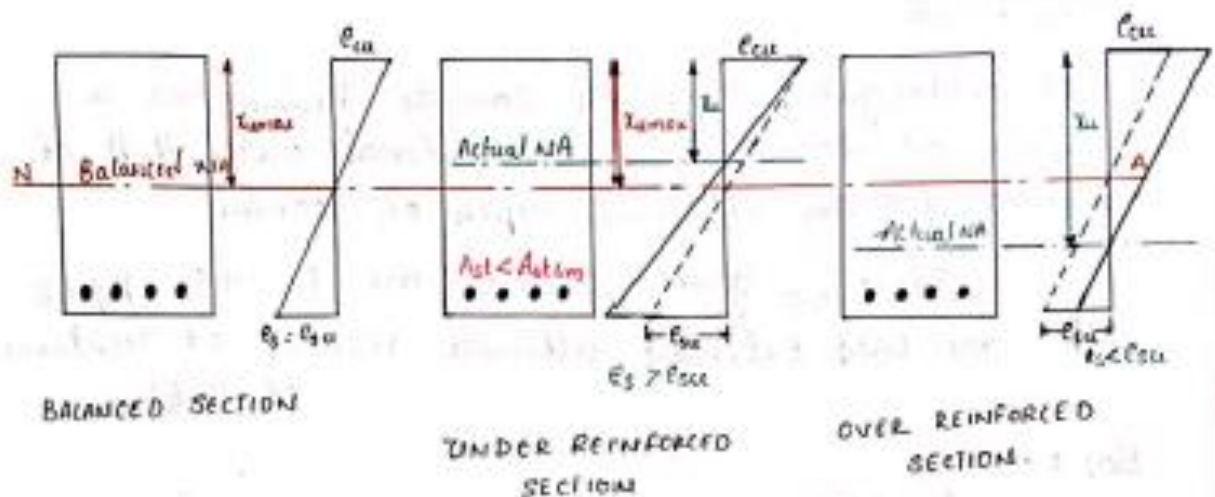


Answer any two full questions

1 (a) Clearly distinguish between under reinforced, balanced reinforced and over reinforced sections with neat sketch.



Balanced Section: The strain in steel and strain in concrete reach their maximum value simultaneously. The  $\epsilon_c = \epsilon_{cu}$  &  $\epsilon_s = \epsilon_{su}$ . The % of steel in this section is known as critical. Limiting steel percentage ( $P_{t,lim}$ ). The depth of neutral axis,  $x_u = x_{u,max}$ .

Under Reinforced Section: is one in which  $P_t$  is less than critical. Limiting percentage. Due to this the actual NA is above the balanced NA &  $x_u < x_{u,max}$ . Hence stress in steel reached reaches first than concrete.

Beam fails by excess yielding of steel. Before beam fails it gives sufficient warning.

Over Reinforced Section: In this type of beam etc., the % of steel is greater than what is required for balanced section. Hence stress in concrete reaches first than steel. Beam fails by crushing of concrete in compression zone. Hence this type of failure is sudden & it won't give warning before it fail.

$\therefore$  IS 456: is not permitting over reinforced design.

- (b) A rectangular RCC beam 300mm wide and 500mm deep is reinforced with 4 bars of 16mm dia. It is freely supported on an effective span of 6m. Determine the max permissible imposed service load. Assuming M20 grade concrete & Fe-500 steel.

Given

$$b = 300 \text{ mm}$$

$$D = 500 \text{ mm}$$

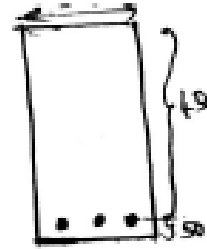
$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.25 \text{ mm}^2$$

$$\text{SSB} \Rightarrow l = 6 \text{ m}$$

$$w_L = ?$$

$$M_{20} \Rightarrow f_{ck} = 20 \text{ MPa}, \quad F_{500} \Rightarrow f_y = 500 \text{ MPa}$$

$$\text{Assume } d' = 50 \text{ mm} \Rightarrow d = D - d' = 500 - 50 = 450 \text{ mm}$$



Compare  $x_u$  &  $x_{u,max}$

$$\frac{x_{u,max}}{d} = 0.46$$

$$x_{u,max} = 0.46 \times 450$$

$$x_{u,max} = 207 \text{ mm}$$

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d} = \frac{0.87 \times 500 \times 804.25}{0.36 \times 20 \times 300 \times 450} = 0.36$$

$\therefore \frac{x_u}{d} < \frac{x_{u,max}}{d} \Rightarrow$  It behaves like under-reinf. section.

Therefore use Eq. 11.1(b)

$$M_u = 0.87 f_y A_{st} b d \left( 1 - \frac{0.44 x_u}{d} \right)$$

$$M_u = 0.87 f_y A_{st} d \left( 1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$= 0.87 \times 500 \times 804.25 \times 450 \left( 1 - \frac{804.25 \times 500}{300 \times 450 \times 20} \right)$$

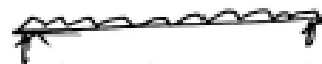
$$M_u = 133.98 \text{ kNm}$$

we know,

$$\frac{8 M_u}{1.5} = \frac{w_g l^2}{8} + \frac{w_L l^2}{8}$$

$$\frac{133.98 \times 8}{1.5} = \frac{2.25 \times 6^2}{8} + \frac{w_L \times 6^2}{8}$$

$$w_L = 16.09 \text{ kN/m}$$



$$w_D = 60 \times 0.5$$

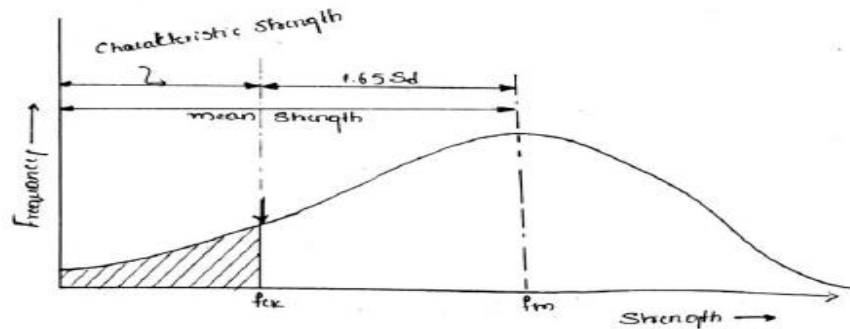
$$= 30 \text{ kN/m}$$

$$w_0 = 3.975 \text{ kN/m}$$

2 (a) Explain the characteristic strength, characteristic load, design strength and design load

### Characteristic strength of materials

Value of strength of material below which not more than a minimum acceptable percentage of test results are expected to fall. Most of design codes adopted the minimum acceptable percentage as 5% for reinforced concrete structures. This implies that there is only 5% probability or chance of the actual strength being less than the characteristic strength or in other words, the characteristic strength has 95% reliability.



$$\text{Characteristic strength} = [\text{Mean strength}] - K \times [\text{Standard deviation}]$$

$$f_k = f_m - K S_d$$

$f_k$  = Characteristic strength of Material

$f_m$  = mean strength       $K$  = constant = 1.65

$S_d$  = Standard deviation for a set of test results.

The value of standard deviation ( $S_d$ )

$$S_d = \sqrt{\frac{\sum \delta^2}{n-1}}$$

$\delta$  = Deviation of the individual test strength from the average strength of  $n$  samples.

$n$  = no. of test results.

Design strength of Material:

The design strength of material ( $f_d$ ) is given by:

$$f_d = \frac{f_k}{\lambda_m}$$

$f_k$  = Characteristic strength of material ( $f_{ck}$  - concrete,  $f_y$  - steel)

$\lambda_m$  = Partial safety factor of material.

### Characteristic load & Design load.

A characteristic load is defined as the value of load which has a 95% probability of not being exceeded during the life of structure.

Thus the characteristic value of a particular load can be calculated theoretically. However research for determining actual loading on structures has not yielded adequate data to enable us to compute theoretical values of variations for arriving @ the actual loading on a structure. Code states that since the data are not available to express loads in statistical terms, the loads given in respective code books are assumed as the characteristic loads:

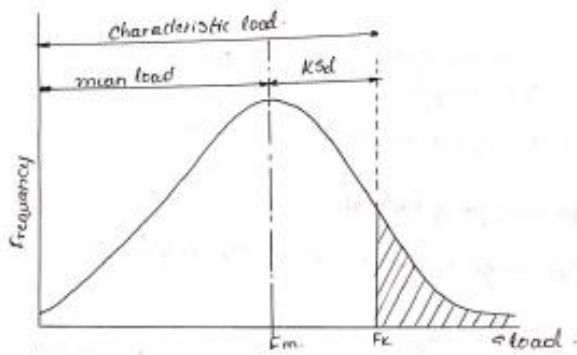
$$F_k = F_m + K S_d$$

$F_k$  = Characteristic load.

$F_m$  = Mean load.

$K$  = constant = 2.585  $\approx$  2.55

$S_d$  = Standard deviation for load.



Design loads: The design load ( $F_d$ ) is given by

$$F_d = F_k \cdot k_f \quad (\text{also known as Factored load})$$

$F_k$  = Characteristic load.

$k_f$  = Partial Safety factor.

(b) Derive the all the stress block parameter for a singly reinforced rectangular beam.

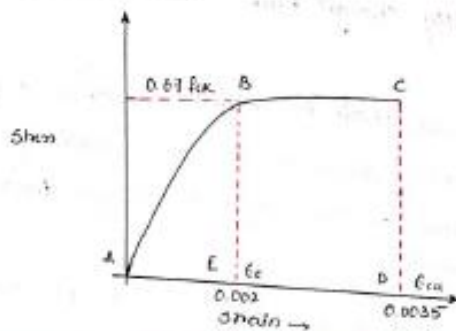
### Stress Block parameter:

The stress strain behaviour of concrete under compression are generally obtained from cylinder or cube of concrete subjected to compression test at loading.

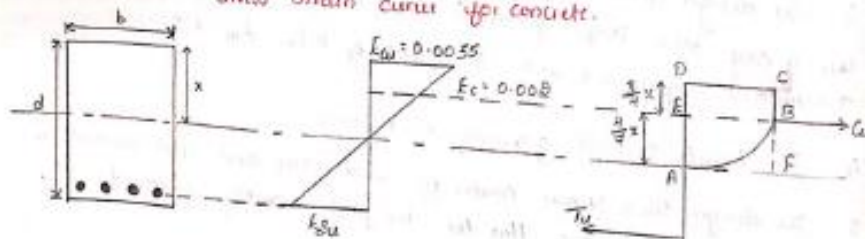
where as the stress and strain are uniform for a cube, they vary across the depth of bending members.

The IS code recommends the compressive strength of concrete in structure =  $0.67 f_{ck}$

The stress strain diagram is as shown in fig.



Stress strain curve for concrete.



Strain and Stress variation across section

fig (b) Shows strain diagram.

fig (c) Shows stress diagram across the section.

The stress block ABCDEA has parabolic part same as stress-strain curve from fig (b) i.e. AB part, & then CB linear part. From CB part of graph = stress of 0.67fc constant.

The total compressive force =  $C_u$  which is below the top fiber & can be expressed in stress block parameter  $K_1, K_2, K_3$ .

$$\text{Then } K_1 = \frac{\text{Shape factor} \times \text{Area of stress block}}{\text{Area of Rectangle}} = \frac{ABCDE}{AFCD}$$

$$K_1 = \frac{\text{Area of ABCD}}{\text{Area of } x \times d} \quad \text{--- (1)}$$

Ultimate strain in concrete =  $0.0035 = \epsilon_{cu} = AD$

Strain at yielding in concrete =  $0.002 = \epsilon_c = AE$  @ stress of  $0.67 f_{ck}$ .

$$\text{The ratio of } \frac{\epsilon_{cu}}{\epsilon_c} = \frac{AE}{AD} = \frac{0.002}{0.0035} = \frac{4}{7}$$

$$\therefore \boxed{AE = \frac{4}{7} AD}$$

$$\text{By } \frac{EP}{AD} = \frac{0.0035 - 0.002}{0.0035} = \frac{3}{7} \quad \therefore \boxed{ED = \frac{3}{7} AD}$$

$$\begin{aligned} \text{Now area ABCD} &= \text{Area of } \triangle ABE + \text{Area BCDE} \\ &= \left(\frac{2}{3} AE \times BE\right) + (ED \times CD) \\ &= \left(\frac{2}{3} \times \frac{4}{7} AD \times CD\right) + \left(\frac{3}{7} AD \times CD\right) \\ &= \frac{8}{21} (AD \times CD) + \frac{3}{7} AD \times CD \\ &= AD \times CD \left(\frac{8}{21} + \frac{3}{7}\right) = \frac{17}{21} AD \times CD \end{aligned}$$

Substituting in eq (1)

$$K_1 = \frac{ABCD}{x \times d} = \frac{AD \times d \times \frac{17}{21}}{AD \times d} = \boxed{\frac{17}{21} = K_1}$$

Resultant compressive force is located @ depth of  $K_2 x$

$$K_2 x = K_2 AD = \frac{\text{Area of parabola} \times \bar{x}_1 + \text{Area of Rectangle} \times \bar{x}_2}{\text{Area of ABCD}}$$

$$= \frac{(\frac{3}{21})(ED + \frac{3}{8}AE) + (\frac{3}{7})(\frac{1}{2}ED)}{\frac{AD \times CD}{AD \times EB}}$$

$$= \frac{(\frac{3}{21})(\frac{3}{4} + (\frac{3}{8} \times \frac{1}{4})AD) + (\frac{3}{7})(\frac{1}{2} \times \frac{3}{7})AD}{\frac{17}{21}}$$

$$= \frac{99}{238} = 0.416 \approx 0.42$$

$$K_2 x = 0.42 AD$$

$$K_2 = 0.42$$

\* To find compressive force:

$$\frac{\text{Force @ Load}}{\text{Area}} = \text{Stress}$$

$$C_u = \text{Stress} \times \text{Area}$$

$$= f_{cc} \times K_1 x \times b$$

$$= 0.67 f_{cc} \times 0.81 x \times b$$

$$C_u = 0.542 f_{cc} b x_u$$

for design =  $\frac{0.542 f_{cc} b x_u}{1.5}$

$$C_u = 0.36 f_{cc} x_u b$$

\* Moment Resistance is given by: when  $x_u/d$  is equal to  $\frac{x_{u,lim}}{d}$

$$M_u = C_u (d - K_2 x_u)$$

$$= 0.36 f_{cc} x_u b (d - K_2 x_u)$$

$x_u = d$

$$M_u = 0.36 f_{cc} x_u b d (1 - \frac{K_2 x_u}{d})$$

$$M_u = 0.36 f_{cc} \frac{x_u b d^2}{d} (1 - \frac{0.42 x_u}{d})$$

Ultimate moment of Resistance in terms of concrete compressive strength.

Explain the design philosophy and design principle.

### Design Philosophies:

with regard to reinforced concrete design, a design philosophy is built upon a few fundamental premises (assumption) and is reflective of way of thinking.

The earliest codified design philosophy is the "working stress method of design (WSM)". close to a hundred years old, this traditional method of design, based on "linear elastic theory" it is used in some countries, although it is now sidelined by modern "Limit state method". In IS 456: 2000 The Working stress method shifted from main text side to (Annex B) to emphasis more on "Limit state Design Method".

3 (a)

Historically, the design procedure to follow the WSM was Ultimate Load method of design, which was developed in early 1950's.

Based on strength of reinforced concrete @ ultimate loads, this method was introduced as an alternative to WSM in ACI code in 1956 and British code in 1957, and subsequently in IS 456:1964.

The philosophy was based on the theory that the various uncertainties in design could be handled more rationally in mathematical framework of probability theory. This risk involved in design was quantified in terms of a probability of failure. Such probabilistic method came to be known as "reliability based".

In order to gain code acceptance, the probabilistic reliability-based approach had to be simplified and reduced to a deterministic format involving multiple (partial) safety factors. European committee for concrete and International Federation for protection of steel were among earliest to introduce the philosophy of "limit state method" of design which is reliability based concept.

Thus, the past several decades have witnessed an evolution in design philosophy from traditional 'Working Stress Method', through 'Ultimate Load method' to modern 'Limit State Method' of design.

- (b) A continuous T beam has the cross-sectional dimension of breadth of flange 1500mm, depth of flange=100mm, web width=300mm. Factored moment is 800kN-m. Determine the flexural reinforcement at mid span of the beam considering M<sub>25</sub> grade concrete and Fe 415 steel. Adopt the beam exposed to moderated exposure condition

$$\begin{aligned} x_{umax} &= 0.48 \times 618 \\ &= 296 \text{ mm} \end{aligned}$$

$$\begin{aligned} \therefore d &= 700 - 40 - 41.6 \\ &= 618 \text{ mm} \end{aligned}$$

- As  $x_{umax} > D_f = 100 \text{ mm}$ , the condition  $x_u = D_f$

Satisfy  $x_u \leq x_{umax}$

- Assuming M<sub>25</sub> concrete,  $f_{ck} = 25 \text{ MPa}$ .

$$\begin{aligned} (M_{uR})_{x_u = D_f} &= 0.36 \times 25 \times 1500 \times 100 \times (618 - 0.42 \times 100) \\ &= 782.5 \times 10^6 \text{ N-mm} < M_u = 800 \text{ kN-m} \end{aligned}$$

$$\Rightarrow x_u > D_f \text{ and } M_u = C_{us}(d - 0.42x_u) + C_{uf}(d - y_f/2)$$

$$\text{where } C_{us} = 0.36 f_{ck} b_w x_u = 0.36 \times 25 \times 300 x_u = (2715 x_u) \text{ N}$$

$$\begin{aligned} C_{uf} &= 0.45 f_{ck} (b_f - b_w) y_f = 0.45 \times 25 \times (1500 - 300) y_f \\ &= (13410 y_f) \text{ N} \end{aligned}$$

- Considering  $x_u = \frac{7D_f}{3} = 233 \text{ mm}$  ( $< x_{u\max} = 296 \text{ mm}$ )

$$y_f = D_f = 100 \text{ mm}$$

$$\Rightarrow (M_{uR})_{x_u} = \frac{7D_f}{3} = (2715 \times 233) (618 - 0.42 \times 233) + (13410 \times 100) \times (618 - \frac{100}{2})$$

$$= 1091.3 \times 10^6 \text{ Nmm} > M_u = 800 \text{ kNm}$$

- Evidently,  $D_f < x_u < \frac{7}{3} D_f$ , for which  $y_f = 0.15x_u + 0.65D_f$ .

$$C_{uf} = 13410(0.15x_u + 0.65 \times 100) = (2011.5x_u + 871650) \text{ N}$$

$$M_u = 800 \times 10^6 = (2715x_u) (618 - 0.42x_u) + (2011.5x_u + 871650) \times [618 - (0.15x_u + 65)/2]$$

$$= -1280.3x_u^2 + 2790229.5x_u + 510.35 \times 10^6$$

Solving this quadratic equation.

$$x_u = 109.3 \text{ mm} < x_{u\max} = 296 \text{ mm}$$

$$y_f = 0.15x_u + 65 = 81.4 \text{ mm}$$

- Applying  $T_u = 0.87 f_y A_{st} = C_{uo} + C_{uf}$ .

$$(A_{st})_{req} = \frac{(2715 \times 109.3) + (13410 \times 81.4)}{0.87 \times 415} = 3845 \text{ mm}^2$$

i.e. The reinforcement (5-32 $\phi$ ,  $A_{st} = 4020 \text{ mm}^2$ )