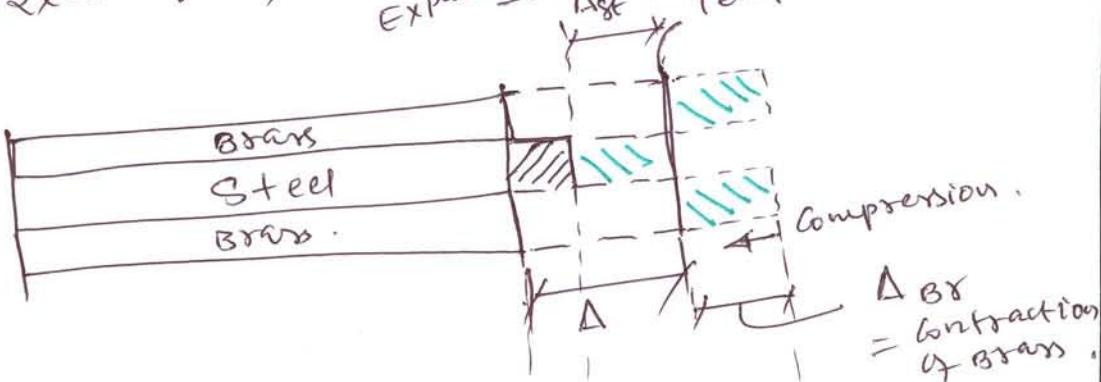
$$\sigma_{st} = 1.67 \times 54.51 = \underline{\underline{91.02 \text{ N/mm}^2}}$$

Average stress
= 1B

10 CR 33

Dec 17 / Jan 18

Q 11
 A steel bar 25mm in diameter is enclosed in a brass tube 25mm internal diameter and 50mm external diameter. Both the bars are of 50mm length 1000mm and rigidly fixed to each other. The composite bar is subjected to rise in temp of 60°C . Determine the stresses due to temp change if in addition to temp change, the bar is subjected to a pull of 60 kN. determine stress due to temp change. $E_{\text{st}} = 200 \text{ kN/mm}^2$, $E_{\text{Br}} = 100 \text{ kN/mm}^2$, $\alpha_{\text{Br}} = 18 \times 10^{-6} / ^{\circ}\text{C}$, $\alpha_{\text{st}} = 12 \times 10^{-6} / ^{\circ}\text{C}$.

Part I

Let σ_{st} & σ_{br} be the stresses in Steel and Brass respectively due to Temperature change.

Since $\alpha_{\text{st}} < \alpha_{\text{Br}}$, Steel is

subjected to compression and Brass is subjected to tension.

Subjected to free expansion of Steel & Brass.

Let Δ be the combined

Steel & Brass.

$$\text{Now } \Delta_{\text{st}} = \Delta - \Delta_{\text{st}} = \Delta - \alpha_{\text{st}} l_{\text{st}} t \rightarrow ①$$

$$\Delta_{\text{Br}} = \Delta_{\text{Br}} - \Delta = \alpha_{\text{Br}} l_{\text{Br}} t - \Delta \rightarrow ②$$

$$\text{Adding } ① + ②, \Delta_{\text{st}} + \Delta_{\text{Br}} = \alpha_{\text{Br}} l_{\text{Br}} t - \alpha_{\text{st}} l_{\text{st}} t \\ = l * t \{ \alpha_{\text{Br}} - \alpha_{\text{st}} \}$$

$$\therefore l_{\text{Br}} = l_{\text{st}}$$

$$\therefore \Delta_{St} + \Delta_{Br} = 1000 \times 60^\circ C \times (18 - 12) \times 10^{-6}$$

$$\Rightarrow \frac{\sigma_{St, \text{yout}}}{E_{St}} + \frac{\sigma_{Br, \text{yout}}}{E_{Br}} = \frac{6000 \times 10^{-6} \times 6}{1 \times 10^5} = 36000 \times 10^{-6}$$

$$\Rightarrow \boxed{\frac{\sigma_{St, \text{yout}}}{Q \times 10^5} + \frac{\sigma_{Br, \text{yout}}}{1 \times 10^5} = 36000 \times 10^{-6} = 1.00036} \quad \rightarrow A.$$

NOW, There is no external load applied.

$$P_{St} + P_{Br} = 0.$$

$$\sigma_{St} A_{St} + \sigma_{Br} A_{Br} = 0.$$

$$\sigma_{St} A_{St} = \sigma_{Br} A_{Br}$$

$$\Rightarrow \sigma_{St} = \sigma_{Br} \cdot \frac{A_{Br}}{A_{St}}$$

$$\Rightarrow \sigma_{St} = \left(\frac{50^2 - 25^2}{25^2} \right) \sigma_{Br} = \frac{(2500 - 625)}{625} \sigma_{Br},$$

$$\boxed{\sigma_{St} = 3\sigma_{Br}.}$$

NOW $\left(\frac{3}{Q \times 10^5} + \frac{1}{1 \times 10^5} \right) \sigma_{Br} = 36000 \times 10^{-6}.$

$$\sigma_{Br} = 14.4 \text{ N/mm}^2 \text{ comp.}$$

$$\boxed{\sigma_{St} = 3 \times 14.4 = 43.2 \text{ N/mm}^2}$$

Temp stresses \rightarrow $\boxed{\begin{aligned} \sigma_{St} &= +43.2 \text{ N/mm}^2 \\ \sigma_{Br} &= -14.4 \text{ N/mm}^2 \end{aligned}}$

$$A_{Br} = \frac{\pi}{4} \times (50^2 - 25^2)$$

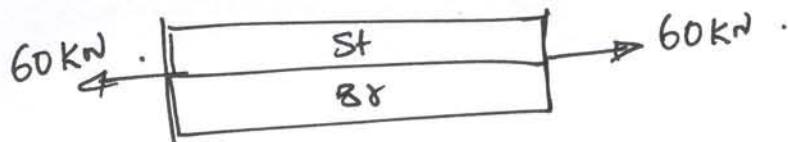
$$A_{St} = \frac{\pi}{4} \times 25^2 \approx 490$$

1473.27
1472.1

0.00036

Part II

Let σ_{St2} & σ_{Br2} be the stresses created in Steel & Brass due to application of external load 60 kN.



Now from strain compatibility condition.

$$\text{Strain of Steel} = \text{Strain of Brass}.$$

(Both Steel & Brass are subjected to Tension.)

$$\epsilon_{Steel} = \epsilon_{Brass}.$$

$$\frac{\sigma_{Steel}}{E_{Steel}} = \frac{\sigma_{Brass}}{E_{Brass}}$$

$$\sigma_{St2} = \frac{E_{st}}{E_{Brass}} \sigma_{Br2}$$

$$\therefore \sigma_{St2} = \frac{2 \times 10^5}{1 \times 10^5} \sigma_{Br2} = 2 \sigma_{Br2}$$

Now Total Load $P = 60 \text{ kN}$ applied, needs to be carried by both Steel & Brass.

$$\therefore 60 \times 10^3 = \sigma_{St2} A_{St} + \sigma_{Br2} A_{Br}$$

$$= 2 \cdot \sigma_{Br2} A_{st} + \sigma_{Br2} A_{Br}$$

$$= \sigma_{Br2} [2 A_{st} + A_{Br}]$$

$$\therefore \sigma_{Br2} = \frac{60 \times 10^3}{(2 \times 490.9 + 1472.6)} = +30.55 \text{ N/mm}^2$$

$$\sigma_{St2} = 2 \times 30.55 = +61.12 \text{ N/mm}^2$$

$$\text{Steel} = \sigma_{st} = \sigma_{St2} + \sigma_{Br2} = 43.2 + 61.12 = 104.32 \text{ N/mm}^2$$

$$\text{Brass} = -14.4 + 30.55 = +16.15 \text{ N/mm}^2$$

NET Stress