

USN



Internal Assessment Test 3 – Nov. 2018

Sub:	Fluid Mechanics				Sub Code:	15CV33	Branch:	Civil	
Date:	22/11/2018	Duration:	90 mins	Max Marks:	50	Sem/Sec:	III A and B	OBE	
<u>Answer any FIVE FULL Questions</u>							MARKS	CO	RBT
1 (a)	Define Hydraulic Gradient Line and Total Energy Line. Explain with a neat sketch.					[05]	CO5	L2	
(b)	Water flowing through a rigid pipe of diameter 500 mm with 1.5 m/s is suddenly brought to rest. Find the instantaneous pressure rise if $K_{\text{water}} = 2 \text{ GPa}$.					[05]	CO5	L2	
2	Derive an expression for pressure rise due to sudden closure of valve when the pipe is elastic.					[10]	CO3	L2	
3 (a)	Derive an expression for head loss due to sudden enlargement in a pipe flow.					[05]	CO4	L2	
(b)	Write a note on cippolletti weir.					[05]	CO5	L2	
4	Derive the expression for discharge through a trapezoidal notch.					[10]	CO5	L2	

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- 5 Two tanks are connected with help of two pipes in series. The lengths of pipes are 1000m and 800m respectively. The coefficient of friction for both the pipes is 0.008. The difference of water level in two tanks is 15m. Find the rate of flow through pipes, considering all losses.
- 6 (a) A rectangular notch 40 cm long is used for measuring a discharge of 30 lps. An error of 1.5 mm was made while measuring the head over the notch. Calculate the percent error in the discharge. Take $C_d = 0.6$.
- (b) Derive the expression for discharge through a small orifice of area ' a ' under a head ' h ' measured above the centre of the orifice.

[10]

CO5	L4
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[05]

[05]

CI

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[05]

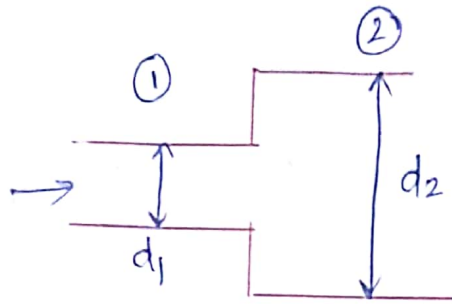
[05]

CI

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HOD

9(c) (8)



$$Q = 0.025 \text{ m}^3/\text{s}$$

$$d_1 = 0.2 \text{ m}$$

$$d_2 = 0.4 \text{ m}$$

$$a_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$a_2 = \frac{\pi}{4} \times 0.4^2 = 0.125 \text{ m}^2$$

$$p_1 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$$

$$v_1 = \frac{Q}{a_1} = \frac{0.025 \text{ m}^3/\text{s}}{0.0314} = 0.796 \text{ m/s}$$

$$v_2 = \frac{Q}{a_2} = \frac{0.025}{0.125} = 0.199 \text{ m/s}$$

(i) Loss of head due to sudden enlargement, h_e

$$h_e = \frac{(v_1 - v_2)^2}{2g} = \frac{(0.796 - 0.199)^2}{2 \times 9.81} = \boxed{0.01816 \text{ m}}$$

(ii) Pressure intensity at larger pipe, P_2
Applying Bernoulli's eqn at (1) & (2)

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_e \quad (z_1 = z_2, \text{ horizontal pipe})$$

$$\left(\frac{P_1}{\rho g} + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} - h_e \right) \rho g = P_2$$

$$P_2 = \left(\frac{11.772 \times 10^4}{10^4 \times 9.81} + \frac{0.796^2}{2 \times 9.81} - \frac{0.199^2}{2 \times 9.81} - 0.01816 \right) \times 10^3 \times 9.81$$

$$P_2 = 117338 \text{ N/m}^2 = 11.73 \text{ N/cm}^2$$

10 (a) (4)

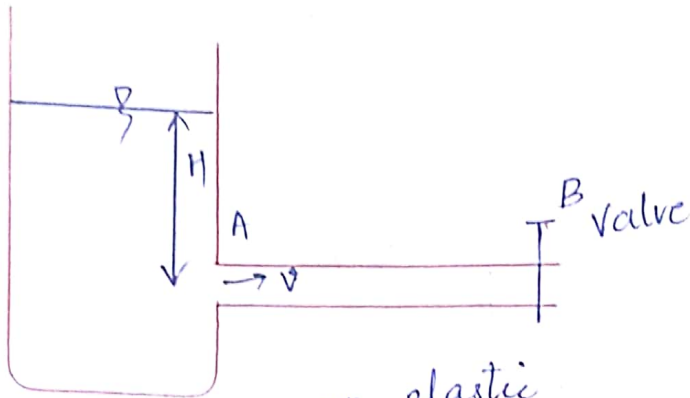
(i) Hydraulic gradient line (HGL) - It is defined as the line which gives the sum of pressure head $\left(\frac{p}{\rho g}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line. It is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head $\left(\frac{p}{\rho g}\right)$ of a flowing fluid in a pipe from the centre of the pipe.

(ii) Total Energy line (TEL) - It is defined as the line which gives the sum of pressure head $\left(\frac{p}{\rho g}\right)$, kinetic head or velocity head $\left(\frac{v^2}{2g}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line.

It is the line which is obtained by joining the top of all vertical ordinates showing the pressure head $\left(\frac{p}{\rho g}\right)$ and kinetic head $\left(\frac{v^2}{2g}\right)$ from the centre of pipe.

10 (B)

Pressure loss due to sudden closure of valve when the pipe is elastic (8)



Consider the pipe AB in which water is flowing as shown in figure.

The loss of kinetic energy, $E_1 =$
 gain of strain energy of water, E_2
 + gain strain energy stored in pipe material E_3 .

$$E_1 = E_2 + E_3 \quad \text{--- (1)}$$

Let

ρ - density of water
 d - diameter of flow (pipe)
 A - Area of flow

L - length of pipe

t - thickness of pipe wall

v - velocity of flow

P - Increase of pressure due to water hammer

K - Bulk modulus of water

E - Modulus of elasticity of pipe material

$1/m = 1/4$ Poisson's ratio

Calculation of Energy (Kinetic Energy)

1) Loss of K.E, E_1

$$E_1 = \frac{1}{2} m v^2 = \frac{1}{2} \rho A L v^2 \quad \text{--- (1)}$$

2) Gain of strain energy of water, E_2

$$E_2 = \frac{1}{2} \left(\frac{P^2}{K} \right) \times \text{vol}$$

$$= \frac{1}{2} \left(\frac{P^2}{K} \right) A L \quad \text{--- (2)}$$

3) Strain energy stored in pipe material, E_3

$$E_3 = \frac{1}{2E} \left[f_l^2 + f_c^2 - \frac{2f_l f_c}{m} \right] \times \text{vol} \quad \text{(3)}$$

where

f_l - longitudinal stress in pipe = $\frac{pd}{4t}$

f_c - circumferential stress in pipe = $\frac{pd}{2t}$

$\text{vol} = \pi d L t$ $\frac{1}{m} = \frac{1}{4}$, $\text{vol} = \pi d L t$ ~~$\text{vol} = \pi d L t$~~

Sub f_l , f_c and $1/m$ in (3)

$$E_3 = \frac{1}{2E} \left[\frac{p^2 d^2}{16 t^2} + \frac{p^2 d^2}{4 t^2} - \frac{2 p^2 d^2}{4 \times 4 \times 2 t^2} \right] \pi d L t$$

$$= \frac{p^2 d^2}{E t^2} \frac{1}{2} \left[\frac{1}{16} + \frac{1}{4} - \frac{1}{8} \right] \pi d L t$$

$$= \frac{p^2 d^3 \pi L}{8Et} \quad \text{Sub } A = \frac{\pi d^2}{4}$$

$$= \frac{p^2 d AL}{2Et} \quad \text{--- (4)}$$

Sub (1), (2) and (4) in eqn

$$E_1 = E_2 + E_3$$

$$\frac{1}{2} p AL v^2 = \frac{1}{2} \left(\frac{p^2}{k} \right) AL + \frac{1}{2} \frac{p^2 d AL}{Et}$$

dividing by $\frac{AL}{2}$

$$p v^2 = \frac{p^2}{k} + \frac{p^2 d}{Et}$$

$$p v^2 = p^2 \left(\frac{1}{k} + \frac{d}{Et} \right)$$

$$p^2 = \frac{p v^2}{\left(\frac{1}{k} + \frac{d}{Et} \right)}$$

$$p = \frac{v \sqrt{p}}{\sqrt{\left(\frac{1}{k} + \frac{d}{Et} \right)}}$$

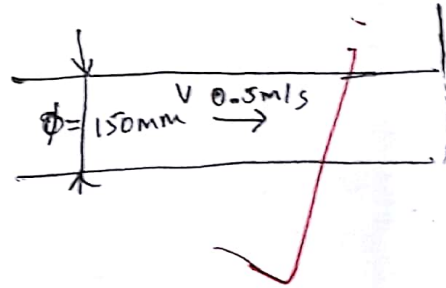
6) Water is flowing in a pipe of 150mm dia with a velocity of 0.5 m/s when it is suddenly brought to rest by closing valve. Find the pressure rise assuming the pipe is elastic $E = 20667 \text{ N/m}^2$. Poisson's ratio = 0.25 ~~K for~~

K for water 6 N/m^2

Given $E = 206 \times 10^3 \text{ N/m}^2$

$\mu = 0.25$

$K = 2.06 \times 10^3 \text{ N/m}^2$



$D = 150 \times 10^{-3}$

$$P = \frac{V \rho}{\sqrt{\frac{1}{K} + \frac{\rho}{ET}}} = \frac{2.5 \sqrt{1000}}{\sqrt{\frac{1}{2.06 \times 10^9} + \frac{150 \times 10^{-3}}{206 \times 10^9}}}$$

$$P = 35.85 \times 10^3 \text{ N/m}^2$$

... 100mm of 0.4 and 600

neglect 0

$$\text{At sec 1} \Rightarrow \frac{P_1}{\rho g} + z_1 = 30.67 + 0 = 30.67 \text{ m}$$

$$\text{At sec 2} \Rightarrow \frac{P_2}{\rho g} + z_2 = 0 + 15 = 15 \text{ m}$$

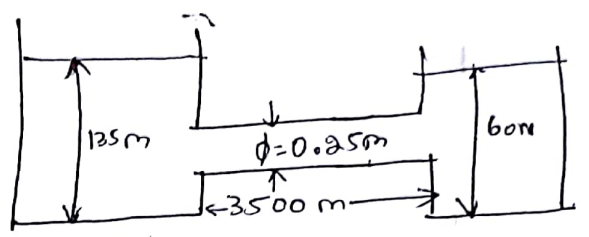
Q) A 250mm diameter, 3km long straight pipe runs b/w two reservoirs of surface elevation 135m & 60m. A 15km long 300mm diameter is laid parallel to 250mm diameter pipe from its mid point to lower reservoir. Find the increase in discharge caused by addition of 300mm diameter pipe.

∴ $H = 135 - 60 = 75 \text{ m}$

$$Q_B = AV = \frac{\pi \times 0.25^2}{4} \times V = 0.049 \text{ m}^2$$

$$h_f = \frac{fLV^2}{2gD}$$

$$h_f = H = 135 - 60 = 75 \text{ m}$$

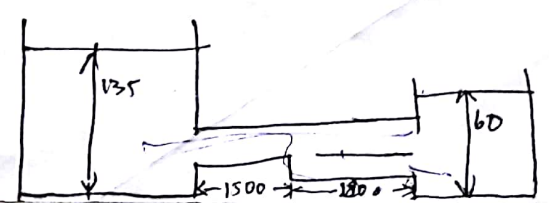


$$75 = \frac{0.02 \times 3000 \times V^2}{2 \times 9.81 \times 0.25}$$

$$V_B = 2.47 \text{ m/s}$$

$$Q_B = A_1 V = 0.049 \times 2.47 = 0.121 \text{ m}^3/\text{s}$$

$$Q_A = Q_{A1} + Q_{A2}$$



$$V = \frac{4fL A_1 V_{A1}^2}{2g D A_1} = \frac{V_{A2}^2 4fL A_1}{2g D A_2}$$

$$\frac{V_A^2}{0.25} = \frac{V_{A2}^2}{0.3}$$

$$V_{A2}^2 = \sqrt{\frac{0.3}{0.25}} V_{A1} \quad \text{--- (1)}$$

$$Q = Q_{A1} + Q_{A2}$$

$$\frac{\pi}{4} \times 0.25^2 V_A = \frac{\pi}{4} \times 0.25^2 V_{A1} + \frac{\pi}{4} \times 0.3^2 V_{A2} \quad \text{--- (2)}$$

sub (1) in (2)

$$\text{we have } 0.25^2 V_A = 0.25^2 V_{A1} + 0.3^2 \sqrt{\frac{0.3}{0.25}} V_A$$

$$\Rightarrow V_A = 2.57 V_{A1}$$

$$V_{A1} = 0.389 V_A$$

considering pipe XYZ $H = h_{fA} + h_{fB}$

$$H = \frac{fL A V_A^2}{2g D_A} + \frac{fL_1 V_{A1}^2}{2g D_1} \Rightarrow 75 = \frac{0.02 \times 1500 V_A^2}{2 \times 9.81 \times 0.25} + \frac{0.02 \times 1500 (0.389 V_A)^2}{2 \times 9.81 \times 0.25}$$

$$75 = \frac{0.02 \times 1500}{2 \times 9.81} \left[\frac{V_{A1}^2 + V_{A2}^2}{0.25} \right]$$

$$V_{A1}^2 + 0.389 V_{A1}^2 = \frac{75}{6.162}$$

$$V_A = \sqrt{\frac{10.2262}{1.389}} = 3.26 \text{ m/s}$$

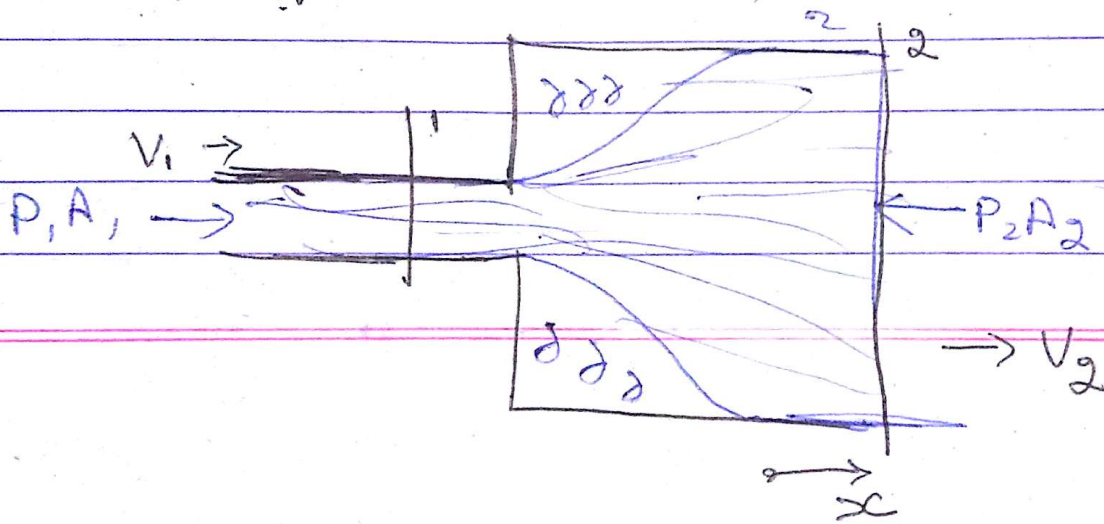
$$Q_A = \frac{\pi}{4} \times 0.25^2 \times 3.26$$

$$= 0.16 \text{ m}^3/\text{s}$$

$$\% \text{ increase} = \frac{Q_A - Q_B}{Q_B} \times 100 = \frac{0.16 - 0.21}{0.12} \times 100$$

$$= 32.26\%$$

Minor loss
loss of head due to sudden Expansion



Consider a liquid flowing through a pipe which has a sudden enlargement of diameter as shown in figure. Consider two sections 1 & 2 before and after the enlargement. Let P_1 and P_2 be the pressure section at ① & ②. V_1 & V_2 be the velocity at ① & ②, A_1 & A_2 be the area of pipe at ① & ②. Due to sudden change in area from A_1 to A_2 the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary of the pipe. Let the flow separates from the boundary and turbulent eddies are formed as shown in figure. The loss of Energy or head takes place due to the formation of eddies.

Let P' = The pressure intensity of the liquid eddies in the liquid area $(A_2 - A_1)$

force due to this pressure is given by $P' = P(A_2 - A_1)$
 from lots of experiment it is given that $P' \approx P_1$

Let h_e be the loss of head due to sudden Expansion.

Applying Bernoulli equation at ① & ② we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

Since pipe is horizontal $z_1 = z_2$

$$h_e = \left(\frac{P_1 - P_2}{\rho g} \right) + \left(\frac{V_1^2 - V_2^2}{2g} \right) \quad \text{--- ①}$$

Consider a control volume b/w section ① & ② applying law of motion we have $\sum F_x = M \frac{dv_x}{dt} = \text{Change in momentum per sec}$
 (dt/sec)

$$\Sigma F_x = \text{Change in momentum / sec} \quad \text{--- (2)}$$

$$\begin{aligned} \Sigma F_x &= P_1 A_1 + P_1 (A_2 - A_1) - P_2 A_2 \\ &= P_1 A_1 + P_1 A_2 - P_1 A_1 - P_2 A_2 \quad P_1 \approx P_2 \\ &= A_2 (P_1 - P_2) \end{aligned}$$

$$\Sigma F_x = A (P_1 - P_2) \quad \text{--- (3)}$$

$$\frac{\text{Change in momentum}}{\text{sec } t} = \frac{\text{moment at section 2}}{t} - \frac{\text{moment at section 1}}{t}$$

$$\frac{\text{momentum at sec 2}}{t} = \rho A_2 V_2 V_2 = \rho A_2 V_2^2$$

$$\left(\frac{\rho A V}{t} \right)$$

$$\rho A_1 V_1^2$$

$$\rho A_2 V_2^2 - \rho A_1 V_1^2$$

Equating (3) & (4)

$$(P_1 - P_2) A_2 = \rho A_2 V_2^2 - \rho A_1 V_1^2$$

$$Q = A_1 V_1 = A_2 V_2 \Rightarrow A_1 = \frac{A_2 V_2}{V_1}$$

$$(P_1 - P_2) A_2 = \rho A_2 V_2^2 - \rho A_2 V_2 \frac{V_1^2}{V_2}$$

$$(P_1 - P_2) = \rho (V_2^2 - V_2 V_1)$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{g} - \frac{V_2 V_1}{g} \quad \text{--- (5)}$$

sub (5) in (1)

$$\frac{V_2^2}{2g} - \frac{V_2 V_1}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V_2^2 - 2V_2 V_1 + V_1^2 - V_2^2}{2g} = \frac{(V_1 - V_2)^2}{2g} //$$