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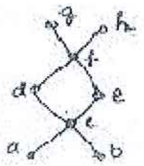
CMRIT CMR INSTITUTE OF TECHNOLOGY, BENGALURU. ACCREDITED WITH A+ GRADE BY NAAC

Internal Assessment Test 2 – Oct. 2018

Sub:	Discrete Mathematical Structures				Sub Code:	17CS36	Branch:	IS	
Date:	17/10/2018	Duration:	90 minutes	Max Marks: 50	Sem/Sec:	III A and B		OBE	

	Question 1 is compulsory and answer any six from Q.2 to Q.10	MARKS	CO	RBT
1	Let $A = \{1,2,3,4,5\}$. Define a relation R on $A \times A$ by $(x_1, y_1) R (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$. Determine the equivalence classes of [(1,3)],[(2,4)],[(1,1)] and the partition of $A \times A$ induced by R.	[08]	CO3	L3
2	Draw the Hasse diagram for the relation defined by aRb if and only if a|b on the set containing positive divisors of 36.	[07]	CO3	L2
3	Consider the Hasse diagram of a POSET (A, R) given below:  Find maximal, minimal, greatest and least elements. If $B_1=\{c, d, e\}$ find all upper bounds, lower bounds, least upper bound, greatest lower bound of B_1.	[07]	CO3	L3
4	Find the negation of "All integers are rational numbers and some rational numbers are not integers."	[07]	CO3	L2

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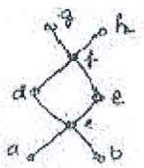
3	Consider the Hasse diagram of a POSET (A, R) given below:  Find maximal, minimal, greatest and least elements. If $B_1=\{c, d, e\}$ find all upper bounds, lower bounds, least upper bound, greatest lower bound of B_1.	[07]	CO3	L3
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5	Let $f: A \to B$ be defined by $f(x) = \begin{cases} 3x-5 \; for \; x > 0 \\ -3x+1 \; for \; x \le 0 \end{cases}$ (i) Find $f(0), f(5/3), f^{-1}(3), f^{-1}(-6), f^{-1}(0)$. (ii) Determine $f^{-1}([-5,5])$.	[07]	CO3	L3
6	Let f, g, h be functions from Z to Z defined by $f(x) = x-1, g(x) = 3x, h(x) = \begin{cases} 0, \; if \; x \; is \; even \\ 1, \; if \; x \; is \; odd \end{cases}$. Determine $(f \circ (g \circ h))(x) \; and \; ((f \circ g) \circ h)(x) \; and \; verify \; that \; f \circ (g \circ h) = (f \circ g) \circ h$.	[07]	CO3	L3
7	Test the validity of the following argument: $p \to q$, $q \to (r \wedge s)$, $\neg r \vee (\neg t \vee u)$, $p \wedge t$, $\therefore u$	[07]	CO1	L3
8	Determine the truth value of the following quantified statements, the universe being the set of all non – zero integers. (i) $\exists x, \exists y, [xy = 1]$ (ii) $\exists x, \forall y, [xy = 1]$ (iii) $\forall x, \exists y, [xy = 1]$ (iv) $\exists x, \exists y, [(2x + y = 5) \wedge (x - 3y = -8)]$ (v) $\exists x, \exists y, [(3x - y = 17) \wedge (2x + 4y = 3)]$	[07]	CO1	L3
9	Prove by direct method, indirect method and the method of contradiction that "If n is an odd integer then n+9 is an even integer."	[07]	CO4	L3
10	ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm.	[07]	CO1	L2

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IAT- 2 Scheme and Sol^n

Discrete Mathematical Structures - 17CS36

1. $[(1,3)] = \{(x,y) \mid (1,3)\, R\, (x,y)\}$

$= \{(x,y) \mid 1+3 = x+y\}$

$= \{(1,3), (2,2), (3,1)\}$

III^{ly} $[(2,4)] = \{(2,4), (4,2), (1,5), (5,1), (3,3)\}$

$[(1,1)] = \{(1,1)\}$

Now, to find the partition induced by R,

$[(1,1)] = \{(1,1)\}$

$[(1,2)] = \{(1,2), (2,1)\}$

$[(1,3)] = \{(1,3), (3,1), (2,2)\}$

$[(1,4)] = \{(1,4), (4,1), (2,3), (3,2)\}$

$[(1,5)] = \{(1,5), (5,1), (2,4), (4,2), (3,3)\}$

$[(2,5)] = \{(2,5), (5,2), (3,4), (4,3)\}$

$[(3,5)] = \{(3,5), (5,3), (4,4)\}$

$[(4,5)] = \{(4,5), (5,4)\}$

$[(5,5)] = \{(5,5)\}$

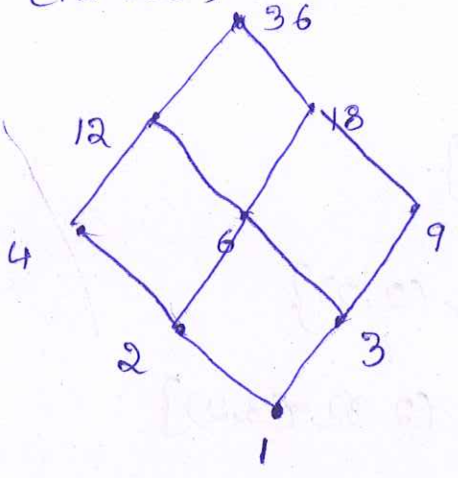
$\therefore$ the partition induced by R is

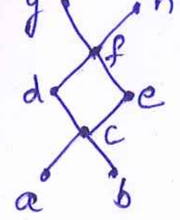
$P = \{[(1,1)], [(1,2)], [(1,3)], [(1,4)], [(1,5)], [(2,5)],$
$[(3,5)], [(4,5)], [(5,5)]\}$

2. Let the set of +ve divisors of 36 be A.

$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,9), (1,12), (1,18), (1,36), (2,2), (2,4), (2,6), (2,12), (2,18), (2,36), (3,3), (3,6), (3,9), (3,12), (3,18), (3,36), (4,4), (4,12), (4,36), (6,6), (6,12), (6,18), (6,36), (9,9), (9,18), (9,36), (12,12), (12,36), (18,18), (18,36), (36,36)\}$



3. 

Maximal Elements : g & h

Minimal Elements : a & b

Greatest Elements : –

Least Elements : –

$B_1 = \{c, d, e\}$

Upper Bound of B_1 = f, g, h

Least UB of B_1 = f

Lower Bound of B_1 = a, b, c

Greatest LB of B_1 = c

5. $f(x) = \begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x \leq 0 \end{cases}$

$f(0) = -3(0)+1 = 1$

$f(5/3) = 3(5/3) - 5 = 0$

$f^{-1}(3) = x$

$\Rightarrow f(x) = 3$

$3x-5 = 3$	$-3x+1 = 3$
$3x = 8$	$-3x = 2$
$x = 8/3$	$x = -2/3$

$\therefore f^{-1}(3) = \{8/3, -2/3\}$

Let $f^{-1}(-6) = x$

$f(x) = -6$

$3x-5 = -6$	$-3x+1 = -6$
$3x = -1$	$-3x = -7$
$x = -1/3$	$x = 7/3$
#	#

$f^{-1}(-6) = \{\ \}$

Let $f^{-1}(0) = x$

$f(x) = 0$

$3x-5 = 0$	$-3x+1 = 0$
$x = 5/3$ ✓	$-3x = -1$
	$x = 1/3$
	#

$\therefore f^{-1}(0) = 5/3$

$$f^{-1}([-5,5]) = \{x \mid f(x) \in [-5,5]\}$$

$$= \{x \mid -5 \le f(x) \le 5\}$$

$-5 \le (3x-5) \le 5$	$-5 \le (-3x+1) \le 5$
$0 \le 3x \le 10$	$-6 \le -3x \le 4$
$0 \le x \le 10/3$	$2 \ge x \ge -4/3$
	$-4/3 \le x \le 2$

$$\therefore f^{-1}([-5,5]) = [-4/3, 10/3]$$

Only for CS D

7. Let $A = B = R$.

Consider $(gof)(x) = g[f(x)]$

$$= g[2x^3 - 1]$$

$$= \left\{\frac{1}{2}(2x^3-1+1)\right\}^{1/3}$$

$$= \left\{\frac{1}{2}(2x^3)\right\}^{1/3}$$

$$= (x^3)^{1/3} = x$$

$\Rightarrow gof = I_A$

Consider $(fog)(y) = f[g(y)] = f\left\{\left[\frac{1}{2}(y+1)\right]^{1/3}\right\}$

$$= 2\left\{\left[\tfrac{1}{2}(y+1)\right]^{1/3}\right\}^3 - 1$$

$$= (y+1) - 1 = y$$

$\therefore fog = I_B$ $\quad\therefore$ f and g are inverses of each other

Only for CS D

8. Let $A = \{1, 3, 5\}$, $B = \{2, 3\}$, $C = \{4, 6\}$

(i) $(A \cup B) \times C$

$A \cup B = \{1, 2, 3, 5\}$

$(A \cup B) \times C = \{(1,4), (1,6), (2,4), (2,6), (3,4), (3,6)$
$(5,4), (5,6)\}$

(ii) $(A \times B) \cap (B \times C)$

$A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

$B \times C = \{(2,4), (2,6), (3,4), (3,6)\}$

$(A \times B) \cap (B \times C) = \phi$

Only for CS D

9. $S = \{1, 2, 3\}$

Subsets of S are

$S_0 = \phi$, $S_1 = \{1\}$, $S_2 = \{2\}$, $S_3 = \{3\}$,

$S_4 = \{1, 2\}$, $S_5 = \{1, 3\}$, $S_6 = \{2, 3\}$, $S_7 = S$

Here, $S_0 \subseteq S_1, S_2, \dots S_7$

$S_1 \subseteq S_1, S_4, S_5, S_7$

$S_2 \subseteq S_2, S_4, S_6, S_7$

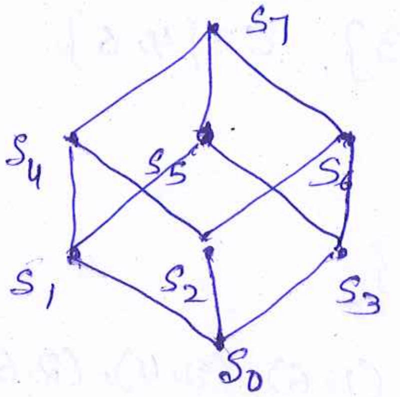
$S_3 \subseteq S_3, S_5, S_6, S_7$

$S_4 \subseteq S_4, S_7$

$S_5 \subseteq S_5, S_7$

$S_6 \subseteq S_6, S_7$

$S_7 \subseteq S_7$



Only for IS A and B

7.

$$\begin{array}{l} p \to q \\ q \to (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ p \wedge t \\ \hline \therefore u \end{array} \Rightarrow \begin{array}{l} p \to (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ p \wedge t \\ \hline \therefore u \end{array}$$

rule of syllogism

$$\Rightarrow \begin{array}{l} p \to (r \wedge s) \\ r \to (t \to u) \\ p \\ t \\ \hline \therefore u \end{array}$$

rule of conjunctive simplification.

$$\Rightarrow \begin{array}{l} (r \wedge s) \\ r \to (t \to u) \\ t \\ \hline \therefore u \end{array}$$

Modus pones

$$\Rightarrow \begin{array}{l} t \\ r \\ r \to (t \to u) \\ \hline \therefore u \end{array}$$

Conjunctive simplification

$$\Rightarrow \begin{array}{l} t \\ t \to u \\ \hline \therefore u \end{array}$$

Modus pones

This is valid in view of Modus pones.

Coeff. of x^{12} which corresponds to $r=1$ is — $\left(\frac{1}{2}\right)$

$= \binom{10}{1}(-2)^9 =$ — (1)

(4)

~~8.~~

$A = \{1, 2, \ldots\ldots 12\}$

$(x,y) \in R$ iff $x-y$ is a multiple of 5

i.e. $x-y = 5m$ (say)

Reflexive: Let $x \in A$

then $x-x = 0 = 5\times 0$

$\therefore$ xRx or $(x,x) \in R$

$\therefore$ R is reflexive — (1)

Symmetric: Let $x, y \in A$

If $(x,y) \in R$ i.e. $x-y = 5m$

or $y-x = -5m = 5(-m)$

so $(y,x) \in R$

$\therefore$ R is symmetric. — (1)

Transitive: Let $x, y, z \in A$

If $(x,y) \in R$ & $(y,z) \in R$

i.e. $x-y = 5m$, $y-z = 5n$ — (1)

$\Rightarrow (x-y)+(y-z) = 5(m+n)$ or $x-z = 5(m+n)$

$\therefore$ $(x,z) \in R$

$\therefore$ R is transitive. $\therefore$ R is an equivalence relation.

— (1)

Equivalence classes:

$[1] = \{1, 6, 11\} = [6] = [11]$

$[2] = \{2, 7, 12\} = [7] = [12]$

$[3] = \{3, 8\} = [8]$

$[4] = \{4, 9\} = [9]$

$[5] = \{5, 10\} = [10]$ — (2)

$\therefore$ Partition of A induced by R is

$$P = \{[1], [2], [3], [4], [5]\} \quad \text{——} (1)$$

(6)

$$f(x) = x-1, \quad g(x) = 3x, \quad h(x) = \begin{cases} 0, & x \text{ is even} \\ 1, & x \text{ is odd} \end{cases}$$

$$\begin{aligned}(f\circ(g\circ h))(x) &= f[g\circ h(x)] \\ &= f[g(h(x))] \\ &= f(3h(x)) = 3h(x) - 1 = 3\begin{cases} 0, & \text{even} \\ 1, & \text{odd} \end{cases} - 1\end{aligned}$$

$$= \begin{cases} -1, & x \text{ is even} \\ 2, & x \text{ is odd} \end{cases} \quad \text{——} (3.5)$$

$$\begin{aligned}((f\circ g)\circ h)(x) &= f\circ g[h(x)] \\ &= f[g(h(x))] = f(3h(x)) = 3h(x) - 1\end{aligned}$$

$$= \begin{cases} -1, & x \text{ is even} \\ 2, & x \text{ is odd} \end{cases} \quad \text{——} (3.5)$$

$$\therefore f\circ(g\circ h) = (f\circ g)\circ h$$

Only for IS A and B

(10) Consider the triangle DEF formed by the mid pts of the sides AB, AC & BC of ΔABC. ΔABC is partitioned into four small equilateral triangles each of which has sides equal to $\frac{1}{2}$ cm. Treating each of these four

—— (2)

portions as a pigeonhole and five pts chosen inside the Δ as pigeons. By pigeonhole principle at least one portion must contain two or more points. Clearly, distance b/w such points is less than $\frac{1}{2}$ cm. —— (5)

Symbolic form : $(\forall x \in \mathbb{Z}, p(x)) \wedge (\exists x \in \mathbb{Q}, \neg q(x))$ — (1)

Negation : $\neg((\forall x \in \mathbb{Z}, p(x)) \wedge (\exists x \in \mathbb{Q}, \neg q(x)))$ — (1)

$\equiv (\exists x \in \mathbb{Z}, \neg p(x)) \vee (\forall x \in \mathbb{Q}, q(x))$ (De morgan's & double negation) — (2)

∴ Some integers are not rational no. or all rational no. are integers. — (1)

3. Given $\mathbb{Z}^{+} - \{0\}$ as universe

(i) $\exists x, \exists y, [xy = 1]$ (T) (for $x = 1, y = 1$) — (1)

(ii) $\exists x, \forall y, [xy = 1]$ (F) (for fixed x, $xy = 1$ is not true $\forall y$) — (1.5)

(iii) $\forall x, \exists y\ [xy = 1]$ (F) (for $x = 2$, there is no y) — (1.5)

(iv) $\exists x, \exists y, [(2x + y = 5) \wedge (x - 3y = -8)]$ — (1.5)
(T) (for $x = 1, y = 3$)

(v) $\exists x, \exists y\ [(3x - y = 17) \wedge (2x + 4y = 3)]$ — (1.5)
(F) (As equations don't have a common integer solution)

4. (i) $$\left(3x^2 - \frac{2}{x}\right)^{15} = \sum_{r=0}^{15} \binom{15}{r} (3x^2)^r \left(-\frac{2}{x}\right)^{15-r}$$ — (1)

$$= \sum_{0}^{15} \binom{15}{r} 3^r (-2)^{15-r} x^{3r-15}$$ — (1)

Coeff. of x^0 which corresponds to $r = 5$ is — (1/2)

$$\binom{15}{5} (3)^5 (-2)^{10}$$ — (1)

(ii) $$x^3(1-2x)^{10} = x^3 \sum_{r=0}^{10} \binom{10}{r} (1)^r (-2x)^{10-r}$$ — (1)

$$= \sum_{0}^{10} \binom{10}{r} (-2)^{10-r} (x)^{13-r}$$ — (1)

IAT-2, Solution - DMS (2018)

(1.) p: n is an odd integer

q: $n+9$ is an even integer —— (1)

Direct Method: Let p be true

$\therefore n = 2K+1$

then $n+9 = 2K+1+9 = 2(K+5) =$ even int.

$\therefore q$ is true.

Hence $p \to q$ is true —— (2)

Indirect: $\neg p$: n is an even int.

$\neg q$: $n+9$ is an odd int.

Let $\neg q$ be true

$\therefore n+9$ is an odd int or $n+9 = 2K+1$

$n = 2K+1-9 = 2(K-4) =$ even

$\therefore \neg p$ is true $\Rightarrow \neg q \to \neg p$ is true —— (2)

$\therefore p \to q$ is true

Method of contradiction: Let $p \to q$ be false

so p is true & q is false

Let q is false then $\neg q$ is true

$n+9 = 2K+1 \Rightarrow n = 2(K-4) =$ even

but n is odd as p is true

which is a contradiction

$\therefore p \to q$ is true —— (3)

(2.) $\mathbb{Z}$: Set of integers, $\mathbb{Q}$: Set of rational no.

$p(x)$: x is a rational no.

$q(x)$: x is an integer —— (2)