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CMRIT — CMR INSTITUTE OF TECHNOLOGY, BENGALURU. ACCREDITED WITH A+ GRADE BY NAAC — CELEBRATING 25 YEARS

Internal Assessment Test 2 – Oct. 2018

Sub:	Discrete Mathematical Structures				Sub Code:	17CS36	Branch:	CS
Date:	17/10/2018	Duration:	90 mins.	Max Marks: 50	Sem / Sec:	III - A, B & C		OBE

Question 1 is compulsory and answer any six from Q.2 to Q.10

		MARKS	CO	RBT
1	Prove by direct method, indirect method and the method of contradiction that "If n is an odd integer then n+9 is an even integer."	[08]	CO4	L3
2	Find the negation of "All integers are rational numbers and some rational numbers are not integers."	[07]	CO1	L2
3	Determine the truth value of each of the following quantified statements, the universe being the set of all non – zero integers. (i) $\exists x, \exists y, [xy = 1]$ (ii) $\exists x, \forall y, [xy = 1]$ (iii) $\forall x, \exists y, [xy = 1]$ (iv) $\exists x, \exists y, [(2x + y = 5) \wedge (x - 3y = -8)]$ (v) $\exists x, \exists y, [(3x - y = 17) \wedge (2x + 4y = 3)]$	[07]	CO1	L3
4	Find the coefficients of (i) x^0 in the expansion of $(3x^2 - (2/x))^{15}$ (ii) x^{12} in the expansion of $x^3(1-2x)^{10}$.	[07]	CO3	L3
5	Let A= {1, 2, …… 12 } let R be a relation on A defined by (x,y)ϵR if and only if x-y is a multiple of 5. Verify that R is an Equivalence relation. Find the partition of A induced by R.	[07]	CO3	L3
6.	Draw the Hasse diagram for the relation defined by aRb if and only if a|b on the set containing positive divisor of 36.		CO3	L2

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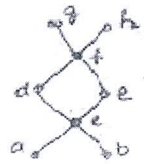
7	Let $f: A \rightarrow B$ be defined by $f(x)=\begin{cases} 3x-5 \text{ for } x>0 \\ -3x+1 \text{ for } x \leq 0 \end{cases}$ (i) Find $f(0), f(5/3), f^{-1}(3), f^{-1}(-6), f^{-1}(0)$. (ii) Determine $f^{-1}([-5,5])$.	[07]	CO3	L3
8	Consider the Hasse diagram of a POSET (A, R) given below:  Find maximal, minimal, greatest and least element If B_1={c, d, e}, find (if they exist) all upper bounds , lower bounds, least upper bound, greatest lower bound of B_1.	[07]	CO3	L3
9	Let f, g, h be functions from Z to Z defined by $f(x)=x-1, g(x)=3x, h(x)=\begin{cases} 0, \text{ if } x \text{ is even} \\ 1, \text{ if } x \text{ is odd} \end{cases}$. Determine $(f \circ (g \circ h))(x)$ and $((f \circ g) \circ h)(x)$ and verify that $f \circ (g \circ h) = (f \circ g) \circ h$..	[07]	CO3	L3
10	ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm.	[07]	CO3	L2

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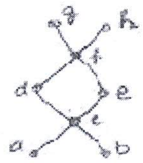
8	Consider the Hasse diagram of a POSET (A, R) given below:  Find maximal, minimal, greatest and least element If B_1={c, d, e},find (if they exist) all upper bounds , lower bounds, least upper bound, greatest lower bound of B_1.	[07]	CO3	L3
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IAT-2, Solution - DMS (2018)

1. p: n is an odd integer

   q: $n+9$ is an even integer —— (1)

Direct Method: Let p be true

$\therefore \; n = 2K+1$

then $n+9 = 2K+1+9 = 2(K+5) =$ even int.

$\therefore$ q is true.

Hence $p \to q$ is true. —— (2)

Indirect: $\neg p$: n is an even int.

$\neg q$: $n+9$ is an odd int.

Let $\neg q$ be true

$\therefore$ $n+9$ is an odd int or $n+9 = 2K+1$

$n = 2K+1-9 = 2(K-4) =$ even

$\therefore$ $\neg p$ is true $\Rightarrow$ $\neg q \to \neg p$ is true —— (2)

$\therefore$ $p \to q$ is true

Method of contradiction: Let $p \to q$ be false

so p is true & q is false

Let q is false then $\neg q$ is true

$n+9 = 2K+1 \Rightarrow n = 2(K-4) =$ even

but n is odd as p is true

which is a contradiction

$\therefore$ $p \to q$ is true —— (3)

2. $\mathbb{Z}$: Set of integers, $\mathbb{Q}$: Set of rational no.

$p(x)$: x is a rational no.

$q(x)$: x is an integer —— (2)

Symbolic form: $(\forall x \in \mathbb{Z}, p(x)) \wedge (\exists x \in \mathbb{Q}, \neg q(x))$ — (1)

Negation: $\neg((\forall x \in \mathbb{Z}, p(x)) \wedge (\exists x \in \mathbb{Q}, \neg q(x)))$ — (1)

$\equiv (\exists x \in \mathbb{Z}, \neg p(x)) \vee (\forall x \in \mathbb{Q}, q(x))$ (De morgan's & double negation) — (2)

$\therefore$ Some integers are not rational no. or all rational no. are integers. — (1)

(3.) Given $\mathbb{Z}^{+} - \{0\}$ as universe

(i) $\exists x, \exists y. [xy = 1]$ (T) (for $x=1, y=1$) — (1)

(ii) $\exists x, \forall y. [xy = 1]$ (F) (for fixed x, $xy=1$ is not true $\forall y$) — (1.5)

(iii) $\forall x, \exists y\ [xy = 1]$ (F) (for $x=2$, there is no y) — (1.5)

(iv) $\exists x, \exists y. [(2x+y=5) \wedge (x-3y=-8)]$ — (1.5)
(T) (for $x=1$, $y=3$)

(v) $\exists x, \exists y\ [(3x-y=17) \wedge (2x+4y=3)]$ — (1.5)
(F) (As equations don't have a common integer solution)

(4.) (i) $$\left(3x^2 - \frac{2}{x}\right)^{15} = \sum_{r=0}^{15} \binom{15}{r} (3x^2)^r \left(-\frac{2}{x}\right)^{15-r}$$ — (1)

$$= \sum_{0}^{15} \binom{15}{r} 3^r (-2)^{15-r} x^{3r-15}$$ — (1)

Coeff. of x^0 which corresponds to $r = 5$ is — (½)

$$\binom{15}{5} (3)^5 (-2)^{10}$$ — (1)

(ii) $$x^3(1-2x)^{10} = x^3 \sum_{r=0}^{10} \binom{10}{r} (1)^r (-2x)^{10-r}$$ — (1)

$$= \sum_{0}^{10} \binom{10}{r} (-2)^{10-r} (x)^{13-r}$$ — (1)

Coeff. of x^{12} which corresponds to $r=1$ is — $\left(\frac{1}{2}\right)$

$= \binom{10}{1}(-2)^9$ # — (1)

5. $A = \{1, 2, \ldots, 12\}$

$(x,y) \in R$ iff $x-y$ is a multiple of 5

i.e. $x-y = 5m$ (say)

Reflexive: Let $x \in A$

then $x-x = 0 = 5 \times 0$

$\therefore xRx$ or $(x,x) \in R$

$\therefore R$ is reflexive — (1)

Symmetric: Let $x, y \in A$

If $(x,y) \in R$ i.e. $x-y = 5m$

or $y-x = -5m = 5(-m)$

so $(y,x) \in R$

$\therefore R$ is symmetric. — (1)

Transitive: Let $x, y, z \in A$

If $(x,y) \in R$ & $(y,z) \in R$

i.e. $x-y = 5m$, $y-z = 5n$ — (1)

$\Rightarrow (x-y)+(y-z) = 5(m+n)$ or $x-z = 5(m+n)$

$\therefore (x,z) \in R$

$\therefore R$ is transitive. $\therefore R$ is an equivalence relation. — (1)

Equivalence classes:

$[1] = \{1, 6, 11\} = [6] = [11]$

$[2] = \{2, 7, 12\} = [7] = [12]$

$[3] = \{3, 8\} = [8]$

$[4] = \{4, 9\} = [9]$

$[5] = \{5, 10\} = [10]$ — (2)

$\therefore$ Partition of A induced by R is

$P = \{[1], [2], [3], [4], [5]\}$ —— (1)

(9) $f(x) = x-1$, $g(x) = 3x$, $h(x) = \begin{cases} 0, & x \text{ is even} \\ 1, & x \text{ is odd} \end{cases}$

$$(fo(goh))(x) = f[goh(x)]$$
$$= f[g(h(x))]$$
$$= f(3h(x)) = 3h(x) - 1 = 3\begin{cases} 0, & \text{even} \\ 1, & \text{odd} \end{cases} - 1$$

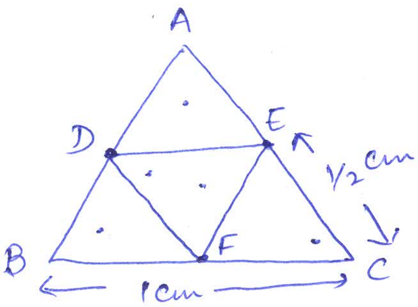
$$= \begin{cases} -1, & x \text{ is even} \\ 2, & x \text{ is odd} \end{cases}$$ —— (3.5)

$$((fog)oh)(x) = fog[h(x)]$$
$$= f[g(h(x))] = f(3h(x)) = 3h(x) - 1$$

$$= \begin{cases} -1, & x \text{ is even} \\ 2, & x \text{ is odd} \end{cases}$$ —— (3.5)

$\therefore fo(goh) = (fog)oh$

(10) Consider the triangle DEF formed by the mid pts of the sides AB, AC & BC of ΔABC. ΔABC is partitioned into four small equilateral triangles each of which has sides equal to $\frac{1}{2}$ cm. Treating each of these four portions as a pigeonhole and five pts chosen inside the Δ as pigeons. By pigeonhole principle at least one portion must contain two or more points. Clearly, distance b/w such points is less than $\frac{1}{2}$ cm. —— (5)

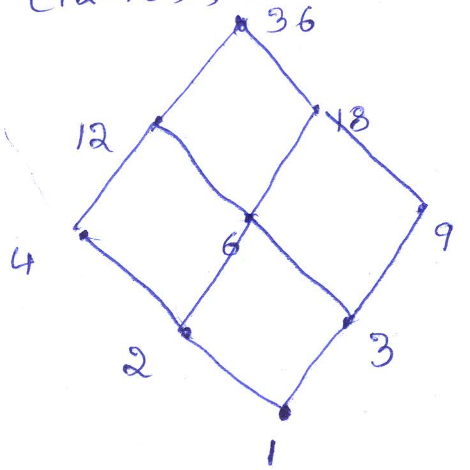


—— (2)

~~2~~. 6 Let the set of +ve divisors of 36 be A.

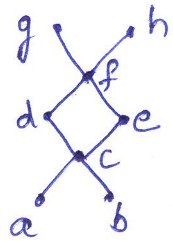
$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ —— (1)

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,9), (1,12), (1,18),$
$(1,36), (2,2), (2,4), (2,6), (2,12), (2,18), (2,36), (3,3),$
$(3,6), (3,9), (3,12), (3,18), (3,36), (4,4), (4,12),$
$(4,36), (6,6), (6,12), (6,18), (6,36), (9,9), (9,18),$
$(9,36), (12,12), (12,36), (18,18), (18,36), (36,36)\}$ —— (2)



—— (3)

8 ~~3~~.



Maximal Elements : g & h —— (1)

Minimal Elements : a & b —— (1)

Greatest Elements : — —— (1)

Least Elements : — —— (1)

$B_1 = \{c, d, e\}$

Upper Bound of B_1 = f, g, h —— (1)

Least U B of B_1 = f —— (1½)

Lower Bound of B_1 = a, b, c —— (1)

Greatest L B of B_1 = c —— (1½)

7

~~5~~. $f(x) = \begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x \le 0 \end{cases}$

$f(0) = -3(0)+1 = 1$ —— 1

$f(5/3) = 3(5/3) - 5 = 0$ —— 1

$f^{-1}(3) = x$

$\Rightarrow f(x) = 3$

$3x-5=3$	$-3x+1=3$
$3x=8$	$-3x=2$
$x=8/3$	$x=-2/3$

$\therefore f^{-1}(3) = \{8/3, -2/3\}$ —— 1

Let $f^{-1}(-6) = x$

$f(x) = -6$

$3x-5=-6$	$-3x+1=-6$
$3x=-1$	$-3x=-7$
$x=-1/3$	$x=7/3$
#	#

$f^{-1}(-6) = \{\}$ —— 1

Let $f^{-1}(0) = x$

$f(x) = 0$

$3x-5=0$	$-3x+1=0$
$x=5/3$ ✓	$-3x=-1$
	$x=1/3$
	#

$\therefore f^{-1}(0) = 5/3$ —— 1

$$f^{-1}([-5,5]) = \{x \,/\, f(x) \in [-5,5]\}$$

$$= \{x \mid -5 \le f(x) \le 5\}$$

$-5 \le (3x-5) \le 5$	$-5 \le (-3x+1) \le 5$
$0 \le 3x \le 10$	$-6 \le -3x \le 4$
$0 \le x \le 10/3$	$2 \ge x \ge -4/3$
	$-4/3 \le x \le 2$

$$\therefore f^{-1}([-5,5]) = [-4/3, 10/3] \longleftarrow 2$$